Hedge fund excess returns under time-varying betas

Ron Bird*, Harry Liem and Susan Thorp

University of Technology, Sydney
Sydney, Australia

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Abstract

In this paper we construct a time-varying factor model of hedge fund returns. Prior to analysis we investigate database biases arising from voluntary self-reporting and suggest how to minimise these biases. Using a constant beta model, we find that between 1994 and 2009 the average hedge fund manager realises an excess return of 3.7 percent per annum and the average manager outperforms in nine of the hedge fund strategy types that we examine. However, we find that almost all this added value comes from security selection skills with only a small subsample of our universe displaying market timing skills. These conclusions are robust to the inclusion of time-varying beta, volatility clustering and leverage effects. Finally, we suggest how the reward to risk for hedge fund investing can be improved through the adoption of a least risk portfolio approach and the inclusion of managed futures.

Keywords: hedge funds; time-varying beta; GARCH.

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* Corresponding author.
E-mail: Ron.Bird@uts.edu.au
Phone: +61 2 9514 7716
Fax: +61 2 9514 7711
Room: CM05D.03.22B
PO Box 123, Broadway NSW 2007, Australia

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1. INTRODUCTION

Investors are attracted to the prospect of finding managers of highly successful hedge funds with long term track records, such as Bridgewater, Paulson & Co or Brevan Howard. The Yale Endowment, for example, targets 21 percent of its assets towards hedge funds. Yet all of this is happening at a time when academic researchers are questioning the ability of these investments to deliver excess returns, once returns are adjusted for systematic exposure to style factors that are not reflective of manager skill (Asness et al., 2001; Malkiel and Saha, 2005). Thus, hedge funds’ apparent excess returns may be due to exposure to a wider array of systematic risk factors (so-called “alternative beta”).

The apparent fragility of excess returns to hedge funds mirrors earlier results for traditional assets. A long literature demonstrates no positive excess returns to mutual funds and connects higher management fees with poorer, not better, performance, see Carhart (1997), Elton et al. (1996), Gruber (1996) and Malkiel (1995). French (2008) and Wermers (2000) conclude that the mutual fund literature has reached a quasi consensus where aggregate abnormal performance is negative after fees and positive before fees. This matches the rational model of financial intermediation of Berk and Green (2004) that predicts that financial intermediaries provide zero excess returns to their investors while capturing a rent that is commensurate with their abilities. In other words, there is 'no free lunch' for investors, and managers do not generate excess returns above their fees.

Here we test the Berk and Green proposition by evaluating whether hedge fund investors realise sustainable excess returns. Our approach builds on the pioneering
work on style analysis by Fung and Hsieh (1997, 2002, 2004), Sharpe (1992) and Treynor and Mazuy (1996) and on the conditional style models of Ferson and Schadt (1996). In so doing we make the following three contributions to the literature. First, we combine several techniques to account for the illiquidity and tail risk existent in hedge fund strategies, starting from the Fama-French model. This improves our understanding of the fundamental drivers of hedge fund excess returns. Second, we test the robustness of the modelled outcome by allowing for time varying conditional beta and we apply an asymmetric GARCH model to better understand observed volatility clustering in excess returns. Traditionally hedge fund researchers have applied constant beta and variance models: for example Capocci and Hubner (2004), Chen et al. (2010) and Malkiel and Saha (2005). Third, we separately examine evidence of market timing skill as a source of excess returns. Our results indicate that hedge funds do create excess returns, but also contain an element of repackaged ‘style tilts’ (or systematic factor exposures). These results are consistent with findings by Chen et al. (2010) who suggest hedge funds represent a mix of alpha ('skill') and beta ('market exposure'). Further, our results give insight into the nature of hedge fund excess returns, and suggest how both hedge fund managers and institutional investors can adapt portfolios going forward. Finally, we propose a method to improve the reward to risk ratio for hedge fund investing through the adoption of a least risk portfolio approach and the inclusion of managed futures.

The paper is organised as follows. In section 2 we review background literature on style analysis, prior to discussing the model in section 3 and data limitations particular to the hedge fund industry in section 4. Section 5 presents our empirical results. Section 6 provides concluding remarks and suggestions for further research.
2. BACKGROUND LITERATURE

The linear factor modelling approach using empirically observed variables remains the most widely used and successful method for empirical asset pricing studies. Fung and Hsieh (2004) use Sharpe’s (1992) framework for mutual funds to introduce a seven factor model to explain hedge fund returns using market related factors.¹ Their model aims to account for the vast number of different hedge fund strategies by using an array of factors. Sharpe’s style model, applied to hedge funds, is presented below.

\[ R_{it} - R_{f} = \alpha_i + \sum_{k=1}^{n} \beta_{ik} F_{kt} + \varepsilon_{it} \]  

Where \( R_{it} \) is the rate of return to hedge fund \( i \) at time \( t \), \( R_{f} \) is the risk free rate at time \( t \), \( \alpha_i \) is the excess return (alpha) to hedge fund \( i \) uncorrelated with other factor (beta) sources, \( \beta_{ik} \) is the coefficient of returns to hedge fund \( i \) to factor \( k \), \( F_{kt} \) is the value of risk factor \( k \) at time \( t \) and the error is independently and identically distributed, \( \varepsilon_{it} \sim i.i.d (0,\sigma^2_\varepsilon) \).

Hedge funds actively engage in market timing: one of the perceived benefits of hedge fund investing is the fact that hedge funds are able to profit regardless of the direction of the equity markets. The ability to generate hedged or uncorrelated returns is referred to by industry practitioners as ‘market neutral’ or ‘non-directional’. This was the concept of the original hedge fund set up by Alfred Winslow Jones in 1949. Since then, the number of strategies has substantially expanded. Many strategies (e.g. ‘long

¹ These are the return on the S&P 500 minus the risk free rate, three trend following factors on currencies, bonds and commodities, the large-cap versus small-cap spread, the return on 10 year US Treasuries, and credit spreads (Moody’s Baa rated bond yields minus the 10-year Treasury yield). More recently, the return on the International Finance Corporation (IFC) emerging market index was added as an eighth factor. For data refer [http://faculty.fuqua.duke.edu/~dah7/HFRFData.htm](http://faculty.fuqua.duke.edu/~dah7/HFRFData.htm). [Accessed: 23 September 2011]
short’ and ‘emerging markets’) are now long biased rather than market neutral. In a sense, strategies have evolved while perceptions have lagged. To test market timing ability, Treynor and Mazuy (1966) include a quadratic term in the market excess return. For a portfolio manager with forecasting ability, the return on the managed portfolio will not be linearly related to the excess market return. This arises because the manager will gain more than the market return when the market return is forecast to rise and he will lose less than the market when it is forecast to fall. Thus, his portfolio returns will be a convex function of market returns. Equation (2) sets out the original Treynor and Mazuy model.

\[ R_{it} - R_{ft} = \alpha_i + \beta_{i,\text{market}} (R_{mt} - R_{ft}) + \beta_{i,\text{timing}} (R_{mt} - R_{ft})^2 + \epsilon_{it} \] (2)

The variables are as defined for equation (1) (excluding all but the market risk factor) but in addition, \( \beta_{i,\text{timing}} \) measures the market timing ability of hedge fund \( i \) by using the quadratic term \((R_{mt} - R_{ft})^2\) to measure convexity. The Henriksson Merton (1981) and Treynor Mazuy (1966) timing models are most often used in literature. While they inform us about a manager’s market timing ability, caution is advised regarding the interpretation of the remaining intercept in terms of security selection (refer Ferson, 2009).

For traditional asset classes, a large body of evidence indicates that market timing ability for mutual funds is rare (Friesen and Sapp, 2007; Henrikson, 1984). Cerrahoglu et al. (2003) and Fung et al. (2002) suggest evidence of negative market timing ability for hedge funds, while Chen and Liang (2007) find evidence of positive market timing ability for only a very small subsample of the hedge fund universe. Care should be taken when applying the Treynor and Mazuy extension to multi-factor
models. Multicollinearity, especially with the equity risk premium, should be avoided where possible. In our model, we reduce the number of factors compared to the approach taken by Fung and Hsieh (2004). In addition, we also investigate market timing ability by testing hedge fund returns against the equity risk premium as a single factor (figures 1 and 2).

Traditional performance evaluation has used unconditional expected returns as the performance measure. As a consequence, information about the changing state of the economy is ignored when forming expectations. Yet hedge fund managers actively tilt portfolio style to account for market conditions, and if expected returns and risks vary over time, the unconditional approach will be unreliable. Ferson and Schadt (1996) find conditional CAPM better explains abnormal returns than unconditional CAPM, and expand the Sharpe (1992) model by incorporating lagged information variables.

$$R_{it} - R_{f} = \alpha_i + \sum_{k=1}^{n} \beta_{ik} F_{kt} + \sum_{j=1}^{m} \beta_{ijk} Z_{j,t-1} F_{kt} + \epsilon_{it} \tag{3}$$

Where the variables are defined as in equation (1), but in addition $\beta_{ijk}$ represents the conditional beta of hedge fund $i$ to factor $k$ dependent on the $j^{th}$ lagged information variable, $Z_{j,t-1}$ by allowing for an interaction effect $Z_{j,t-1} F_{kt}$. For traditional asset classes, studies find that conditioning is relevant and that the intercept (alpha) is smaller in conditional models than in unconditional models (Ferson and Schadt, 1996).

Cerrahoglu et al. (2003) and Patton and Ramodarai (2009) extend the conditioning variable literature to hedge funds, and also find evidence of significant conditioning, reflecting the dynamic nature of hedge fund positions.
3 THE MODEL

In this section we present a multifactor model as a variant of the Intertemporal CAPM (ICAPM), in which the equity risk premium is supplemented by additional state variables associated with alternative assets. Under ICAPM (Merton, 1973), state variables are defined as undesirable outcomes such as illiquidity or tail events, against which investors hedge, thereby creating additional risk premia.

3.1 Market factors

To test excess returns we apply the Fama and French three factor model (Fama and French, 1993) rather than rely on the more extended Fung and Hsieh (2004) seven factor model. We opt for a more parsimonious model augmented by incorporating liquidity and tail risk and develop a model that explains a majority of the variation in excess returns.

For completeness, we also verify our results under the Fung and Hsieh seven factor model, but do not find the additional (mainly trend following) factors significantly impact our results. Another benefit is that when we start to apply conditional asset pricing models, the number of variables to estimate can become quite large under the Fung and Hsieh model. With m factors and n conditioning variables there are m (1+n) + 1 coefficients to estimate.

The Fama-French model applies because most hedge fund strategies are exposed to the equity risk premium through their holdings in direct equities or equity-like instruments such as subordinated debt, and this is captured through the equity market risk factor. While most hedge fund strategies are long biased, our model can also cater
to dedicated short hedge funds by applying a negative sensitivity to the same equity risk premium.

Furthermore, based on 13-F filings, hedge funds are long smaller-capitalised, high growth stocks and short large-capitalised low growth stocks, as there is more shorting stock available for large companies (Brunnermeier and Nagel, 2004). This style tilting behaviour is then captured by the Fama French SMB (Small Minus Big stocks) and HML (High Minus Low book-to-price stocks) factors. SMB (Small Minus Big) represents the average return on three smaller capitalised stock portfolios (with a Value, Style Neutral and Growth style tilt) minus the average return on three large capitalised portfolios. HML (High Minus Low) is the average return on two value portfolios minus the average return on two growth portfolios. See Fama and French (1993).

3.2 Adjustment for illiquidity

Getmansky et al. (2004) explore several sources of serial correlation in returns to hedge funds and show that the most likely explanation is illiquidity exposure. The authors examine market inefficiencies, time varying expected returns, leverage and fee structures as possible sources of serial correlation before reaching their conclusions. Liang and Park (2007) confirm that the illiquidity in hedge funds causes positive first order serial dependence in the dependent variable. To account for stale

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2 Managers who exercise investment discretion over $100 million or more in Section 13-F securities must report their holdings with the SEC. See [http://www.sec.gov/about/forms/form13f.pdf](http://www.sec.gov/about/forms/form13f.pdf) [Accessed: 23 September 2011]

3 Data can be downloaded from [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html) [Accessed: 23 September 2011]
pricing due to illiquidity effects we add lagged market variables on the right hand side of the estimated equations as a liquidity factor, as per Asness et al. (2001).

3.3 Adjustment for tail events

If investors were evaluating the performance of a manager by return-risk measures like the Sharpe ratio, then a manager selling put options on the index will appear to be a superior performer under normal market conditions. This is because under normal market conditions, a manager who is long equities and sells put options receives an additional option premium compared to a manager who is just long equities. These put options are exercised against the manager only during extreme market conditions so this strategy may inflate return-risk measures during normal market conditions and overstate performance. Performance evaluation must be robust to simple manipulation of this type.

Agarwal and Naik (2000) are among the first to model this idea and find evidence that hedge fund strategies are highly correlated with synthetic option-writing strategies. In particular, they find a strategy of writing uncovered put options to be an important factor in a majority (73 per cent) of cases. Similar evidence has been produced by Mitchell and Pulvino (2000) and Lo (2001). Lo argues that the performance fees embedded in hedge funds encourage risk taking behaviour. Hedge funds create a profile similar to a put option (where most of the time the hedge fund generates superior performance, but occasionally goes into a severe drawdown), thereby increasing the probability of large tail events. We use the PUT index, introduced by the Chicago Board of Exchange (CBOE) in 2007 to measure the risk premium received for exposure to tail events. The PUT index strategy is designed to
mechanically sell a sequence of one-month, at-the-money, S&P 500 Index puts and invest the proceeds in Treasury Bills. The PUT index is highly negatively correlated with left tail events. A large market decline results in losses for sellers of put options.⁴

3.4 Conditioning variables
Research has shown a number of public information variables to be relevant for predicting asset class returns. Examples include interest rates (Fama and French, 1976, 1989), and the CBOE Volatility Index or VIX (Patton and Ramodarai, 2009). As defined in equation (3), we test the following conditioning variables \((Z_{p,t-1})\) at a one-period lag: the VIX, the term spread (the spread between the 10 year US Treasury bond yield and the 3 month US T-bill rate) and to proxy financing conditions we apply the credit spread (the spread between US Aaa and Baa bond yields as defined by Moody’s).⁵ Given the limited data history of hedge funds, and also to reduce the number of regressors, we employ an iterative backward elimination procedure to find the best combinations of conditioning variables and factors (refer table 5 for additional explanation). We restrict the number of factors and conditioning variables by fitting the most general model and removing insignificant regressors stepwise, until the five most significant remain.

3.5 Mean equation
Having identified the independent and conditioning variables, we use three different models, represented by equations (4), (5) and (6), to measure the significance of excess returns created by hedge funds. Equation (4) represents the augmented Fama French model which includes an extension for illiquidity risk and tail events.

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Where \( R_{it} \) is the rate of return of manager \( i \) at time \( t \), \( R_{ft} \) is the risk free rate at time \( t \) and \( \alpha_i \) is the intercept term not explained by the multi-factor model. In terms of factor exposure, \((R_{mt} - R_{ft})\) represents the equity risk premium, \((R_{m,t-1} - R_{f,t-1})\) the illiquidity premium based on the lagged market variable and \((R_{PUT}^T - R_{ft})\), the tail event premium. Coefficients \( \beta_{i,m} \), \( \beta_{i,L} \) and \( \beta_{i,tail} \) measure the sensitivity of excess returns of manager \( i \) to the market, illiquidity and tail event premium. The Fama French factors are represented by \( SMB \), reflecting the differential return between smaller and larger capitalised companies, and \( HML \) reflecting the differential return between high and low book to price companies. Coefficients \( \beta_{i,s} \) and \( \beta_{i,v} \) measure the sensitivity of excess returns of manager \( i \) to the small cap and value premium.

Equation (5) extends equation (4) to include the Treynor and Mazuy market timing component.

\[
R_{it} - R_{ft} = \alpha_i + \beta_{i,m} (R_{mt} - R_{ft}) + \beta_{i,L} (R_{m,t-1} - R_{f,t-1}) + \beta_{i,tail} (R_{PUT}^T - R_{ft}) + \beta_{i,s} * SMB + \beta_{i,v} * HML + \beta_{i,timing} (R_{mt} - R_{ft})^2 + \varepsilon_{it} \tag{5}
\]

Equation (5) allows differentiation between the skill of manager \( i \) in terms of market timing (\( \beta_{i,timing} \)) and security selection (\( \alpha_i \)).
Finally, equation (6) allows for testing whether the factors from equation (5) are conditional on selected information variables $Z_{j,t-1}$.

$$R_{it} - R_{ft} = \alpha_i + \beta_{i,m} (R_{mt} - R_{ft}) + \beta_{i,L} (R_{mt,t-1} - R_{ft,t-1}) + \beta_{i,tail} (R^{PUT}_{t} - R_{ft}) + \beta_{i,s} * SMB + \beta_{i,v} * HML + \sum_{j=1}^{n} \left[ \beta_{ij,m} Z_{j,t-1} * (R_{mt} - R_{ft}) + \beta_{ij,L} Z_{j,t-1} * (R_{mt,t-1} - R_{ft,t-1}) + \beta_{ij,tail} Z_{j,t-1} * (R^{PUT}_{t} - R_{ft}) + \beta_{ij,s} Z_{j,t-1} * SMB + \beta_{ij,v} Z_{j,t-1} * HML \right] + \epsilon_{it} \quad (6)$$

$Z_{j,t-1}$ represents lagged conditioning variable $j$ at $t-1$ influencing the different factors for manager $i$.

We apply three conditioning variables (the VIX, the term spread and the credit spread), $\beta_{ij,m}$, $\beta_{ij,L}$, $\beta_{ij,tail}$, $\beta_{ij,s}$ and $\beta_{ij,v}$ for each lagged information variable measure the impact on hedge fund strategy $i$ of the conditional coefficients for the market risk premium, illiquidity premium, tail event premium, small cap and value premium coefficients.

### 3.7 Variance equation

Elyasiani, Getmansky and Mansur (2008) are among the first to investigate the risk-return behaviour of different styles of hedge funds using a multifactor model of asset pricing within a GARCH framework. They find evidence of ARCH effects for hedge fund styles. As hedge funds use leverage, which is likely to amplify asymmetric volatility effects, we use maximum likelihood estimation to apply GJR-GARCH (Glosten, Jagannathan and Runkle 1993) which caters for asymmetry in the GARCH process. The volatility model is estimated simultaneously with the factor model.
Under the standard GJR approach, given the information set $\Omega$ at $t-1$ then $\epsilon_{it} \mid \Omega_{t-1} \sim N(0, h_{it})$ and the conditional volatility is

$$h_{it} = \gamma_0 + \gamma_1 \epsilon_{i,t-1}^2 + \gamma_2 h_{i,t-1} + \gamma_3 \epsilon_{i,t-1}^2 I_{t-1}$$

(7)

where $I_{t-1} = 0$ if $\epsilon_{i,t-1} \geq 0$, and $I_{t-1} = 1$ if $\epsilon_{i,t-1} < 0$.

In this case, $\gamma_0$ represents a constant intercept impacting the long run unconditional volatility, $\gamma_1$ a weighting to the previous period’s squared shock, $\gamma_2$ a weighting to the previous period’s predicted volatility and $\gamma_3$ a sensitivity to negative returns shocks.

Note that in our model we estimate t-distributed errors rather than normal distributed errors to cater for fat tailedness in hedge fund return distributions.

4. THE DATA

Several authors have noted biases in hedge fund data due to the way they are collected and reported (Chen et al., 2010; Fung and Hsieh, 2000; Malkiel and Saha, 2005).

Return series are provided by managers on a voluntary and net-of-fees basis and not independently verified. This creates a number of biases that are especially pertinent in the case of hedge funds, given their short average life span of around 5 years (Brown et al., 1999). Fung and Hsieh (2000) distinguish between natural and spurious biases.

Natural biases arise from the natural birth, growth and death processes of managed funds (e.g. survivorship bias), while spurious biases such as 'self-selection' bias or 'backfill' bias arise from sampling from an unobservable universe of funds and from the way data vendors collect hedge fund information.

Understanding these biases is important. While biases cannot be completely avoided, it is important to control for these biases prior to further analysis, as they may skew
results. *Backfill bias* refers to the fact that managers enter databases with instant backfilled history (most commonly 12 to 24 months), and only enter if they have a good track record. *Survivorship bias* refers to the fact that some databases or surveys report only ‘live’ managers and no longer include returns of ‘graveyard’ managers, thereby omitting poor performers. Finally, *self selection* bias exists because underperformers do not wish to make their performance known, or because large successful funds with sufficient institutional following have less incentive to report to data vendors when they are closed to investors in terms of capacity. Funds have no incentive to report if they have a poor track record to begin with. Similarly, funds may elect to no longer report their final negative return if they go out of business.

It is essential to note that biases in the database depend on the provider, and that database providers can create indices which control for some of these biases. Well constructed indices do not allow retroactive adjustment in their return numbers and so (unlike the underlying databases) avoid backfill bias. In this study we use the appropriately constructed HFR indices which are calculated using underlying manager data from the Hedge Fund Research (HFR) database. HFR is considered to be one of the leading databases for academic studies, see Agarwal and Naik (2000), Capocci and Hubner (2004) and Patton and Ramodarai (2009). Goltz, Martellini and Vaissie (2007) conclude it possible to construct robust representative indices that capture strategy essentials.

Chen et al. (2010) suggest the return biases in such indices are much smaller than in the historical databases, which are continually amended. The HFR indices (HFRI) represent an equal weighted net of fees monthly time series. As to the constituents of these indices, as at December 2009, the HFR database contains 2,481 single manager funds which are included in the HFR indices, of which 1,930 are classified as active
funds and 551 as ‘graveyard’ funds (funds that have gone out of business between January 1994 and December 2009 but whose track record remains in the database). HFR includes dead funds but not prior to 1994, which is why we exclude any data before that date for our analysis. Capocci and Hubner (2004) use data from HFR and MAR and raise serious concerns about the data from 1984-1993 based on the statistical reliability of the observations during this period. Using these indices controls for backfill bias, and captures the performance of ‘graveyard’ funds (survivorship bias). While we consider the HFR indices appropriately constructed from the overall HFR database, for completeness, we test the backfill and survivorship bias for individual hedge funds in the HFR database. Backfill bias we calculate by first deleting the first 12 months, and then 24 months, of reported returns. We find a backfill bias of 1.9 to 2.9 per cent per annum, in line with Chen et al. (2010) and Fung and Hsieh (2000). For survivorship bias we test the difference between the equally-weighted performance of live funds only (1,930 funds) and live and dead funds (2,481 funds) over their life time. Without the inclusion of the ‘graveyard funds’ a survivorship bias of 0.9 percent per annum would have emerged in the database. While the use of these indices removes some unnecessary biases, the final, and perhaps most important bias, self-selection bias, cannot be measured since we cannot compare hedge funds that choose to incorporate returns in a database with those that do not. This also ties in to another issue. By relying on equal weighted indices we do not represent the industry experience on an asset weighted basis. Cross referencing the managers in the HFR database with the top 20 largest hedge fund managers confirms this is an important limitation of our research. Based on a table from Pensions and Investments (2010) we find that only 6 out of the top 20 largest hedge fund managers
have provided return series in the HFR database. This top 20 accounts for $531 million out of $1.6 trillion in industry funds under management (HFR estimate as of Dec 2009). Since we cannot access the true universe, selection bias means our sample returns can still be biased downwards (as large successful managers are underrepresented), or upwards (if a large group of unprofitable hedge fund managers fails to report).

5. EMPIRICAL RESULTS

5.1 Descriptive statistics

Table 1 presents descriptive statistics for the HFR hedge fund index returns series. The data for all tables is collected on a monthly basis, and net of fees, over the period from 1994 to 2009. A description of the basic mechanics of each individual strategy is provided in Appendix A. For a more extensive primer on the various strategies, refer to Fung and Hsieh (1999).

Based on table 1, average excess returns over the period are positive for all categories of hedge funds. The only exception is dedicated short. This is a useful first indication that in general, hedge funds are not market neutral in terms of ‘excess returns’. Funds with a long bias towards the equity risk premium have produced higher reported excess returns than those with a short bias. The augmented Fama French model we propose builds on this fundamental concept by adding additional risk premia. Furthermore, the estimated beta of the overall industry to the S&P 500 is only 0.4,

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with the highest beta found in the 'long-short' and 'emerging market' styles. 'Dedicated short' funds produce returns that are negatively impacted by equity market exposure. In other words, they suffer when markets rise. On the other hand, strategies such as ‘event driven’ and ‘managed futures’ produce impressive returns while having a low equity beta of 0.2 to 0.3. However, there seems to be a cost to the excess returns produced, with most strategies exhibiting a negative skew and positive kurtosis. This suggests funds engage in strategies equivalent to writing put options (Agarwal and Naik 2000; and Lo 2001).

5.2 Performance of GARCH models

Tables 2, 3, 4 and 5 show the estimated excess returns under the GJR GARCH model. Table 2 shows that under the augmented Fama French model, the industry has, on average, produced excess returns of 0.31 per cent per month or 3.7 per cent per annum, with an adjusted $R^2$ of 0.78.\(^7\) Table 2 further indicates that sensitivity to equity risk ($\beta_m$) and liquidity risk ($\beta_L$) are important drivers of returns. Persistent shocks occur, as ARCH ($\gamma_1$) and GARCH ($\gamma_2$) effects are large and significant for many strategies. This is consistent with findings from Elyasiani et al. (2008) who apply a GARCH (1,1) model. The positive sensitivity to $\gamma_3$ is significant for 4 out of 10 hedge fund strategies as well as for the aggregate hedge fund industry. This suggests that there is higher volatility during negative return shocks due to the leverage. The positive $\gamma_3$ for ‘macro’ suggests that for macro based strategies, the opposite occurs: volatility declines. Thus, macro managers are perhaps able to reduce leverage in time to avoid negative returns shocks.

\(^7\) For reference we verify that, under the GJR GARCH approach, the 7 factor Fung and Hsieh (2004) model generates a comparable annual excess return of 3.6 percent per annum with an $R^2$ of 0.72.
Based on sensitivity to tail events ($\beta_{\text{tail}}$), both ‘macro’ and ‘managed futures’ funds hedge against tail events. On the other hand, ‘dedicated short’ funds underperform during periods of high systemic risk. Arguably, the introduction of (infrequent) shorting bans by regulators negatively impacts performance of short sellers during the times of crises covered by our sample period.

<< INSERT TABLE 2 >>

As reported in Table 2, the augmented Fama French model provides an explanation for close to 80 per cent of the variation in excess returns produced. We find over the period, the Fama and French SMB and HML factors were positive (smaller capitalized and value stocks outperformed) but as hedge funds overweight growth rather than value stocks, this offset the additional returns from their bias towards smaller capitalized stocks. This is consistent with findings in a study by Brunnermeier and Nagel (2004) that examined hedge fund portfolio stock holdings over the period from 1998 to 2000 and found hedge funds held long and leveraged positions in technology stocks.

Table 3 examines the evidence of the Treynor and Mazuy market timing model by separating excess returns into skills from security selection ($\alpha$) and from market timing ($\beta_{\text{timing}}$). Under Treynor and Mazuy, a positive and significant $\beta_{\text{timing}}$ coefficient implies a positive convex relationship between market returns and fund returns. A hedge fund should outperform in both rising and falling markets due to its market timing ability (the ability to go long and sell short).
As can be seen from Table 3, nine out of ten sub-strategies exhibit negative market timing coefficient, of which three are statistically significant. The worst market timing skill is exhibited by 'dedicated short' and 'distressed debt' managers. The notable exception is 'managed futures', where managers engage in automated trend following, and whose strategies are similar to simultaneously buying a call and a put option (also known as a ‘strangle’) on various indices (equities, bonds, currencies and commodities).

<< INSERT TABLE 3 >>

Figures 1 and 2 show a plot of the strategy returns on the y-axis to the equity market returns on the x-axis. A curve is then fitted against the squared market returns. The more positively convex the fitted curve, the better the market timing ability of the managers comprising that particular strategy.

<< INSERT FIGURE 1 >>

As per table 3, figure 1 indicates the hedge fund aggregate industry has negative market timing ability.

<< INSERT FIGURE 2 >>

As can be seen from Figures 1 and 2, the poor market timing ability of most hedge funds leads to negative convexity. Although we find a reasonable fit for most strategies, it could be argued that some of the negative convexity is created by
outliers. On the other hand, market timing ability is important precisely during these outlier events when hedge funds are supposed to protect investors’ capital. To demand negative convexity is in any case not necessary: even a linear relationship suffices as evidence of a lack of market timing abilities. Furthermore, the augmented Treynor Mazuy model confirms a negative and statistically significant market timing beta for most strategies. Thus, in general, hedge fund managers turn out to be very poor market timers, consistent with findings by Fung et al. (2002) and Cerrahoglu et al. (2003).

For the aggregate industry, the negative market timing ability is somewhat offset by the increased alpha from security selection. Separating the market timing impact increases the intercept to 5.0 percent per annum (compared to 3.7 percent per annum under the Fama French model). In this case, the intercept refers purely to the excess returns generated from security selection prior to the removal of negative market timing effects. The existence of market timing ability is harder to prove for hedge funds than for mutual funds, given the low survival rate and the existence of survivorship bias. Survivorship bias leads to perceived convex relationships where none exist. For example, if equity markets decline steeply, poorly performing hedge funds will go out of business and no longer report numbers to the database, thereby also creating a convex relationship between the aggregate hedge fund industry and equity market returns. We now examine the impact of conditioning variables based on the adjusted Ferson and Schadt model.

<< INSERT TABLE 4 >>

---

8 We thank Adrian Pagan for pointing out this effect of outliers on convexity.
Table 4 shows that if we apply the de-meaned and lagged VIX, term spread and credit spread as conditioning variables, market timing beta for the overall industry slightly improves compared to table 3. However, on average, market timing still does not add value for most strategies employed.

<< INSERT TABLE 5 >>

Table 5 shows the relevant conditioning betas, based on VIX, term and credit spreads as conditioning variables. As can be seen, VIX and credit spreads are important conditioning variables for a number of strategies, while the term spread has relatively few occurrences where conditioning is significant. The large negative magnitude of conditional coefficients for equity market risk ($\beta_m$) suggest a higher VIX and credit spread lead to significant reductions in conditional equity market exposure ($\beta_m(Z_{t-1})$) by managers for most hedge fund strategies in the subsequent period.

For conditional illiquidity ($\beta_L(Z_{t-1})$) the results are mixed. This reflects the fact illiquidity is a function of the underlying strategies and liquidity timing is more difficult to execute than equity market timing. In the case of market timing, this is easily implemented by transactions in futures markets. In the case of illiquid positions, the manager may have to wait to roll out of illiquidity as positions expire.

The impact of conditional variables on tail risk sensitivity ($\beta_{tail}(Z_{t-1})$) is also mixed. Some strategies exhibit increased tail risk as credit spreads widen (notably convertible arbitrage). The impact of conditional variables on small cap beta ($\beta_{s,j}(Z_{t-1})$) and value beta ($\beta_{v,j}(Z_{t-1})$) is as expected: as credit spreads increase, managers rotate out of small caps and into value stocks.
5.3 Rolling beta analysis

Figure 3 shows equity beta varies between 0.20 and 0.50. Thus, hedge fund managers always retain some sensitivity to movement in equity markets. A reduction in market exposure coincides with the Iraq invasion in 2003, suggesting that managers protected positions during the bear market preceding the invasion, but then failed to increase exposure to catch the subsequent rally to 2005. The illiquidity beta shows that hedge funds’ sensitivity to illiquidity is somewhat less volatile compared to equity beta, and varies between 0 and 0.10. This reflects the fact that illiquidity changes are more difficult to implement than market timing. Tail event beta varies between -0.30 and +0.10, but is, in general, negative. Thus, hedge funds do hedge some of the tail risk but we find this beta to be insignificant at the 95 percent confidence level for the overall industry. From the market timing charts we also found that the tail hedge capability is limited to certain strategies such as managed futures. The rolling small cap (SMB) beta declines over time, suggesting hedge funds are lowering their exposure to small caps. The value (HML) beta is negative most of the time, suggesting hedge funds continue to overweigh low quality (growth) stocks. Market timing beta is negative, suggesting hedge funds as a group are consistently poor at market timing, even more so during financial crises such as the Asia crisis in 1998 or more recently the global financial crisis. On the other hand, three year rolling alpha is surprisingly constant, suggesting that alpha is less dependent on the market environment, though it is worth noting that the confidence band often includes zero, especially during times of crises.
5.4 Risk reduction using managed futures

Our results suggest hedge funds do generate alpha, net of fees, of around 3.7 percent per annum over the period from 1994 to 2009. However, we also conclude that this alpha mainly stems from security selection: hedge funds as an industry exhibit poor market timing skill, with managed futures being a notable exception. Based on figure 2, managed futures is the only strategy to exhibit positive convexity. Rather than investing in the hedge fund aggregate industry (or hedge funds as an asset class), we suggest a method for hedge fund investing which relies on the least risk portfolio approach: an automatic selection of hedge fund strategies including managed futures, with quarterly rebalancing so as to minimise (downside) risk. We apply quarterly rebalancing as a worst case scenario. Many hedge fund strategies offer monthly liquidity, but some only offer quarterly liquidity. This also lowers rebalancing cost. As will be seen in figure 4 the rebalancing cost will be relatively low as the strategy allocations remain fairly stable through time.

To the best of our knowledge, this type of solution for hedge fund investing has not been proposed before. Please note that this approach is not to be confused with hedge fund replication as suggested by Fung and Hsieh (2002) or Hazanhodzic and Lo (2007), which has been the focus of much academic research. Based on the principles set out by Fung and Hsieh, providers such as Merrill Lynch or State Street offer hedge fund replication products through exposure to risk premia (such as the equity or credit premium). Our method relies to a tailored allocation to active hedge fund strategies. Investors can implement the method we propose by buying representative managers for each hedge fund strategy or through Exchange Traded Funds (ETFs). ETFs are available now for most hedge fund strategies from providers such as AQR,
WisdomTree, IQ Hedge, Merrill Lynch and Deutsche Bank. Our method combines the downside protection provided by managed futures with the net alpha provided by other hedge fund strategies.¹⁰

We construct two types of least risk portfolios designed to achieve the least volatility possible given the universe of strategies and compare it against the aggregate hedge fund industry performance. Least risk portfolios are defined as the point on the efficient frontier with the lowest risk, either based on standard deviation or Conditional Value at Risk (CVaR) (Appendix B details the optimisation method). The least risk portfolios is unique in that it is the only portfolio on the frontier without expected returns as inputs. Behr et al. (2008) and Clarke et al. (2006) report that for traditional asset classes many least risk portfolios outperform other type of portfolio structuring methods, e.g. asset weighted or equal weighted approaches.

To set the initial least risk portfolio weights, HFR reported data from January 1991 to December 1997 serves as in-sample period (28 quarterly data points), and the period from January 1998 to December 2009 serves as out of sample period. The weights in the least risk portfolios are then updated based on either variance or Conditional Value at Risk optimisation and applied to the next data point to calculate the out of sample portfolio return. At the end of each period, the portfolio weights are rebalanced using the lengthened window period.

<< INSERT FIGURE 4 >>

¹⁰We did consider downside protection strategies such as Option Based Portfolio Insurance (OBPI) and Constant Proportion Portfolio Insurance (CPPI) to create the desired convexity, but find the results unsatisfactory.
Figure 4 shows the out of sample blanket charts. The blanket charts show for each point in time, the composition of assets that are included in the least risk portfolios. As can be seen, the least risk portfolios are dominated by allocations to multi-strategy, managed futures and emerging markets: managed futures show an especially large allocation when optimising on downside risk (CVaR).

<< INSERT FIGURE 5 >>

Figure 5 shows a comparative drawdown chart for the aggregate hedge fund industry and the least risk portfolios. Maximum drawdown charts are defined as the percentage loss that an asset incurs from its peak value to its lowest subsequent value. For example, an asset that halves in value before returning to its subsequent peak is deemed to have a maximum drawdown of 50 percent. Drawdowns are used to measure defensive ability of absolute return type funds. As can be seen, the optimised portfolios have much less drawdowns than the aggregate industry portfolio.

Out of sample summary statistics are provided in table 6. As can be seen, the optimised portfolios generate lower returns, but with much lower standard deviation, thereby offering higher Sharpe ratios. In practice, return levels could be targeted as an additional constraint for the optimiser. We choose not to do so, so as to reduce the number of parameters to estimate.

<< INSERT TABLE 6>>
To formally test out of sample performance we apply the Jobson Korkie (1981) test in Table 7. This test remains one of the most popular for determining superior risk adjusted performance. Jobson and Korkie note that the statistical power of the test is low. Therefore, observing a statistically significant score between two portfolios can be seen as strong evidence of a difference in risk adjusted performance. Details on the derivation of the Jobson Korkie statistic can be found in Appendix C.

<< INSERT TABLE 7>>

The Jobson Korkie statistics in table 7 suggest that the optimised portfolios significantly outperform an investment in the aggregate industry in terms of reward to risk ratio. This is consistent with Behr et al. (2008), who report that for traditional asset classes, least risk portfolios outperform market capitalisation weighted indices.

6. CONCLUSIONS

Our study increases the robustness of findings from constant beta models used in literature. While we detect excess returns in the order of 3.7 percent per annum for the hedge fund industry, we also find equity and illiquidity risk to be important drivers of hedge fund returns and an augmented Fama-French factor model explains a large part of the variation in hedge fund returns. For completeness, we also test the returns of the HFRX indices which only include funds open to new investors in a given month and are calculated on an asset weighted (not equal weighted) basis. Compared to HFRI based results, we find excess returns reduce by around 1 percent per annum. Based on Table 2 emerging markets managers create excess returns in the order of 6.2 percent per annum. However, if we add an additional portfolio mimicking factor (emerging
market returns minus developed market returns, given that emerging markets tend to be difficult to short), the alpha reduces to a similar magnitude as the other strategies. The size of the excess returns for the industry is similar to that reported by Chen et al. (2010), who report an overall alpha of 3 per cent per annum, and in line with findings by Ang et al. (2009, p.54) who conclude that “so far, the cumulated academic evidence suggests that hedge fund manager skill exists and that the rewards to that skill can be passed on to fund investors, depending upon a judicious manager selection process.” For most hedge fund strategies, we find no evidence of market timing skill which is consistent with the work done by Cerrahoglu et al. (2003) and Fung et al. (2002). In fact, there is evidence to the contrary: most hedge fund managers are poor market timers. Consistent with Patton and Ramodarai (2009) we find evidence for the importance of conditioning information for hedge fund risk factors. Managers mechanically reduce equity and small cap risk exposure and rotate into value stocks as VIX or credit spreads increase. We find that alpha under conditional models is lower, consistent with evidence by Ferson and Schadt (1996) for traditional asset classes and Cerrahoglu et al. (2003) for alternative asset classes. Finally, we demonstrate how least risk portfolios are able to improve the reward to risk ratios for investments in hedge funds.

A few caveats to our study are worth mentioning. First, we noted the importance of self-selection bias, which we cannot measure. Superior hedge funds are unlikely to actively market their efforts through established databases as they are often operating at capacity and closed to new investors. Furthermore, these superior managers tend to be among the largest funds in the industry as their success attracts a large following. As of first half 2010, Hedge Fund Research reports 60 percent of invested money is
now with the larger funds (> $5bn) because of track record, resourcing, infrastructure and credit facilities. Thus perhaps the alpha captured by the databases may not be the alpha experienced by most institutional investors after all. At the same time, inferior hedge fund managers are also unlikely to enter the database with a poor track record. In this sense, our results could still be biased either way.

Second, the excess returns we describe are those earned by the managers on a time-weighted basis, rather than what is actually delivered to investors on a money-weighted basis. Dichev and Yu (2009) find returns to hedge fund investors are 3 to 7 percent lower than corresponding buy and hold returns because of switching behaviour. This reduction in returns is caused as investors ‘follow the hot money.’ Based on the same method, Dichev (2007) finds for mutual funds, dollar-weighted returns are 1.5 percent lower per annum. Similarly, based on the HFRI sample we used to proxy the industry, we find dollar weighted returns reduce results by 5.4 percent. Thus, although hedge funds may have generated returns greater than the fees that they charge, they have not delivered this to their clients due to their clients’ switching behaviour.

A third caveat is that the hedge fund industry is comparatively new and changing dramatically. Thus, any evidence on the existence of excess returns must be seen from a contemporary perspective and be qualified with a concern that as more hedge funds enter the market, in the long run a similar Berk and Green equilibrium could still be reached as for the mutual fund industry. As one example, similar to passive equity funds, ‘hedge fund beta funds’ have recently become available to institutional investors at a comparatively low cost of between 0.5 to 1 percent per annum. Initially
these beta funds tended to focus on mechanically replicating hedge fund operations in narrow breadth strategies such as convertible arbitrage or merger arbitrage where active managers chase the same limited deal flow. As more insights are gained in how hedge fund managers operate, they capture increasingly more hedge fund ‘alpha’.

Some issues warrant further research. Our analysis confirms the findings by Brunnermeier and Nagel (2004) that hedge funds have traditionally opted to underweight value stocks, to their detriment, as, over the period, value stocks have outperformed. Rolling beta analysis suggests this bias towards growth stocks has only increased in recent years. Carhart (1997) and Gruber (1996) find US mutual funds have a similar preference for growth stocks. From that perspective, hedge funds merely amplify behavioural biases. Another possible explanation is that since growth stocks exhibit higher volatility than value stocks they increase the potential value of the asymmetric (option like) performance fees charged by hedge fund managers.

Further, if the average hedge fund manager proves to be as poor in market timing as the average mutual fund manager, the manager could perhaps focus his research efforts on security selection, as most mutual fund managers do as security selection seems to be the main source of value add. Finally, some hedge fund strategies such as managed futures do seem to offer the ability to market time. This conclusion is confirmed by Monarcha (2010) who applies non-parametric tests and finds market timing to be a major performance driver for managed futures. Such types of hedge fund strategies provide a hedge against tail events or stress periods. In this paper we demonstrate how managed futures can contribute to risk reduction for an investment in hedge funds through the use of a least risk portfolio. Managed futures can be
further studied from an overall multi-asset portfolio diversification context for institutional investors.
REFERENCES

Liang, B., and H. Park, 2007. Share restrictions, liquidity premiums and offshore hedge funds, University of Massachusetts at Amherst, working paper.
Appendix A: Hedge Fund Research strategy definitions

The following definitions are sourced from Hedge Fund Research (HFR) and CSFB/Tremont.\(^\text{11}\)

**Convertible arbitrage.** Managers invest in the convertible bond of a company and short the stock to create a hedged position. Positions profit from the higher yield and upgrades of the fixed income security as well as the short sale of the stock.

**Dedicated short.** Managers in this strategy maintain a net short position in equities and equity derivatives.

**Emerging markets.** This strategy involves equity or fixed income investing in emerging markets. As many emerging markets do not allow short selling or offer viable derivative products, managers often employ a long-only bias.

**Market neutral.** Equity market neutral involves being long and short matched equity portfolios. In addition, portfolio managers control for industry, sector, market capitalisation and other style risks.

**Event driven.** This strategy captures price movements generated by anticipated corporate events such as mergers and acquisitions where managers are long the stock of the company being acquired and short the stock of the acquirer. The principal risk is deal risk, should the deal fail to close. This strategy is also known as merger arbitrage or risk arbitrage.

**Distressed debt.** This strategy invests in the debt, equity or trade claims of companies in financial distress or a general bankruptcy. Securities typically trade at a substantial discount to par value pending legal action or restructuring.

**Global macro.** This strategy carries long and short positions in global capital and derivative markets. These positions reflect views on the overall market direction influenced by major economic trends or events. Portfolios include stocks, bond, currencies and commodities.

**Equity long short.** Managers in this strategy maintain long and short positions in equity and equity derivatives. Strategies can be quantitative or based on fundamental analysis. Managers typically maintain at least a 50 percent net exposure to equity markets.

**Managed futures.** This strategy applies algorithmic models to capture trending or momentum characteristics in commodity and currency markets. Managers are usually referred to as Commodity Trading Advisors or CTAs.

**Multi-strategy.** Managers invest in a wide range variety of corporate transactions, including, but not limited to, mergers, restructurings, financial distress, tender offers, buybacks, security issuance or other capital structure adjustments. Fundamental (as opposed to quantitative) analysis of a company’s existing capital structure is employed.

Appendix B Portfolio optimisation

Optimisation under mean-variance

Under the original Markowitz (1952) model, standard deviation is the measure of risk.

\[
\sigma(r_p) = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} x_ix_j \sigma_{ij}} \rightarrow \text{min} \quad \text{(minimise portfolio standard deviation)}
\]

s.t.

\[
\sum_{j=1}^{N} x_j = 1 \quad \text{(weights must sum up to 100 percent)}
\]

\[
\sum_{j=1}^{N} x_j r_j = R \quad \text{(weighted return must sum up to } R)\]

\[
x_j \geq 0 \quad \text{(no negative weights allowed)}
\]

Where

- \( \sigma(r_p) \) = the portfolio standard deviation
- \( x_j \) = weight in asset \( j \)
- \( r_j \) = the return on asset \( j \)
- \( N \) = the number of assets
- \( \sigma_{ij} \) = covariance between asset \( i \) and asset \( j \)

Optimisation under Conditional Value at Risk (CVaR)

Value at Risk (VaR) is used to measure downside risk. For example, if an asset has a 5 percent VaR of minus 10 percent (or the distribution of generated monthly returns at the 5th percentile is minus 10 percent), there is only a 5 percent probability that the asset will fall in value by more than minus 10 percent over the period measured. Alternatively, a 5 percent VaR means that a loss of 10 percent or more on the asset is expected in only 1 year out of 20.

Conditional Value at Risk is an alternative to Value at Risk that is more sensitive to the shape of the loss distribution in the tail of the distribution. For example the expected shortfall at the 5 percent level is the expected return of the asset in the worst 5 percent of cases. Thus, Expected Shortfall measures the average of all observations below the 5th percentile, and considers more than just the single most catastrophic outcome, making it more robust. Conditional Value at Risk is also known as Expected shortfall (ES) and expected tail loss (ETL).

Alexander and Baptista (2004) suggest the CVaR constraint is more effective than VaR constraints for mean-variance optimisation.

\[
\text{CVaR}(r_j) = E(r_j \mid r_j < -\text{VaR}(r_j)) \rightarrow \text{min} \quad \text{(minimise portfolio Conditional Value at Risk)}
\]
Appendix C Jobson Korkie test

To assess the ex-post performance of the alternative portfolios we use the Jobson Korkie (1981) pairwise test of the equality of Sharpe Ratio. The test statistic on portfolios $i$ and $j$ can be formulated as

$$JK_{i,j} = \frac{\hat{\mu}_i \times \hat{\sigma}_j - \hat{\mu}_j \times \hat{\sigma}_i}{\hat{\vartheta}}$$

Where

$$\hat{\vartheta} = \sqrt{\hat{\vartheta}}$$

$$\hat{\vartheta} = \frac{1}{T} \left[ 2\hat{\sigma}_i^2 \hat{\sigma}_j^2 - 2\hat{\sigma}_i \hat{\sigma}_j \hat{\sigma}_o \hat{\sigma}_j + \frac{1}{2} \hat{\mu}_i^2 \hat{\sigma}_j^2 + \frac{1}{2} \hat{\mu}_j^2 \hat{\sigma}_i^2 - \frac{\hat{\mu}_i \hat{\mu}_j}{2\hat{\sigma}_i \hat{\sigma}_j} (\hat{\sigma}_o^2 + \hat{\sigma}_i^2 \hat{\sigma}_j^2) \right]$$

Where $T$ is the number of observations and $\hat{\sigma}_o$ is an estimate of the covariance of the excess returns of the two portfolios over the evaluation period. Jobson and Korkie show the test statistic is approximately normally distributed with a zero mean and a unit standard deviation. A significant test statistic rejects the null hypothesis of equal risk-adjusted performance. Jobson and Korkie (1981) note the statistical power of the test (the probability of rejecting the null hypothesis) is low. Therefore, observing a statistically significant score between two portfolios can be seen as strong evidence of a difference in risk adjusted performance.
Figure 1 Evidence of market timing skill for the aggregate hedge fund index 1994-2009

Hedge Fund Aggregate Index

\[ y = -0.9261x^2 + 0.3224x + 0.0056 \]
\[ R^2 = 0.5794 \]

Equity Risk Premium (S&P 500 - \( R_f \))

Hedge fund strategy excess returns (\( R_i - R_f \))
Figure 2 Hedge fund strategy market timing skill (1994-2009)
Figure 3 Three year rolling beta and alpha experience (1997-2009)

Figure 3 displays estimated beta and alpha coefficients using three-year rolling subsamples of the data. This allows us to examine historical trends and the stability of the model. The dotted lines represent 95 percent confidence bands.
Figure 4 Out of sample blanket charts (1998-2009)

Rolling weights of hedge fund strategies under Mean-Variance

Rolling weights of hedge fund strategies under Mean-Conditional Value at Risk
Figure 5 Out of sample drawdown chart (1998-2009)

Drawdown chart - hedge fund portfolios

-25%
-20%
-15%
-10%
-5%
0%


--- mean variance
--- mean-CVaR
--- hedge fund industry
Table 1

Descriptive hedge fund industry statistics (1994-2009)

Descriptive statistics of the hedge fund industry based on monthly data from January 1994 to December 2009. The data represents net of fees log returns based on the Hedge Fund Research (HFR) indices. The total number of observations is 192 months. The total return is annualised by multiplying monthly results times 12. Excess returns are calculated versus the 3 month t-bill index. The standard deviation is annualised by multiplying monthly standard deviation by the square root of 12. Best month represents the month with the highest monthly return. Worst month represents the month with the lowest monthly return. Beta is measured by calculating the slope of the strategy returns against the S&P 500 index. Skew represents the skew of the monthly returns. Kurtosis refers to the excess kurtosis versus a normal distribution.

<table>
<thead>
<tr>
<th>HFR Hedge Fund Indices</th>
<th>Total Return (%pa)</th>
<th>Excess Return (%pa)</th>
<th>Standard Deviation (%pa)</th>
<th>Best Month (%)</th>
<th>Worst Month (%)</th>
<th>Beta to S&amp;P</th>
<th>Skew</th>
<th>Kurtosis</th>
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<tr>
<td>Aggregate Index</td>
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<td>5.6</td>
<td>7.3</td>
<td>7.4</td>
<td>-9.1</td>
<td>0.4</td>
<td>-1.3</td>
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<td>0.2</td>
<td>-0.2</td>
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<td>19.6</td>
<td>20.6</td>
<td>23.8</td>
<td>-0.9</td>
<td>-1.2</td>
<td>5.7</td>
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<td>5.0</td>
<td>15.1</td>
<td>13.8</td>
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<td>7.3</td>
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<td>1.0</td>
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<td>Distressed</td>
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<td>-8.4</td>
<td>0.2</td>
<td>-3.6</td>
<td>30.3</td>
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</table>
Table 2 Result of GARCH model for monthly hedge fund returns - Augmented Fama and French model (1994-2009)

Table 2 presents the GJR GARCH estimation of the augmented Fama French model based on mean equation (4) and variance equation (7). In the mean equation, the dependent variable \( R_{it} - R_{ft} \) represents the excess return of strategy \( i \) at time \( t \) regressed on the market risk premium \( (R_{mt} - R_{ft}) \), the illiquidity premium \( (R_{V_i} - R_{ft}) \), the option writing premium \( (R_{PUT_{i}} - R_{ft}) \), the small cap premium \( (SMB) \) and the value premium \( (HML) \). In the variance equation, the conditional variance of strategy \( i \) \( (\varepsilon_{i,t}^2) \) is regressed on the error of the previous period \( (\varepsilon_{i,t-1}) \), the variance of the previous period \( (h_{i,t-1}) \) and an additional regressor if the error at t-1 is found to be negative \( (\varepsilon_{i,t-1}I_{t-1}) \).

<table>
<thead>
<tr>
<th>Model</th>
<th>Aggregate Index</th>
<th>Convertible Arbitrage</th>
<th>Dedicated Short</th>
<th>Emerging Markets</th>
<th>Market Neutral</th>
<th>Event Driven</th>
<th>Distressed Debt</th>
<th>Global Macro</th>
<th>Equity Long</th>
<th>Short</th>
<th>Managed Futures</th>
<th>Multistrategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.0031***</td>
<td>0.0043***</td>
<td>-0.0003</td>
<td>0.0052**</td>
<td>0.0020***</td>
<td>0.0052***</td>
<td>0.0049***</td>
<td>0.0038***</td>
<td>0.0035***</td>
<td>0.0026***</td>
<td>0.0026***</td>
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<tr>
<td>( \beta_n )</td>
<td>0.3143***</td>
<td>0.0712***</td>
<td>-0.8433***</td>
<td>0.4492***</td>
<td>0.0512**</td>
<td>0.2551***</td>
<td>0.1988***</td>
<td>0.2113***</td>
<td>0.3183***</td>
<td>0.4177***</td>
<td>0.1005***</td>
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<td>( \delta_n )</td>
<td>0.0718***</td>
<td>0.0493***</td>
<td>0.0174</td>
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<td>0.0335***</td>
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<tr>
<td>( \beta_{id} )</td>
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<td>0.0101</td>
<td>0.2408***</td>
<td>0.1038</td>
<td>-0.0137</td>
<td>-0.0311</td>
<td>-0.0271</td>
<td>-0.1882***</td>
<td>-0.0141</td>
<td>-0.20855***</td>
<td>-0.0120</td>
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</tr>
<tr>
<td>( \beta_1 )</td>
<td>0.2060***</td>
<td>0.0582***</td>
<td>-0.5295***</td>
<td>0.2249***</td>
<td>0.01404</td>
<td>0.2241***</td>
<td>0.1716***</td>
<td>0.1233***</td>
<td>0.2323***</td>
<td>0.1645***</td>
<td>0.0692***</td>
<td></td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>-0.0996***</td>
<td>0.0085</td>
<td>0.1224***</td>
<td>-0.1306***</td>
<td>-0.0568***</td>
<td>0.0005</td>
<td>0.01737</td>
<td>-0.1083***</td>
<td>-0.0894***</td>
<td>-0.0873***</td>
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<tr>
<td>( \gamma_0 )</td>
<td>0.0000</td>
<td>0.0000***</td>
<td>0.0000</td>
<td>0.0000***</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000***</td>
<td>0.0000</td>
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<tr>
<td>( \gamma_1 )</td>
<td>-0.0208</td>
<td>0.6330***</td>
<td>0.1251</td>
<td>-0.1766***</td>
<td>0.2238</td>
<td>0.1686</td>
<td>0.1276</td>
<td>0.1331</td>
<td>-0.0570</td>
<td>0.2252**</td>
<td>0.2497</td>
<td></td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>0.9265***</td>
<td>0.3158**</td>
<td>0.7959***</td>
<td>0.7331***</td>
<td>0.7311***</td>
<td>0.6868**</td>
<td>0.7337***</td>
<td>0.7675***</td>
<td>0.7451***</td>
<td>0.7869***</td>
<td>0.5709***</td>
<td></td>
</tr>
<tr>
<td>( \gamma_3 )</td>
<td>0.1240*</td>
<td>0.0000</td>
<td>0.1016</td>
<td>0.3707***</td>
<td>-0.0765</td>
<td>-0.0629</td>
<td>0.1608</td>
<td>-0.2211**</td>
<td>0.2977***</td>
<td>-0.0711</td>
<td>0.3229*</td>
<td></td>
</tr>
<tr>
<td>( \gamma_1 + \gamma_2 + 0.5 \gamma_3 )</td>
<td>0.9677</td>
<td>0.9478</td>
<td>0.6246</td>
<td>0.7418</td>
<td>0.9167</td>
<td>0.8409</td>
<td>0.9417</td>
<td>0.7900</td>
<td>0.8369</td>
<td>0.9792</td>
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</tr>
<tr>
<td>t-dist errors</td>
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<td>4.81</td>
<td>4.33</td>
<td>36.1</td>
<td>4.87*</td>
<td>4.16***</td>
<td>4.55***</td>
<td>6.45</td>
<td>11.03</td>
<td>7.11*</td>
<td>11.86</td>
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</tr>
<tr>
<td>Adj R²</td>
<td>0.78</td>
<td>0.1687</td>
<td>0.71</td>
<td>0.48</td>
<td>0.09</td>
<td>0.60</td>
<td>0.47</td>
<td>0.20</td>
<td>0.75</td>
<td>0.27</td>
<td>0.31</td>
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<tr>
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<td>470.46</td>
<td>425.35</td>
<td>666.16</td>
<td>615.53</td>
<td>597.42</td>
<td>522.82</td>
<td>624.26</td>
<td>546.97</td>
<td>682.63</td>
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</tr>
</tbody>
</table>

* significant at the 10 percent level ** significant at the 5 percent level *** significant at the 1 percent level
Table 3 Results of GARCH model for monthly hedge fund returns - Augmented Treynor and Mazuy model (1994-2009)

Table 3 presents the GJR GARCH estimation of the Treynor and Mazuy market timing model based on mean equation (5) and variance equation (7). The variables are as defined for table 2. In addition, in the mean equation, a market timing factor $\gamma_i,t$ is added.

$$R_{it} = \alpha + \beta_{it}(R_m - R_f) + \beta_{ii}(R_{it-1} - R_f) + \beta_{i1}(R_{it-1} - R_f)^2 + \beta_{i2} \gamma_i,t + \beta_{i3} \gamma_i,t^2 + \epsilon_{it}$$

$$\log \text{Likelihood} = \sum_{t=1}^{T} \frac{-\frac{1}{2}(\log \sigma_t^2 + \frac{y_{it}^2}{\sigma_t^2})}{\sqrt{2\pi}}$$

<table>
<thead>
<tr>
<th>Model</th>
<th>Aggregate Index</th>
<th>Convertible Arbitrage</th>
<th>Dedicated Short</th>
<th>Emerging Markets</th>
<th>Market Neutral</th>
<th>Event Driven</th>
<th>Distressed Debt</th>
<th>Global Macro</th>
<th>Equity Long Short</th>
<th>Managed Futures</th>
<th>Multistrategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.0042***</td>
<td>0.0043***</td>
<td>0.0014</td>
<td>0.0064**</td>
<td>0.0025***</td>
<td>0.0056***</td>
<td>0.0061***</td>
<td>0.0038**</td>
<td>0.0038***</td>
<td>0.0004</td>
<td>0.0032***</td>
</tr>
<tr>
<td>$\beta_{it}$</td>
<td>0.3255***</td>
<td>0.0698***</td>
<td>-0.8122***</td>
<td>0.5096***</td>
<td>0.0537**</td>
<td>0.2588***</td>
<td>0.2156***</td>
<td>0.2115***</td>
<td>0.3284***</td>
<td>0.4008***</td>
<td>0.1043***</td>
</tr>
<tr>
<td>$\beta_{it}$</td>
<td>0.0678***</td>
<td>0.0487***</td>
<td>0.0031</td>
<td>0.1236**</td>
<td>0.0282**</td>
<td>0.0560***</td>
<td>0.0690***</td>
<td>0.0326</td>
<td>0.0640***</td>
<td>0.0172</td>
<td>0.0388***</td>
</tr>
<tr>
<td>$\beta_{it}$</td>
<td>-0.0440</td>
<td>0.0118</td>
<td>0.1540*</td>
<td>0.0232</td>
<td>-0.0247</td>
<td>-0.0394</td>
<td>-0.0618</td>
<td>-0.1898**</td>
<td>-0.0304</td>
<td>-0.1558**</td>
<td>-0.0248</td>
</tr>
<tr>
<td>$\beta_{it}$</td>
<td>0.2143***</td>
<td>0.0619***</td>
<td>-0.5173***</td>
<td>0.2322***</td>
<td>0.0186</td>
<td>0.2212***</td>
<td>0.1686***</td>
<td>0.1235***</td>
<td>0.2253***</td>
<td>0.1589***</td>
<td>0.0690***</td>
</tr>
<tr>
<td>$\beta_{it}$</td>
<td>-0.0904***</td>
<td>0.0112</td>
<td>0.1349***</td>
<td>-0.1382***</td>
<td>-0.0533***</td>
<td>-0.0008</td>
<td>0.0201</td>
<td>-0.1082***</td>
<td>-0.0862***</td>
<td>-0.0976***</td>
<td>0.0293***</td>
</tr>
<tr>
<td>$\beta_{it}$</td>
<td>-0.4479**</td>
<td>0.0484</td>
<td>-0.8957***</td>
<td>-0.8689</td>
<td>-0.2579</td>
<td>-0.3245</td>
<td>-0.9761***</td>
<td>-0.0225</td>
<td>-0.3170</td>
<td>1.4671***</td>
<td>-0.2887**</td>
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<tr>
<td>$\gamma_i$</td>
<td>0.0000*</td>
<td>0.0000*</td>
<td>0.0000**</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000***</td>
<td>0.0020</td>
<td>0.0000**</td>
<td></td>
</tr>
<tr>
<td>$\gamma_{i1} (ARCH)$</td>
<td>0.0938*</td>
<td>0.6277***</td>
<td>0.1458</td>
<td>-0.1020</td>
<td>0.2366</td>
<td>0.2532</td>
<td>0.1272</td>
<td>0.1337</td>
<td>0.0121</td>
<td>0.1948**</td>
<td>0.3146***</td>
</tr>
<tr>
<td>$\gamma_{i2} (GARCH)$</td>
<td>0.8605***</td>
<td>0.3203***</td>
<td>0.7827***</td>
<td>0.7782***</td>
<td>0.7390***</td>
<td>0.6017**</td>
<td>0.7721***</td>
<td>0.7672***</td>
<td>0.7680***</td>
<td>0.8437***</td>
<td>0.6564***</td>
</tr>
<tr>
<td>$\gamma_{i3}$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.1036</td>
<td>0.2798***</td>
<td>-0.1157</td>
<td>-0.1602</td>
<td>0.1436</td>
<td>-0.2215***</td>
<td>0.2138*</td>
<td>-0.1109</td>
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</tr>
<tr>
<td>$\gamma_{i4} + \gamma_{i5} + \gamma_{i6}$</td>
<td>0.9544</td>
<td>0.9480</td>
<td>0.9803</td>
<td>0.9177</td>
<td>0.9747</td>
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<td>0.8870</td>
<td>0.3548</td>
<td>0.9710</td>
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</tr>
<tr>
<td>$t$-dist errors</td>
<td>12.67</td>
<td>4.76***</td>
<td>5.50**</td>
<td>76.51</td>
<td>4.22*</td>
<td>4.31***</td>
<td>5.37***</td>
<td>6.44</td>
<td>12.96</td>
<td>20.30</td>
<td>7.01</td>
</tr>
<tr>
<td>Adj $R_t^2$</td>
<td>0.78</td>
<td>0.15</td>
<td>0.72</td>
<td>0.51</td>
<td>0.09</td>
<td>0.61</td>
<td>0.54</td>
<td>0.19</td>
<td>0.76</td>
<td>0.35</td>
<td>0.35</td>
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<tr>
<td>Log Likelihood</td>
<td>631.41</td>
<td>604.21</td>
<td>473.21</td>
<td>425.43</td>
<td>667.08</td>
<td>615.95</td>
<td>600.37</td>
<td>522.82</td>
<td>624.65</td>
<td>555.04</td>
<td>681.52</td>
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</tbody>
</table>

* significant at the 10 percent level ** significant at the 5 percent level *** significant at the 1 percent level

* Variance targeting used to impose a long-run variance estimate for cases where numerical convergence was not reached under the standard GJR model.
Table 4 Results of GARCH model for monthly hedge fund returns - Augmented Ferson and Schadt model (1994-2009)

Table 4 presents the unconditional beta subset of the Ferson and Schadt model based on mean equation (6) and variance equation (7).

\[
\begin{align*}
\gamma_t &= \alpha + \beta_{1}\text{Mkt} - \text{R}_{t-1} + \beta_{2}\text{SMB} + \beta_{3}\text{HML} + \beta_{4}\text{CMA} - \text{R}_{t-1} + \beta_{5}\text{SMB} + \beta_{6}\text{HML} + \epsilon_t \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>Model</th>
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<th>Convertible</th>
<th>Dedicated</th>
<th>Short</th>
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<th>Market Neutral</th>
<th>Event Driven</th>
<th>Distressed Debt</th>
<th>Global Macro</th>
<th>Equity Long</th>
<th>Managed Futures</th>
<th>Multistrategy</th>
</tr>
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<tbody>
<tr>
<td>(\alpha)</td>
<td>0.0041***</td>
<td>0.0036***</td>
<td>0.0016</td>
<td>0.0077***</td>
<td>0.0020***</td>
<td>0.0048***</td>
<td>0.0062***</td>
<td>0.0036**</td>
<td>0.0039***</td>
<td>0.0002</td>
<td>0.0034***</td>
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<tr>
<td>(\beta_1)</td>
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<td>-0.8396***</td>
<td>0.5210***</td>
<td>0.0489**</td>
<td>0.2650***</td>
<td>0.1816***</td>
<td>0.2385***</td>
<td>0.3242***</td>
<td>0.3960***</td>
<td>0.0984***</td>
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</tr>
<tr>
<td>(\beta_2)</td>
<td>0.0615***</td>
<td>0.0420***</td>
<td>-0.0066</td>
<td>0.1177**</td>
<td>0.0227**</td>
<td>0.0628***</td>
<td>0.0744***</td>
<td>0.0332</td>
<td>0.0633***</td>
<td>-0.0117</td>
<td>0.0382***</td>
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</tr>
<tr>
<td>(\beta_4)</td>
<td>-0.0207</td>
<td>0.0482</td>
<td>0.1602**</td>
<td>-0.0259</td>
<td>0.0228</td>
<td>0.0570</td>
<td>-0.0124</td>
<td>-0.1071</td>
<td>0.0125</td>
<td>-0.0405</td>
<td>-0.0329</td>
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</tr>
<tr>
<td>(\beta_5)</td>
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<td>0.0617***</td>
<td>-0.5192***</td>
<td>0.2007***</td>
<td>0.0323**</td>
<td>0.2238***</td>
<td>0.1522***</td>
<td>0.0989**</td>
<td>0.1971***</td>
<td>0.1623***</td>
<td>0.0692***</td>
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</tr>
<tr>
<td>(\beta_6)</td>
<td>-0.0380**</td>
<td>0.0261*</td>
<td>0.1393***</td>
<td>-0.1497***</td>
<td>-0.0171</td>
<td>0.0531***</td>
<td>0.0464***</td>
<td>-0.0691**</td>
<td>-0.0697***</td>
<td>-0.0587**</td>
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<tr>
<td>(\beta_{\text{CMA}})</td>
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<td>1.6841**</td>
<td>0.0651</td>
<td>0.3971**</td>
<td>0.00204</td>
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<td>1.5639**</td>
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<tr>
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<td>0.0000*</td>
<td>0.0000</td>
<td>0.0000*</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000*</td>
<td>0.0000*</td>
<td>0.0000*</td>
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</tr>
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<td>0.4564***</td>
<td>0.0970</td>
<td>0.0567</td>
<td>0.4935</td>
<td>-0.0487***</td>
<td>0.0931</td>
<td>0.1212</td>
<td>-0.0662</td>
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<td>0.8489***</td>
<td>0.5060***</td>
<td>0.8270***</td>
<td>0.7594***</td>
<td>0.6737***</td>
<td>1.0202***</td>
<td>0.8132***</td>
<td>0.7531***</td>
<td>0.7048***</td>
<td>0.8125***</td>
<td>0.6834***</td>
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<tr>
<td>(\gamma_3)</td>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.1103</td>
<td>0.2566*</td>
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<td>0.0158</td>
<td>0.0708</td>
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<td>0.3685**</td>
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<td>(\gamma_4 + \gamma_5 + 0.5 \gamma_3)</td>
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<tr>
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<td>4.99***</td>
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<td>3.48**</td>
<td>19.97</td>
<td>3.33***</td>
<td>19.22</td>
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<td>14.64</td>
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</tr>
<tr>
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<td>0.07</td>
<td>0.75</td>
<td>0.57</td>
<td>0.30</td>
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<tr>
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<td>429.15</td>
<td>674.35</td>
<td>650.27</td>
<td>612.70</td>
<td>534.90</td>
<td>639.15</td>
<td>572.08</td>
<td>689.59</td>
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</tbody>
</table>

* significant at the 10 percent level ** significant at the 5 percent level *** significant at the 1 percent level

* Variance targeting was used to impose a long-run variance estimate for cases where numerical convergence was not reached under the standard GJR model
Table 5 Conditioning betas based on backward elimination (1994-2009)

Table 5 presents the selection of conditioning coefficients based on automatic backward elimination. All possible conditioning variables are first included in the model and then iteratively removed based on remaining p-values until the five most important coefficients remain.

\[
R_t - R_f = \alpha_i + \beta_{i1}(R_{mt} - R_f) + \beta_{i2}(R_{mt-1} - R_f) + \beta_{i3}^* \text{SMB} + \beta_{i4}^* \text{HML} + \beta_{i5}^* (R_{me} - R_f) + \sum \beta_{ij}(Z_{jt-1}) (R_{mt} - R_{ft}) + \beta_{ij}(Z_{jt-1}) (R_{mt-1} - R_{ft}) + \beta_{ij}(Z_{jt-1}) (R_{mt-2} - R_{ft}) + \beta_{ij}(Z_{jt-1}) * \text{SMB} + \beta_{ij}(Z_{jt-1}) * \text{HML} + \varepsilon_{it}
\]

<table>
<thead>
<tr>
<th>Model</th>
<th>Aggregate Index</th>
<th>Convertible Arbitrage</th>
<th>Dedicated Short</th>
<th>Emerging Markets</th>
<th>Market Neutral</th>
<th>Event Driven</th>
<th>Distressed Debt</th>
<th>Global Macro</th>
<th>Equity Long Short</th>
<th>Managed Futures</th>
<th>Multistrategy</th>
</tr>
</thead>
<tbody>
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<td>Z_{VIX}</td>
<td>(\beta_{i1}(Z_{t-1}))</td>
<td>-0.0012</td>
<td>-0.0135***</td>
<td>0.0168</td>
<td>-0.0219***</td>
<td>-0.0055**</td>
<td>-0.0181***</td>
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<tr>
<td></td>
<td>(\beta_{i2}(Z_{t-1}))</td>
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<td></td>
<td>0.0024</td>
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<td>(\beta_{i3}(Z_{t-1}))</td>
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<td>-0.0271</td>
<td>-0.0046**</td>
<td>0.010</td>
<td>0.0011</td>
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<td>(\beta_{i4}(Z_{t-1}))</td>
<td>0.0132***</td>
<td>-0.0274***</td>
<td>0.0110***</td>
<td>0.0063</td>
<td>0.0091**</td>
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<td>(\beta_{i5}(Z_{t-1}))</td>
<td>0.0037</td>
<td>0.0206**</td>
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<td>Z_{TERM}</td>
<td>(\beta_{i1}(Z_{t-1}))</td>
<td>-2.1868*</td>
<td>-0.2179</td>
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<td>2.2169</td>
<td>1.6666*</td>
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<td>(\beta_{i2}(Z_{t-1}))</td>
<td>-3.9382</td>
<td>6.2837**</td>
<td>6.5550*</td>
<td>-5.7848*</td>
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<td>(\beta_{i3}(Z_{t-1}))</td>
<td>1.8698</td>
<td>-6.5550*</td>
<td>-4.2778***</td>
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<td>(\beta_{i4}(Z_{t-1}))</td>
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<tr>
<td>Z_{CREDIT}</td>
<td>(\beta_{i1}(Z_{t-1}))</td>
<td>-6.3009***</td>
<td>-29.3261**</td>
<td>-8.2170***</td>
<td>-6.1331***</td>
<td>7.4756*</td>
<td>-5.7133*</td>
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<td>(\beta_{i2}(Z_{t-1}))</td>
<td>1.2534</td>
<td>-2.9273**</td>
<td>6.8795***</td>
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<td>(\beta_{i3}(Z_{t-1}))</td>
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<td>35.9217</td>
<td>-1.1553</td>
<td>0.011</td>
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<td>(\beta_{i5}(Z_{t-1}))</td>
<td>-8.5803</td>
<td>10.2497</td>
<td>4.7235*</td>
<td>4.4301**</td>
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</table>

* significant at the 10 percent level ** significant at the 5 percent level *** significant at the 1 percent level
Table 6 Out of sample optimised portfolio statistics (1998-2009)

<table>
<thead>
<tr>
<th>Hedge fund returns</th>
<th>Return (%pa)</th>
<th>Standard deviation (%pa)</th>
<th>CVaR (%)</th>
<th>Sharpe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate industry</td>
<td>7.4</td>
<td>9.3</td>
<td>-7.7</td>
<td>0.8</td>
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<tr>
<td>Mean-variance optimised portfolio</td>
<td>5.5</td>
<td>3.0</td>
<td>-0.2</td>
<td>1.9</td>
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<tr>
<td>Mean-CVaR optimised portfolio</td>
<td>5.6</td>
<td>3.1</td>
<td>-0.2</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Performance is presented net of fees in USD.
Table 7 Out of sample Jobson Korkie test results (1998-2009)

Jobson and Korkie (1981) show that the test statistic $Z$ is approximately normally distributed with a zero mean and a unit standard deviation. A significant $Z$ statistic rejects the null hypothesis of equal risk-adjusted performance. A 90 percent confidence level is suggested by an absolute $Z$-score greater than 1.645. A 95 percent confidence level starts from 1.96, while a 99 percent confidence level starts from 2.576. The table above shows we can be 99 percent confident that the mean-variance and mean-CVaR optimised portfolios exhibit higher Sharpe ratios than the aggregate hedge fund industry.

<table>
<thead>
<tr>
<th>Method</th>
<th>Z statistic</th>
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<tbody>
<tr>
<td>Mean-variance</td>
<td>4.22</td>
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<tr>
<td>Mean-CVaR</td>
<td>3.96</td>
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