Marking-to-Market, Liquidity Provision and Asset Prices*

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ABSTRACT

We propose a tractable model to analyze the effect of liquidity shocks on asset prices when banks are subject to capital adequacy requirement and mark their assets to market. We show that a negative liquidity shock can bind banks’ capital requirement and increase the volatility, price impact, and expected return of risky assets, and can even lead to a market crash and bank insolvency. We further discuss how these problems can be potentially avoided.
Many market participants have suggested that the marking-to-market rules played a significant role in the recent banking crisis. A negative liquidity shock decreases the market prices of risky assets. This affects the behavior of both liquidity providers and liquidity demanders, and in some cases an agent may even switch between these two roles. For instance, the decline in asset values may force a bank to sell its risky asset holdings to meet the regulatory capital requirement if it cannot raise additional capital. The financial institution that normally functions as liquidity provider will find itself demanding liquidity. At the same time, wealth-deprived investors can become more risk averse and unwilling to accommodate the bank’s liquidity needs. As a result, liquidity provision will likely deteriorate and even a market crash may occur in the extreme. We propose a simple model to examine these issues.

The model features risk-averse investors and a representative bank subject to a capital adequacy requirement. There are multiple risky assets and a bond available for investment. One of the risky assets reflects the price of the asset that the investors and the bank commonly hold. In the simplest form, one can think of it as asset-backed securities (ABS) or mortgage-backed securities (MBS) that are paid by the cash flows from the bank’s loans, or collateralized debt obligations (CDO) that repackage them. The commonly held risky asset is subject to exogenous supply shocks, which can result, for example, from the sale by liquidity constrained traders outside of the model. By the usual supply and demand argument, a positive supply shock will put a downward pressure on the risky asset’s price. When the bank’s assets are marked to the market, this will reduce its capital-to-asset ratio. If the capital-to-asset ratio falls below the required level, the bank must either sell the asset or raise capital to comply with the capital adequacy requirement.

We first analyze the former case of asset sales, in which the bank effectively turns to a
liquidity demander despite its role as liquidity provider during normal times. The sale of the risky asset adds to the supply shock. If the investors have enough risk-bearing capacity with sufficient risk-tolerance, their elastic demand can meet the aggregate supply, which is the sum of the supply shock and bank sales. There is no market crash, but the price impact of the supply shock is larger due to the addition of the bank sales. This makes the price more sensitive to the change in supply and hence more volatile, which also implies that the investors require a larger price discount. Thus, binding capital requirement combined with marking to market amplifies the price volatility, illiquidity, and the risk premium.

An extreme event can occur if the investors are sufficiently risk-averse so that they have only a limited risk-bearing capacity. In this case, they require too large a price discount to accommodate the aggregate supply. A positive supply shock can dislocate the equilibrium and cause a spiral of bank selling and falling price leading to a market crash. To see this, suppose the economy is in equilibrium. A small positive supply shock creates an excess supply and puts a downward pressure on the price until it falls enough to meet the concession that the investors require. However, at the new price level the bank must further reduce its risky asset position to comply with the capital requirement, adding to the supply pressure. This initiates another round of price drop, and the whole cycle repeats until the bank has no more risky asset to sell. At that point, the bank would not be able to continue without an additional, exogenous remedy such as a government bailout. The result is a market crash and a bank failure. Even if the bank obtains a government aide to avoid bankruptcy, there is still a discontinuous drop in the market price of the risky asset. All these points can be shown in an analytically tractable setting with normal payoffs and negative exponential utility that exhibits constant absolute risk aversion.

An even serious consequence can result when the risk attitude of investors changes with their
wealth. A positive supply shock can put the economy into an initial phase with deteriorated liquidity, high volatility, and low valuation. If investors become more risk averse due to deprived wealth, an additional supply shock can lead to a market crash because of investors inability to accommodate further risk. Worse, the economy looks healthy during normal times when investors are wealthier and willing to accommodate risk. We demonstrate this point using power utility investors with a wealth effect.

These issues may be avoided if banks can raise capital rather than sell the risky assets to meet the capital requirement. An increasing body of researchers proposes the use of debt or hybrid securities that can be converted into equity (French et al. (2010), Mankiw (2010), Rajan (2009)). In particular, contingent convertible bonds can limit the cost of debt while insuring banks from a deteriorating capital-to-asset ratio. Our analysis, however, implies that the contingency clause should be carefully crafted. It should kick in well before the capital requirement binds, and once in effect the bond should perhaps be automatically converted irrespective of the holder’s will.

Our work builds on the models of marking-to-market and crash in the finance and accounting literature. Cifuentes, Ferrucci, and Shin (2005) analyze the effect of liquidity risk when banks are subject to regulatory capital requirements and mark their assets to the market. Assuming an exogenously given demand curve, they show that small shocks can result in contagious failures of interconnected financial institutions. In contrast, we omit the interbank obligations and yet show the occurrence of not only a crash but also deteriorated liquidity and volatile markets. Our model also allows us to analyze the risk premium because the pricing kernel is endogenously determined and can be affected by the banks’ capital requirement. Allen and Carletti (2008) point out that mark-to-market accounting can make banks insolvent due to the
devaluation of their assets caused by liquidity shocks in the insurance sector, while historic cost
accounting can avoid that. With continuous payoffs and prices, our model is more suitable to
the analysis of crash than their discrete state model. Finally, the idea that a backward bending
demand curve can lead to a market crash goes back at least to Gennotte and Leland (1990).
They propose an asymmetric information model in which hedgers, who follow a schedule akin
to portfolio insurance, reduce their stock holdings as the stock price decreases. Like our bank,
these hedgers unload their risky asset holdings deterministically as the market price falls. In
contrast to Gennotte and Leland (1990), however, we show that a crash can occur without
information asymmetry, and as a result of seemingly bona fide bank regulation.

The rest of paper is organized as follows. The next section sets up the model and presents
the analysis. A proof for the deteriorated liquidity provision is given and numerical examples
illustrate the occurrence of a market crash. The final section concludes with suggestions for
future research.

1. Banks’ capital requirement and market liquidity

1.1 Investors

The model for the investors is standard and general. Our objective is to show that a
seemingly innocuous setting, coupled with bona fide bank regulation, can lead to deteriorated
liquidity provision and even a disastrous market crash.

Throughout the paper, we use bold letters for vectors and matrices and normal letters for
scalars. There are multiple risky securities and a riskless bond available for investment. The
risky assets pay a vector of normally distributed terminal dividends, \( \mathbf{d} \), with mean vector \( \bar{\mathbf{d}} \) and
variance-covariance matrix $\Sigma$. We take the riskless bond as the numeraire of the economy and therefore it always sells for the price of unity. The bond is in perfectly elastic supply so that the gross interest rate is fixed at $R_f$.

Investors will optimally invest in a mix of the risky assets and the bond. Let $x$ and $b$ be their holdings of the risky assets and the riskless bond, respectively. Then the investors’ terminal wealth is

$$W = d'x + R_fb.$$  

Their budget constraint is

$$p'x + b = w_0,$$

where $w_0$ is their exogenously specified endowment. Eliminating the bond position from these two equations yields

$$W = (d - R_fp)'x + R_fw_0.$$  

(1)

Investors maximize their expected utility defined over the terminal wealth,

$$\max_x E[u(W)],$$

where, as usual, $u(\cdot)$ is increasing, concave, and twice-differentiable. The first order conditions are given by

$$E[u'(W)(d - R_fp)] = 0.$$  

(2)
1.2 Banks and Equilibrium

The set-up for the bank is a stripped down version of Cifuentes, Ferrucci, and Shin (2005) in which interbank obligations are omitted. If we include such obligations, the result will be stronger. Since there is no interaction among banks, we aggregate them and consider a representative bank endowed with $e$ units of a risky asset, debt value $h$, and liquid asset value $c$. The unit price of the risky asset is marked to market at $p$, an element of the price vector $\mathbf{p}$ that satisfies Equation (2). For commercial banks subject to regulatory capital requirement, the risky asset would typically be loans, and one can think of $p$ as the market price of asset-backed securities (ABS) or mortgage-backed securities (MBS) that are paid by the cash flows from those loans, or collateralized debt obligations (CDO) that repackage such securities. For expositional purposes, we assume here that the dividend of the risky asset is uncorrelated to the other assets’. With normality, this implies that the market for the risky asset is independent from the other $K-1$ assets’, and in the analysis below we will omit them. However, it is easy to see that all assets would generally matter if the dividends were correlated and there would be a link between the loan or mortgage markets and stock markets. Under the current assumptions, the bank’s equity value is given by $pe - (h - c)$. For simplicity, we assume that the risk weight in calculating the asset value for regulatory purposes is 100% for the risky asset and 0% for the liquid asset. Let the bank’s risky asset holding be $e - s$, where $s \geq 0$ represents the sale of the risky asset. If the bank needs to sell the risky asset to comply with the regulatory capital requirement, $s > 0$ and otherwise $s = 0$. We further assume that the bank cannot short sell the risky asset, $s \leq e$. Then the risk-weighted asset value is given by $p(e - s)$, because the liquid asset does not count by assumption. Now the regulatory requirement that the bank must
comply with can be written as follows, which requires that the bank’s capital-to-asset ratio be no less than a prespecified level, ρ:

\[
\frac{pe - (h - c)}{p(e - s)} \geq ρ.
\]  (3)

We assume that the bank’s debt exceeds its liquid asset, \( h > c \), so that the capital requirement can bind as the risky asset price falls. Note that only the debt in excess of the liquid asset, \( h - c \), is relevant in determining the above capital-to-asset ratio. Assuming that the inequality is binding, solve for the bank’s sale function:

\[
s = \min \left( \max \left( 0, \frac{1}{ρ} \left[ \frac{h - c}{p} - (1 - ρ)e \right] \right), e \right),
\]  (4)

where the minimum and maximum operators ensure that \( 0 \leq s \leq e \). In equilibrium, the market for the risky asset clears:

\[
z \equiv x - s = q,
\]  (5)

where \( q \) is the supply of the risky asset, \( z \) is the aggregate excess demand function and \( x \) is the relevant element of the investors’ demand vector \( x \) that satisfies Equation (2).

1.3 Banks’ asset sale and market prices

In the subsequent sections we explore two classes of investors’ risk attitude, constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA). With an analytical solution the former is convenient to illustrate the adverse effect of the bank’s capital requirement such as deteriorated liquidity provision and market breakdown, while the latter demonstrates the
importance of changing risk aversion due to a wealth effect.

Let us start with the CARA case. Assume that investors have negative exponential utility,

\[ u(W) = -\exp(-\alpha W). \]

The derivation in the appendix shows that the solution to Equation (2) is

\[ x = \frac{1}{\alpha} \Sigma^{-1}(\overline{d} - R_f p). \]  

(6)

Our point is best presented by a numerical example. We normalize the mean dividend level at \( \overline{d} = 1 \) and assume that the interest rate is zero \( (R_f = 1) \) for simplicity. As long as the horizon is finite, a positive interest rate will not change the qualitative results to follow. This implies that the fundamental value of the risky asset is \( \overline{d}/R_f = 1 \), which is the discounted expected dividend. The dividend volatility is assumed to be 10% \( (\Sigma = 0.01) \) and the risk aversion parameter \( \alpha = 2 \). The minimum capital-to-asset ratio is set at \( \rho = 0.07 \), as recently agreed in the Basel III Accord. For illustrative purposes we set the bank’s balance sheet parameters so that it starts selling the risky asset at a price below the fundamental value of 1, say at \( p = 0.97 \). Using Equation (4), it is easy to see that one such choice is that the bank’s risky asset endowment \( e = 2.74 \) and that the debt in excess of the liquid asset \( h - c = 2.47 \). Note that \( w_0 \) is irrelevant in determining the investors’ demand as it drops from Equation (6) due to the property of the CARA utility. However, it will play a pivotal role in the CRRA case because of the wealth effect.

Figure 1 plots the investors’ demand curve, \( x \) (the blue line with empty circles), the bank’s sale function, \( s \) (the pink, kinked line with squares), and the aggregate excess demand curve,
z (the red curve with stars). The price of the risky asset is given by the intersection of the excess demand curve and a particular supply level, $q$. When there is no supply shock ($q = 0$), the price equals the fundamental value, 1. As $q$ decreases, the investors command a premium in selling the asset to meet the exogenous demand (negative supply, moving to the left), while as $q$ increases they require a price discount for bearing risk in meeting the exogenous supply (moving to the right). Meanwhile, as the price decreases, the bank’s capital-to-asset ratio worsens because the asset value in the denominator of Equation (3) shrinks faster than the equity value in the numerator. Once the capital requirement binds, the bank must either raise equity or sell the risky asset. These two remedies against a deteriorating capital-to-asset ratio can have contrasting effects on the risky asset price. If the bank can raise equity to avoid asset sales, there will be no change in excess demand and the asset price will decrease along the demand curve. However, it is unclear whether banks can do so in a timely manner amid supply pressure, and the further discussion of this issue is delayed till Section 1.5. Instead, we consider the effect of asset sales here. When the capital-to-asset ratio dips below the required level, the bank can shed some of the risky asset to support the ratio. This is so because the left hand side of Equation (3) increases in $s$. The sale, however, adds to the supply pressure and can further decrease the market price of the risky asset. Formally, consider the market clearing condition in Equation (5). The bank’s sale of the risky asset counters the investors’ demand and creates a kink at Point A on the excess demand curve as in Figure 1. Since the bank’s sale function is less price-elastic than the investors’ demand, the rate of the sale is modest and does not exceed the rate of demand change. This implies that the excess demand is still downward sloping and can meet the supply continuously, but with deteriorated liquidity provision; the slope of the excess demand curve is now steeper, implying an increased price impact. Due to the convexity
of the sale function in (4) and the linearity of the demand function in (6), the aggregate excess demand curve is concave and the price impact keeps increasing with the supply between Points A and B (albeit mildly as represented by the slight increase in the magnitude of the negative slope in the figure). When the sale equals $e$ at the right dotted vertical line, the bank has no more risky asset to sell, and cannot continue without an additional, exogenous remedy such as a government bailout. Here, we assume that a bailout infuses just enough capital into the bank to meet the capital requirement. With the bailout the bank’s sale is capped, and as the price falls below Point B, the excess demand again increases at the same rate as the investors’ original demand.

We summarize these results in the following proposition:

**PROPOSITION 1** (Bank sale) Suppose the aggregate excess demand curve is downward sloping. Then, for a given supply level, (i) the price impact of a supply shock, (ii) the expected return, and (iii) the return volatility all increase when the capital requirement binds and the bank elastically sells the risky asset to comply with it.

*Proof.* All proofs can be found in the appendix. ■

(i) should be obvious from the above analysis, and (ii) and (iii) simply follow from the lower price under the stated conditions, i.e., from the fact that the aggregate excess demand curve lies strictly below the investors’ demand curve in the relevant region (between Points A and B in Figure 1).
1.4 Market crash

When the bank’s sale function is more price-elastic than the investor’s demand curve, a different picture emerges. Figure 2 illustrates such a case. The only difference in the parameter values is that the bank’s risky asset endowment $e = 4.02$ and that the debt in excess of the liquid asset $h - c = 3.62$. That is, with increased excess debt and risky asset holding, the bank now has a less deep pocket and its capital-to-asset ratio is more sensitive to the fluctuations in market price. For a small supply shock around $q = 0$, the bank does not sell the risky asset and the story remains the same. However, once the bank starts doing so it now sells more of the risky asset than the investor can accommodate. Said differently, the rate of the bank’s asset sale exceeds the rate of the investors’ demand change. Thus, at Point A the aggregate excess demand curve becomes backward bending, and remains so until a government bailout finally occurs at Point B. From there on, the excess demand curve slopes down parallel to the investors’ demand curve. In this case what happens at Point A? When the supply shock is just large enough to induce bank selling, the price discontinuously drops from Point A to Point C. A market crash occurs, and the price will not settle until a little after the government has bailed out the ailing bank. Note that to get the economy back on track, the government needs to infuse more than enough capital to just save the bank at Point B; a slightly more capital is necessary until the price further drops and the investors’ demand recovers to meet the supply level at Point C. Loosely speaking, the economy has incurred a sort of dead-weight loss.

It is best to prevent such a crash, but once it occurs, can’t we still stop somewhere before the price falls all the way down to Point C? The answer is unfortunately no. The bank sells the risky asset till it has none to. This process is depicted in Figure 3, which plots the investors’
demand and the aggregate supply, which equals the sum of the bank’s sale and the supply shock. A slight increase in the supply shock at Point A will dislocate the equilibrium point to the right, say to the supply level at Point $A_1$, exaggerated and plotted in the middle of the figure for expositional purposes. This creates an excess supply and puts a downward pressure on price until it falls to Point $A_2$, where the price meets the concession that the investors require. However, at that price level the bank must further reduce its risky asset position to comply with the capital requirement, bringing the supply level to Point $A_3$. This initiates another round of price drop to Point $A_4$, and the whole cycle repeats until the equilibrium reaches Point C where the demand and the supply meet again. While we started this process with a positive supply shock, it is easy to see that a downward revision in the price, rather than an increase in the supply, can also lead to the same consequence, as pointed out by Cifuentes, Ferrucci, and Shin (2005, Figure 1). Thus, Point A is stable to neither quantity nor price tatonnement. This leads to the following proposition:

PROPOSITION 2 (Crash) Consider a price decrease caused by a positive supply shock. If the bank sells the risky asset by more than is demanded by the investors, a discontinuous drop in the market price occurs.

We are not the first to point out the possible occurrence of a market crash due to a backward-bending demand curve. For example, Gennotte and Leland (1990) propose an asymmetric information model in which hedgers, who follow a schedule akin to portfolio insurance, reduce their stock holdings as the stock price decreases. This can make the aggregate excess demand curve backward bending and lead to a market crash. Like our bank, these hedgers unload their risky asset holdings deterministically as the market price falls. In contrast to Gennotte and
Leland (1990), however, we show that a crash can occur without information asymmetry; unlike their rational expectations equilibrium model our model features no uninformed investors who infer information from the market price, and the agents’ expectations play a minimal role here.

1.5 **Wealth effect and contingent-convertible debt**

The above examples illustrate cases with either an increased price impact without a crash or a crash without an increased price impact. However, in the real market a market crash is typically preceded by periods of deteriorated liquidity provision. We now illustrate such an example. This is possible with the CARA utility in principle, but it is best demonstrated by a power utility with a wealth effect. Since there is no analytical solution, Equation (2) is solved numerically using a technique known as Gaussian quadrature to compute the expected value.¹

Figure 4 shows two examples with different levels of initial wealth: \( w_0 = 1 \) in Panel A and \( w_0 = 3 \) in Panel B. The parameter values remain the same except for \( e = 2.49 \) and \( h - c = 2.25 \).

In the low initial-wealth case (Panel A), as the supply shock increases the bank starts selling its risky asset at Point A. Past that point, the price impact increases as indicated by a steeper excess demand curve between Points A and D. Then the crash occurs at its endpoint and the price drops to Point C. Again, Point C is feasible only after a government bailout has occurred at the price level corresponding to Point B *and* the government infuses some more capital into the bank.

Panel B demonstrates that a crash can be avoided with high initial wealth. With power utility, investors’ risk-bearing capacity increases with wealth (in the sense of decreasing absolute

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¹A quadrature rule approximates a definite integral by a weighted sum of the function values at specific nodes. Gaussian quadrature chooses \( n \) nodes designed to produce an exact result for polynomials of order \( 2n - 1 \) or less. When the weights are points on a probability density function, this yields the expected value of the function of a random variable. In our numerical exercise, we generate seven Gaussian quadrature nodes and weights \( (n = 7) \) for a univariate normal random variable, \( d \sim N(\bar{d}, \Sigma) \), using Miranda and Fackler’s (2002, p.96) Matlab code.
risk aversion), and they will command a lower price discount for a given level of supply (note that the two panels are drawn on the same scale of the horizontal axis). Thus, while the bank starts shedding the risky asset at Point A and keeps doing so until it exhausts its risky asset holding and calls for the government bailout at Point B, the excess demand curve is nowhere backward bending and a crash does not occur.

While this might sound encouraging, our intention to show this example is indeed the opposite. An apparently healthy bank (as in Panel B) can suddenly find it short of capital during bad times when investors become poorer and effectively more risk-averse (Panel A); note that the only difference between the two panels is the investors' wealth. Clearly, the bank should not rely on a costly government bailout to insure itself against possible bankruptcy in an economic downturn. Raising capital would be better, but it is unclear whether the bank can issue equity during periods of supply pressure in a timely manner. One way to address this issue is for banks to raise debt or hybrid securities that can be converted into equity. In particular, contingent convertible (CoCo) bonds can limit the cost of debt while insuring banks from a deteriorating capital-to-asset ratio (French et al. (2010), Mankiw (2010), Rajan (2009)). Our analysis, however, implies that the contingency clause should be carefully crafted. It should kick in well before the capital requirement binds, and once in effect the bond should perhaps be automatically converted irrespective of the holder's will. For example, the Basel III Accord includes a 2.5% buffer by which a bank's capital-to-asset ratio can dip below the 7% threshold. A bank with the capital ratio within this buffer may be required to retain more earnings than it could pay out as dividends or executive compensation (The Wall Street Journal, September 13, 2010). Such a buffer should not be used to justify a contingency clause that will not trigger, say, until the capital-to-asset ratio falls to 6%.
2. Conclusion and Future Agenda

The recent turmoil in financial markets generated discussions and proposals on how to regulate financial institutions. While intuitive, these debates often lack a quantitative tool to back their analysis. The current paper is an attempt to offer one. A negative liquidity shock decreases the market prices of risky assets and hence can put a bank in breach of regulatory capital requirement. If the bank indeed must sell the risky assets to comply with the capital requirement, it demands, rather than provides, the liquidity. At the same time, investors are also deprived of their wealth and likely have only limited risk-taking ability. This leads to an increased price impact, volatility, and a higher risk premium. When the investors cannot accommodate the bank’s liquidity needs, a market crash can occur. We have developed a model that demonstrates these points. Perhaps the most surprising is the model’s simplicity: a plain standard setting with power or CARA utility investors can produce a market crash when combined with the bank’s capital adequacy requirement in the presence of marking to market.

This preliminary draft leaves many interesting research agenda for future revision. First, we have not endogenized the bank’s behavior. Currently, it mechanically follows the capital requirement and does nothing else. If banks are indeed liquidity providers in normal times, does that affect its ability to bear negative liquidity shocks during a crisis? This raises a question of what the bank’s objective function is during non-crisis periods. Second, the current single-period model addresses the liquidity provision and risk premium that are concurrent with binding capital requirement. If the model is extended to a multi-period setting, can the possibility of the bank’s future asset sale affect the current liquidity and premium? Third, we have assumed that the asset which the bank and the investors commonly hold is independent
of other risky assets. If their payoffs or liquidity shocks are correlated, would a liquidity shock of one asset cause deteriorated liquidity or crashes in the markets of others? Forth, we have abstracted from the details of the convertible bond to avoid the bank’s asset sale. What contingency conditions will best prevent undesirable market outcomes?

Finally but not least importantly, what are the alternatives to mark banks’ assets? A natural first alternative would be to simply use book values. We can define book values as the unconditional expected value of the the risky assets’ dividends. A second alternative is to use signals about the future dividends along with market prices to calculate the banks’ risk-weighted assets. This is equivalent to marking to market when investors condition their trades on market prices as in noisy rational expectations equilibrium models. We expect that these procedures to mark banks’ book will have different welfare implications for investors. For instance, using book values can give market makers incentives to partially ignore signals about future dividends and to provide excessive liquidity at the cost of increased likelihood of default. Signal-based marking-to-market, on the other hand, can decrease the likelihood of market makers’ default at the expense of curbing the ability of investors to hedge their idiosyncratic shocks.
Appendix: Proofs

Derivation of Equation (6). Rewrite (2) using the covariance formula:
\[
cov(u'(W), d) + E[u'(W)](\bar{d} - R_fp) = 0.
\]
Thus, generally the price vector is given by
\[
p = \frac{1}{R_f} \left[ \bar{d} + \frac{cov(u'(W), d)}{E[u'(W)]} \right].
\]
Under normality, by Stein's lemma we have \(cov(u'(W), d) = E[u''(W)]cov(W, d)\). Therefore,
\[
p = \frac{1}{R_f} \left[ \bar{d} + \frac{E[u''(W)]}{E[u'(W)]} \Sigma x \right].
\]
Further for a CARA utility, \(\alpha = -u''(W)/u'(W)\) (the Arrow-Pratt measure of absolute risk aversion), which yields Equation (6) upon substitution. ■

Proof of Proposition 2. When the bank sells the risky asset by more than is demanded by the investors, \(\partial s/\partial p < \partial x/\partial p < 0\) and therefore \(\partial z/\partial p = \partial x/\partial p - \partial s/\partial p > 0\). This implies that \(\partial p/\partial z > 0\), i.e., the aggregate excess demand function is backward bending. By the mechanism described in the main text, a discontinuous drop in the market price occurs at the smallest such supply level. ■

Proof of Proposition 1. When the bank elastically sells the risky asset, strictly \(\partial s/\partial p < 0\) from Equation (3). Thus, \(\partial x/\partial p < \partial x/\partial p - \partial s/\partial p = \partial z/\partial p < 0\), where the last inequality follows from the assumption that the aggregate excess demand curve is downward sloping. Taking the reciprocal and then the absolute value yields \(|\partial p/\partial z| > |\partial p/\partial x|\). This proves (i). Next, assuming that the price is positive, the expected return on the risky asset is \(\bar{d}/z^{-1}(\bar{q})\) when the bank elastically sells the risky asset and is otherwise \(\bar{d}/x^{-1}(\bar{q})\), where \(z^{-1}(\bar{q}) = p\) denotes the inverse aggregate demand function and \(x^{-1}(\bar{q}) = p\) the investors’ inverse demand function. Since \(s(p) > 0\), we have \(z(p) = x(p) - s(p) < x(p)\), i.e., the graph of \(q = z(p)\) lies below that of \(q = x(p)\). This implies that \(z^{-1}(\bar{q}) < x^{-1}(\bar{q})\) and hence \(\bar{d}/z^{-1}(\bar{q}) > \bar{d}/x^{-1}(\bar{q})\). Similarly, the return volatility with and without capital requirement satisfies the relation \(\Sigma/(z^{-1}(\bar{q}))^2 > \Sigma/(x^{-1}(\bar{q}))^2\). ■
References


Figure 1: Increased price impact. Investors possess CARA utility, \( u(W) = -\exp(-\alpha W) \). The figure plots the inverse functions of the investors’ demand \( x(p) \) in Equation (6), the bank’s sale \( s(p) \) in Equation (4), and the aggregate excess demand \( z(p) = x(p) - s(p) \). The right vertical dotted line represents \( q = e \). Parameter values: \( \bar{d} = 1, \Sigma = 0.01, R = 1, \alpha = 2, \rho = 0.07, e = 2.74, \) and \( h - c = 2.47 \).
Figure 2: A market crash. Investors possess CARA utility, $u(W) = -\exp(-\alpha W)$. The figure plots the inverse functions of the investors’ demand $x(p)$ in Equation (6), the bank’s sale $s(p)$ in Equation (4), and the aggregate excess demand $z(p) = x(p) - s(p)$. The right vertical dotted line represents $q = e$. Parameter values: $\alpha = 1$, $\Sigma = 0.01$, $R = 1$, $\alpha = 2$, $\rho = 0.07$, $e = 4.02$, and $h - e = 3.62$. 
Figure 3: Market crash. See the caption to Figure 2 for parameter values.
Figure 4: Wealth effect. Investors possess power utility, $u(W) = W^{1-\alpha}/(1 - \alpha)$. The figure plots the inverse functions of the investors’ demand $x(p)$, the bank’s sale $s(p)$ in Equation (4), and the aggregate excess demand $z(p) = x(p) - s(p)$, numerically solved to satisfy Equations (2) and (5). The right vertical dotted line represents $q = e$. Parameter values: $\bar{d} = 1$, $\Sigma = 0.01$, $R = 1$, $\alpha = 2$, $\rho = 0.07$, $e = 2.49$, and $h - c = 2.25$. Panel A: $w_0 = 1$, Panel B: $w_0 = 3$. 