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The return-volatility relation in commodity futures markets

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Abstract

By employing a continuous time stochastic volatility model, we analyse the dynamic relation between price returns and volatility changes in the commodity futures markets. We use an extensive daily database of gold and crude oil futures and futures options to estimate the model that is well suited to assess the return–volatility relation for the entire term structure of futures prices. Our empirical results indicate a positive relation in the gold futures market and a negative relation in the crude oil futures market, especially over periods of high volatility principally driven by market-wide shocks. However, the opposite reaction occurs over quiet volatility periods when typically commodity-specific effects dominate. As leverage effect and volatility feedback effect do not adequately explain this reaction especially for the crude oil futures, we propose the convenience yield effect. We demonstrate that commodity futures markets in normal backwardation entail a positive relation, while futures markets in contango entail a negative relation.

Keywords: Return-volatility relation; Commodity futures returns; Gold futures volatility; Crude oil futures volatility; Contango; Backwardation
JEL: G13, E32, Q40

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1. Introduction

Over the last decade, commodity markets have attracted increasing attention due to the significant volatility that has occurred in these traditionally tranquil markets. Additionally, the performance of the equity markets has been disappointing and due to the historically low correlation between equity markets and commodity markets, the commodity markets have become a viable alternative to investors. Consequently, futures commodity markets have become very liquid and far more volatile. To illustrate this, the average open interest of the December crude oil futures was 12,438 contracts in 1995 and has more than tripled to 42,287 contracts in 2010 while it has doubled for the gold futures markets (from 6,533 contracts in 1995 to 11,447 in 2010). Furthermore, as futures prices emerge with greater transparency (compared to spot commodity prices, which depend heavily on grade and location), futures prices could potentially provide a better understanding of what drives the dynamics of prices in commodity markets and their prevailing volatility. This is of critical importance for practical applications related to trading and pricing commodity derivatives that play an important role in investment management and risk management. Subsequently, a comprehensive understanding of the relation between returns and volatility changes in these markets becomes equally important.

Asymmetric volatility is a well known empirical phenomenon in the equity markets describing the negative relation between stock returns and stock return (conditional) volatility. More specifically, negative (positive) return shocks are correlated with positive (negative) volatility shocks with the effect being more robust during periods of market crashes where high volatility is combined with low returns. Extensive literature has been dedicated to explain this relation. The two main accounts put forward are the leverage effect postulated by [8] and the volatility feedback effect proposed by [11], while more recently, a new explanation of the behavioral effect has been introduced by [33].[1] Most of the existing literature deliberates on which effects best explains asymmetric volatility including [4], [49] and [18].

Much less attention has been given to the investigation of the relation between returns and volatility changes in the commodity markets and in particular the commodity futures markets. [25] study the return–volatility relation for commodity indexes and [48] fit an asymmetric power GARCH model in the gold market. [2] fits the asymmetric volatility specification introduced by [26] to the gold market and uses the safe haven property of gold to explain the prevailing positive return–volatility relation. The empirical phenomenon of a positive return–volatility relation is referred to as *inverted asymmetric* volatility. [30] has found an asymmetric volatility reaction to news using a GARCH model in the crude oil market. [43] found that leverage effects in crude oil markets postulate asymmetric volatility (namely bad news in the oil market has the potential of increasing volatility in the oil price than does good news). However all these studies are based on GARCH models and

[1] The leverage effect implies that negative stock return shocks cause an increase in volatility due to the increase in the firm's leverage. A similar reaction in the commodity market is also termed as leverage effect, where negative futures (or spot) return shocks tend to increase volatility. The volatility feedback effect suggests that any change in volatility, but in particular an increase in volatility, will decrease stock returns. Alternatively based on the behavioral concepts of representativeness, affect and extrapolation bias, individuals prefer positions of high return and low risk as these represent a good investment.

investigate the relation in spot commodity markets.

We provide a comprehensive qualitative and quantitative analysis of the return–volatility relation in commodity futures markets under different perspectives. Firstly, we propose a continuous time stochastic volatility model to assess and measure the volatility of the commodity futures markets. We employ a stochastic volatility model within the [31] framework that accommodates multiple volatility factors with flexible volatility term structures ranging from exponential decay to hump-shaped. In addition, the volatility features the empirically observed characteristics of unspanned components from commodity futures (see [47]) and asymmetric responses to news. These two features can be captured by the correlation between the innovations of the futures price returns and the innovations of its volatility. To the best of our knowledge, there has been no research work that studies the return–volatility relation in the futures markets with this type of model and from this perspective. Limited literature exists mainly on futures with short maturities, see [48]. Typically when the entire term structure of futures prices is modelled via stochastic volatility forward models, it is difficult to obtain tractable solutions for derivative prices and the estimation of such models is not always trivial. Consequently, most of the literature uses econometric type of models (GARCH) to assess the asymmetric reaction of volatility. However these econometric models preserve the limitation of allowing only for V or U shaped reactions (as they restrict volatility to be a deterministic function of the return shock) implying that both positive and negative shocks in volatility will decrease asset returns. This contradicts empirical evidence of monotonically decreasing reaction in equity markets, see [50].

Our second contribution relies on the estimated volatility measure used in our analysis. The proposed stochastic volatility model possess finite-dimensional affine realizations for the commodity futures price and quasi-analytical prices for options on commodity futures. Subsequently, the model is estimated by using both futures prices as well as options prices. By selecting purposely the most active commodities markets, namely gold and crude oil, we use an extensive database of daily futures and option prices extending to 31 years for gold and 21 years for crude oil. Thus, our estimated models integrate information from both futures prices and options prices and guarantee a better fit to the observable futures term structures as well as the prevailing (implied) volatility term structure, see [47] for a demonstration in the crude oil futures market. Existing literature on the return–volatility relation, estimates volatility by using either sample return variances or implied volatilities where either commodity futures prices or option prices are taken into account but not both.

Thirdly, we undertake an empirical analysis of the stochastic volatility models in the gold futures and crude oil futures markets for several reasons. Both markets are amongst the most liquid commodity derivatives markets that impact, and are influenced by, macroeconomic and financial conditions, as they are used widely for investment purposes and/or hedging and speculation. Gold is classified as an investment commodity whereas crude oil as a consumption commodity thus we have the opportunity to gauge any noteworthy differences between these two markets. As these two markets are principally different, our study provides insightful findings of the nature of the return–volatility relation in these two distinct futures markets over different market conditions related to volatility levels. To enhance the analysis and verify the significance of different volatility market conditions, we estimate our models over the whole period (as it has been done in most literature) as well as over different periods differentiated by volatility intensity.

Fourthly, we not only identify the nature of the return-volatility relation but we also provide an explanation of the results. By using long-established theories and hypotheses such as the volatility feedback effect, the safe haven property and the theory of storage, we justify our results. A new account also emerges from our analysis, the so called *convenience yield effect* that plays a critical role in explaining the return-volatility relation, especially in the crude oil futures markets. Based on our results on crude oil, one of our key findings is that consumption commodity markets in normal backwardation are characterised by inverted asymmetric volatility, while commodity markets in contango are characterised by asymmetric volatility. Thus the return-volatility relation is connected to the convenience yield of the commodity.

Additionally by selecting the models that best fit futures prices and options prices, we consider a one-factor stochastic volatility model with exponential decay volatility specifications for the gold and a two-factor stochastic volatility model with hump-shaped specifications for the crude oil. Our empirical results indicate that during volatile periods driven by market-wide shocks, the gold futures volatility is inverted asymmetric (implying a positive relation) as explained by the safe haven property of gold, while the crude oil futures volatility is asymmetric (implying a negative relation) as explained by the convenience yield effect. However, during less volatile market conditions, commodity-specific effects dominate, causing a negative relation in the gold futures market and a positive relation in the crude oil futures market. Thus when markets are quiet, gold futures respond similarly to other financial assets like equities, while crude oil preserves a typically high convenience yield that stimulates a positive relation.

We further illustrate that for these two commodity futures markets, when the market uncertainty is high, then market wide shock effects dominate, while when the market is quiet then commodity-specific shock effects dominate. This is consistent with the finding of [18] who have shown that asymmetric (implied) equity volatility is primarily attributed to systematic market-wide factors rather than aggregated firm-level effects. We also confirm that the relation is consistent with the option-derived implied volatility skew.

The paper is organized as follows. Section 2 discusses the relation of futures returns and volatility for the two distinct type of commodities markets, namely the investment commodities and the consumption commodities. Section 3 presents a generalised HJM stochastic volatility model for modelling volatility of commodity markets and explains the method employed to estimate the model. Section 4 describes and analyzes the data of gold and crude oil derivatives and justifies the model choice. Section 5 presents the estimation results of the stochastic volatility models and reflections on the return-volatility relation. Section 6 concludes. Technical details are presented in the Appendix.

2. The relation between price returns and volatility in commodity futures markets

This section considers the characteristics of commodity futures markets in terms of the differences between investment commodities and consumption commodities. Additionally, we discuss commodity specific effects and market-wide shock effects on the return-volatility relation for these two types of commodity futures markets that we analyse further in Section 5 for the gold futures market and the crude oil futures market.

2.1. *Commodity futures markets characteristics*

For the purpose of studying futures prices in commodities markets one has first to distinguish the analysis between investment commodities and consumption commodities. Investment commodities are treated as investment assets, although they involve storage costs like most commodities do. However, ownership of physical consumption commodities provides benefits, which are referred to as the convenience yield, that are not obtained by holders of futures contracts on these commodities. Physical hedgers access the futures markets to reduce the price risk of their positions in the underlying physical commodity, while speculators trade in the markets in the hope of profits. Consequently short positions in futures contracts typically exceed their demand, thus futures prices are trading at lower levels than spot prices (markets are typically inverted).

According to the Keynes's Theory of Normal Backwardation ([44] and [34]), the futures prices are set at a discount to the future spot prices at maturity to reward risk-averse speculators for taking long positions against hedgers. However major international commodity markets such as gold and crude oil have been experiencing excessive price volatility that cannot be solely explained by these commodity driven effects. Recently, some commodity markets, such as crude oil, were in contango. This reverse flow of risk premium may be the result of financial investors seeking portfolio diversification ([28]) or speculative behaviour (as contango is generally associated with a high level of volatility, see [27], or implying a structural shift in inventory management, see [38]). Like financial markets, consumption commodity markets have became increasingly dependent on market manipulation practices. In the second half of 2008, a sharp reduction of financial investment in commodity markets led futures prices and then spot prices to fall significantly. [22] find that maroeconomic news has a swift and significant impact on prices, realized volatility and the volume of metal futures (gold, silver and copper). [38] debates whether macroeconomic shocks have been the main real oil price driver since mid-1980s, financial shocks may had a sizable contribution since the early 2000s, and the macro-finance shocks largely account for the 2007-2008 oil price swing. In the equity market, [18] empirically demonstrates that systematic risk factors become more influential during high volatility regimes.

Consequently, we argue that the relation between futures price returns and volatility should be explained by two main sources, namely, commodity-specific effects and market-wide shock effects. These two effects impact differently on the two distinctive commodities markets, namely, investment commodities and consumption commodities.

2.2. *Commodity-Specific Effects*

For each commodity, the spot as well as the futures prices are determined by regular supply and demand forces driven by commodity specific fundamentals such as inventories, production and consumption.[2]

- Investment commodities
  Gold and silver are monetary metals, consequently they are treated by the majority

[2]Increasing liquidity attributed to the price discovery in futures markets has the effect of relating positive returns with decreasing volatility (the liquidity effect). Note that non-fundamental changes to futures prices typically are transmitted to spot prices even without inventories adjustments, see [38].

of the market participants as financial assets. In the leading financial market, the equity market, there is compelling empirical evidence that volatility feedback effect and/or leverage effect typically generates an *asymmetric* volatility reaction. For investment commodities accordingly, with an increase in conditional volatility, the risk of the underlying commodity increases, leading to lower commodity price return. As the spot commodity price returns are historically positively correlated to the futures price returns (for instance for gold, the correlation coefficient between the 12-month futures returns and 1-month futures returns over thirty years is 0.991), this effect will be transmitted to the futures prices, implying that increasing volatility is associated with negative futures price returns. Additionally, if a volatility shock is anticipated then futures traders are not willing to trade, thus prices drop to balance buying and selling volumes. Thus negative futures return shocks are associated with increasing volatility. Overall due to commodity-specific effects, for investment commodities, a negative relation between futures price returns and volatility should be pertinent.

- Consumption commodities
  For consumption commodities, such as crude oil, the supply and demand forces primarily affect the market via inventory. Inventory and convenience yield, according to the Theory of Storage (see [35]), are negatively correlated. If inventory is low (among other reasons due to a shortage in the commodity), the convenience yield is high and causes spot prices to trade higher than the futures prices. [40] also explains that as volatility increases, the convenience yield increases as a result of an increasing demand for storage; market participants will increase their inventories in order to absorb the anticipated shocks in production and consumption, while at the same time, the commodity spot price ascends more than futures prices. In addition, the higher the convenience yield, the stronger the pressure for a rise in both spot and futures commodity prices is anticipated. The volatility of the spot price returns, the volatility of the futures price returns and the volatility of the convenience yield are also increasing. Thus positive futures return shocks are associated with increasing volatility. However, if inventory is high, the convenience yield is relatively low and commodity prices tend to decrease and the volatility is getting lower, reflecting the decreasing risk of the exhaustion of inventories. Hence negative return shocks would signal lower future volatility. Overall due to commodity-specific effects, for consumption commodities, a positive relation between futures price returns and volatility should be pertinent implying an *inverted asymmetric* volatility reaction. Considering the example of the crude oil market in the 90's, we detect that the crude oil futures market was mostly in backwardation (77.39% of the time in weak backwardation[3]) that consequently implied a high convenience yield. As we will confirm in Section 5.2, the crude oil futures market featured an *inverted asymmetric* volatility in the 90's.

[3]Weak backwardation is defined as the case of "discounted" futures prices being below spot commodity prices. We have computed the percentage of weak backwardation for the 13-month futures contracts.

2.3. Market-wide shock effects

Apart from the regular supply and demand forces, commodity derivative prices, returns and volatility are also affected by market-wide shock factors such as investment growth, interest rates, exchange rates, market contractions and weather.

- Investment commodities
  Most investment commodities, such as gold, have the property of a *safe haven* investment, i.e., investors turn to it during periods of uncertainty driven by market-wide shocks, see [3] and [41]. Positive commodity price changes that are principally associated with safe haven purchases, are signals for increasing risk or uncertainty in macroeconomic and financial conditions. This introduces uncertainty in the market, thus increasing volatility. This effect is consistent with the empirical studies of [25] and [2] in the gold market. Subsequently, as futures commodity prices are historically positively correlated with the spot commodity prices, the same reaction would be anticipated in the gold futures markets. Apart of the safe haven property, according to the *Market Pressure Theory*, ([16]) while futures prices increase, large long speculative trading activity in futures markets would be taken that will lead to further futures price increases enforcing a positive return-volatility relation. Additionally, the connection between inventory and volatility (as low inventory signals high future volatility) can also potentially produce *inverted asymmetries* in gold futures volatility.

- Consumption commodities
  The impact of severe market-wide shocks in the futures prices of consumption commodities and consequently the return–volatility relation is not always definite and straightforward. To illustrate this, we present two examples of consumption commodities with opposite reactions to extreme market conditions. Firstly, we consider the crude oil market. In the last decade, as a result of the 9/11 terrorist attacks, the US invasion to Iraq and Global Financial Crisis (GFC), see [36] and [45], the crude oil market has experienced excessive volatility, not only in terms of volatility of spot price and futures price returns but also in terms of volatility of adjusted spreads (adjusted spreads are considered as a measure of convenience yield, see [24]) and implied volatility. The 2008 oil bubble was attributed to the increasing oil demand amidst stagnant oil production to meet the strong global economic growth that occurred up to 2008, see [37], as well as increasing speculative trading activity. These factors have been causing oil prices to rise until mid of 2008 where intense economic contraction led by the GFC caused oil prices to plunge by 80% between July 2008 to December 2008. While the increasing volatility was predominantly attributed to economic contractions that impacted negatively the demand for crude oil, that led to a noticeable decrease of the spot and futures commodity prices (as observed in the second half of 2008) and a considerable decrease of the convenience yield.[4] The volatility of the convenience

[4] This is not consistent with the fundamentals of consumption commodities as presented in Pindyck (2001) for example, where under increasingly volatile market conditions, the convenience yield increases as a result of an increasing demand for storage.

yield though remained high enforcing an increase in the riskiness of the crude oil futures market that was associated with an immediate decrease in the crude oil futures price. This implies a negative return-volatility relation thus an *asymmetric volatility* response. Major consumption commodity markets were in *contango*[5] and more specifically, the crude oil market underwent an extended period of contango over the last four years following from the economic contraction of the GFC.[6] The second example is the sugar market. The sugar market has experienced extensive volatility over the last five years. The fall of sugar production in 2009–2010 due to poor weather conditions in major production areas combined with the scarcity of investment capital to increase production as a result of the GFC and an increasing demand for sugar from food industries has caused sugar spot prices to soar. This increase was also transmitted to sugar futures prices. As a result the return–volatility relation for sugar has been mostly positive implying an *inverted asymmetric volatility* reaction. Furthermore, the commodity supply shortage (that postulates a strong convenience yield) had led the sugar market into backwardation.

In light of the above observations, we introduce a new effect that has an explanatory power on the return–volatility relation in futures commodity markets that we refer to as the *convenience yield effect* and it is more relevant to consumption commodities. According to this effect, as convenience yield increases (decreases) implying that the commodity market goes into backwardation (contango) mode, then the increasing volatility associated with the market-wide shock effects that impact on the convenience yield will lead to increasing (decreasing) futures returns. Thus commodity futures markets in normal backwardation are characterized by inverted asymmetric volatility, while commodity futures markets that are in contango are characterized by asymmetric volatility. We demonstrate in Section 5.2 that this effect can explain the return-volatility relation especially in the crude oil market.

Following [18], we assume that the volatility can be decomposed into two main components (similarly to the returns); one component could be postulated by systematic market-wide shocks while the second one could be controlled by commodity-specific shocks. In Section 5, we demonstrate that for the two commodity futures markets, namely the gold market and the crude oil market, under high market uncertainty, the market wide shock effects dominate while when the market is quiet, the commodity-specific shock effects are more influential.

3. Model and Method

3.1. A stochastic volatility model for commodity futures prices

We introduce $\mathbf{V} = \{\mathbf{V_t}, t \in [0,T], \}$, a generic stochastic volatility process modelling the uncertainty in the commodity futures market. We denote as $F(t, T, \mathbf{V_t})$, the futures price of

[5]As the convenience yield reflects the flow of services that accrues to an owner of the physical commodity but not to an owner of a contract for future delivery of the commodity, see [9], it is apparent that the benefits of owning crude oil were significantly reduced due to the GFC economic contraction. Thus the considerably lower (mostly negative) convenience yield is reflected by the persistence of the contango in these markets.

[6]It is worth noticing that the Asian financial crisis in 1997-1998 caused a similar extended period of contango that lasted for approximately one and half years.

the commodity at time $t \geq 0$, for delivery at time T, (for all maturities $T \geq t$). The spot price at time t of the underlying commodity, denoted as $S(t, \mathbf{V_t})$ satisfies $S(t, \mathbf{V_t}) = F(t, t, \mathbf{V_t})$, $t \in [0, T]$. We assume that the commodity futures prices are driven by an $n-$dimensional Wiener process $W(t) = \{W_1(t), \ldots, W_n(t)\}$ and the stochastic volatility process $\mathbf{V}_t$, is driven by the $n-$dimensional Wiener process $W^V(t) = \{W_1^V(t), \ldots, W_n^V(t)\}$, for all $t \in [0, T]$.[7] It is well known that the commodity futures price process can be described by a driftless stochastic process under a risk-neutral probability measure Q, since the futures price process is equal to the expected future commodity spot price under this measure, see [20]. Thus we assume that the risk-neutral dynamics of the futures prices are

$$\frac{dF(t, T, \mathbf{V_t})}{F(t, T, \mathbf{V_t})} = \sum_{i=1}^{n} (\kappa_{0i} + \kappa_i (T - t)) e^{-\eta_i (T-t)} \sqrt{\mathbf{V_t}^i} dW_i(t), \tag{1}$$

where κ_{0i}, κ_i and η_i are constants. The volatility process $\mathbf{V_t} = \{\mathbf{V_t^1}, \ldots, \mathbf{V_t^n}\}$ is an $n-$dimensional, [32] type process such that

$$d\mathbf{V_t}^i = \mu_i^V (\nu_i^V - \mathbf{V_t}^i) dt + \varepsilon_i^V \sqrt{\mathbf{V_t}^i} dW_i^V(t), \tag{2}$$

where μ_i^V, ν_i^V, and ε_i^V are constants (they can also be deterministic functions of time) and

$$\mathbb{E}^Q[dW_i(t) \cdot dW_j^V(t)] = \begin{cases} \rho_i dt, & i = j; \\ 0, & i \neq j. \end{cases} \tag{3}$$

By considering $n-$dimensional independent Wiener processes $W^1(t) = W(t)$ and $W^2(t)$, the system (1) and (2) can be expressed in terms of these independent Wiener processes as[8]

$$\frac{dF(t, T, \mathbf{V_t})}{F(t, T, \mathbf{V_t})} = \sum_{i=1}^{n} (\kappa_{0i} + \kappa_i (T - t)) e^{-\eta_i (T-t)} \sqrt{\mathbf{V_t}^i} dW_i^1(t), \tag{4}$$

$$d\mathbf{V_t}^i = \mu_i^V (\nu_i^V - \mathbf{V_t}^i) dt + \varepsilon_i^V \sqrt{\mathbf{V_t}^i} \left(\rho_i dW_i^1(t) + \sqrt{1 - \rho_i^2} dW_i^2(t) \right). \tag{5}$$

These model specifications allow for a variety of shapes for the volatility structure of futures prices, including the exponentially declining stochastic volatility structures and hump-shaped volatility structures. There is an extensive literature that has shown empirical evidence that these are typical volatility structures of interest rate market volatility, see [42], [13] and [46], and of commodity volatility, see [47] and [12].

[7] We consider a filtered probability space $(\Omega, \mathcal{A}_T, \underline{\mathcal{A}}, P)$, $T \in (0, \infty)$ with $\underline{\mathcal{A}} = (\mathcal{A}_t)_{t \in [0,T]}$, and we assume that the filtration $\mathcal{A}_t$ includes $\mathcal{A}_t = \mathcal{A}_t^f \vee \mathcal{A}_t^V$, where

$$(\mathcal{A}_t^f)_{t \geq 0} = \{\sigma(W(s) : 0 \leq s \leq t)\}_{t \geq 0},$$
$$(\mathcal{A}_t^V)_{t \geq 0} = \{\sigma(W^V(s) : 0 \leq s \leq t)\}_{t \geq 0},$$

and σ denotes a sigma algebra.

[8] Equation (4) presents one possible representation.

The correlation structure of the innovations determines the extent to which the stochastic volatility is unspanned. If the Wiener processes $W_i(t)$ are uncorrelated with $W_i^V(t)$ then the volatility risk is unhedgeable by futures contracts. When the Wiener processes $W_i(t)$ are correlated with $W_i^V(t)$, then the volatility risk can be partially spanned by the futures contracts implying that the volatility risk (and consequently options on futures contracts) cannot be completely hedged by using only futures contracts.

The commodity forward model (4) and (5) admits finite dimensional realisations, see [14] and [6].

Proposition 1. *The instantaneous futures prices $F(t,T,\mathbf{V_t})$ are expressed in terms of $6n$ state variables as*

$$F(t,T,\mathbf{V_t}) = F(0,T,V_0)\exp\{-Z(t,T)\} \tag{6}$$

$$Z(t,T) = \sum_{i=1}^{n}\Big(\frac{1}{2}\left(\gamma_{i1}(T-t)x_i(t)+\gamma_{i2}(T-t)y_i(t)+\gamma_{i3}(T-t)z_i(t)\right)+\left(\beta_{i1}(T-t)\phi_i(t)+\beta_{i2}(T-t)\psi_i(t)\right)\Big), \tag{7}$$

where for $i=1,2,\dots,n$

$$\begin{aligned}
\beta_{i1}(T-t) &= (\kappa_{0i}+\kappa_i(T-t))e^{-\eta_i(T-t)},\\
\beta_{i2}(T-t) &= \kappa_i e^{-\eta_i(T-t)},\\
\gamma_{i1}(T-t) &= \beta_{i1}(T-t)^2,\\
\gamma_{i2}(T-t) &= 2\beta_{i1}(T-t)\beta_{i2}(T-t),\\
\gamma_{i3}(T-t) &= \beta_{i2}(T-t)^2.
\end{aligned} \tag{8}$$

The state variables $x_i(t), y_i(t), z_i(t), \phi_i(t)$ and $\psi_i(t)$, $i=1,2,\dots,n$ evolve according to

$$\begin{aligned}
dx_i(t) &= (-2\eta_i x_i(t)+\mathbf{V_t}^i)dt,\\
dy_i(t) &= (-2\eta_i y_i(t)+x_i(t))dt,\\
dz_i(t) &= (-2\eta_i z_i(t)+2y_i(t))dt,\\
d\phi_i(t) &= -\eta_i\phi_i(t)dt+\sqrt{\mathbf{V_t}^i}dW_i(t),\\
d\psi_i(t) &= (-\eta_i\psi_i(t)+\phi_i(t))dt,
\end{aligned} \tag{9}$$

subject to $x_i(0)=y_i(0)=z_i(0)=\phi_i(0)=\psi_i(0)=0$. The above-mentioned $5n$ state variables are associated with the stochastic volatility processes $\mathbf{V_t}^i$ following the dynamics (5).

Proof: Follows along the lines of [12]. ■

The price of options on futures can be obtained in closed form as a tractable expression since the characteristic function exists. By employing Fourier transforms, call and put options on futures contracts can be priced. These results are summarised in the following proposition that is a natural extensions of existing literature and are quoted here for completeness.

Proposition 2. *Under the stochastic volatility specifications (2) and for $t \le T_o \le T$, the transform $\phi(t;v,T_o,T) =: \mathbb{E}_t^Q[\exp\{v\ln F(T_o,T,V_{T_o})\}]$ is expressed as*

$$\phi(t;v,T_o,T) = \exp\{M(t;v,T_o)+\sum_{i=1}^{n}N_i(t;v,T_o)\mathbf{V_t}^i+v\ln F(t,T,\mathbf{V_t})\}, \tag{10}$$

where $M(t) = M(t; v, T_o)$ and for $i = 1, \ldots, n$, $N_i(t) = N_i(t; v, T_o)$ satisfy the Ricatti ordinary differential equations

$$\frac{dM(t)}{dt} = -\sum_{i=1}^{n} \mu_i^V \nu_i^V N_i(t), \tag{11}$$

$$\frac{dN_i(t)}{dt} = -\frac{v^2 - v}{2} (\varphi_i)^2 - \left(\varepsilon_i^V v \rho_i \varphi_i - \mu_i^V\right) N_i(t) - \frac{1}{2} {\varepsilon_i^V}^2 N_i^2(t), \tag{12}$$

subject to the terminal conditions $M(T_o) = N_i(T_o) = 0$, where $\varphi_i = (\kappa_{0i} + \kappa_i(T-t))e^{-\eta_i(T-t)}$.

The price at time t of a European put option maturing at T_o with strike K on a futures contract maturing at time T, is given by

$$\begin{aligned} \mathcal{P}(t, T_o, T, K) &= \mathbb{E}_t^Q[e^{-\int_t^{T_o} r_s ds} (K - F(T_o, T))^+] \\ &= P(t, T_o)[K G_{0,1}(\log(K)) - G_{1,1}(\log(K))] \end{aligned} \tag{13}$$

where $P(t, T_o)$ is the price at time t of a zero-coupon bond maturing at T_o and $G_{a,b}(y)$ is given by

$$G_{a,b}(y) = \frac{\phi(t; a, T_o, T)}{2} - \frac{1}{\pi} \int_0^\infty \frac{Im[\phi(t; a + \mathbf{i}bu, T_o, T)e^{-\mathbf{i}uy}]}{u} du. \tag{14}$$

Note that $\mathbf{i}^2 = -1$.

Proof: Follows along the lines of [21] and [15]. ■

We further assume that the market price of volatility risk can be specified by a "complete" affine representation, see [17] and [19], such as

$$\begin{aligned} dW_i^{\mathbb{P}}(t) &= dW_i(t) - \lambda_i \sqrt{\mathbf{V_t}^i} dt, \\ dW_i^{\mathbb{P}V}(t) &= dW_i^V(t) - \lambda_i^V \sqrt{\mathbf{V_t}^i} dt, \end{aligned} \tag{15}$$

for $i = 1, \ldots, n$, where $W_i^{\mathbb{P}}(t)$ and $W_i^{\mathbb{P}V}(t)$ are Wiener processes under the physical measure $\mathbb{P}$. Then the $9n$ model parameters are; λ_i, λ_i^V, κ_{0i}, κ_i, η_i, μ_i^V, ν_i^V, ε_i^V, ρ_i that we will estimate by fitting to futures and option prices in two commodity markets, gold and crude oil.

3.2. Estimation method

The estimation approach is quasi-maximum likelihood in combination with the extended Kalman filter. The loglikehood function is maximised by using the constrained optimization routine "e04jy" in the NAG library. We begin with several different initial hypothetical parameter values, firstly on monthly data, then on weekly data and finally on daily data, aimed at obtaining global optima.

The ODE's (11) and (12) are solved by a standard fourth-order Runge-Kutta algorithm using complex arithmetic. The integral in (14), is approximated by the Gauss-Legendre quadrature formula with 30 integration points and truncating the integral at 400. By a process of trial and error this has been found to provide sufficient accuracy.

The model is cast into a state-space form, which consists of the system equations and the observation equations. For estimation purposes, a time-homogeneous version of the model (6) is considered, by assuming for all T, $F(0,T) = f_o$, where f_o is a constant representing the long-term futures price (at infinite maturity). This constant is an additional parameter that is also to be estimated. In the estimation we normalized the long run mean of the volatility process, ν_i^V, to one to achieve identification. [9]

The system equations describe the evolution of the underlying state variables. In our case, the state vector is $X_t = \{X_t^i, i = 1, 2, \ldots, n\}$ where X_t^i consists of the six state variables $\mathsf{x}_i(t), \mathsf{y}_i(t), \mathsf{z}_i(t), \phi_i(t), \psi_i(t)$ and $\mathbf{V_t}^i$. The continuous time dynamics (under the physical probability measure) of these state variables are defined by (9), (2) and (15). The corresponding discrete evolution is

$$X_{t+1} = \Phi_0 + \Phi_X X_t + w_{t+1},\ w_{t+1} \sim iid\, N(0, Q_t), \tag{16}$$

where Φ_0, Φ_X and Q_t can be computed in closed form. Details can be found in [12].

The observation equation describes how observed options and futures prices are related to the state variables, namely

$$z_t = h(X_t) + u_t,\ u_t \sim iid\, N(0, \Omega). \tag{17}$$

In particular, the logarithm of the futures prices are linear functions of the state variables (as described in (6)) and the options prices are nonlinear functions of the state variables (as described in (13) and (14)) so the function h will have to vary accordingly.

3.3. The discount function

The discount function $P(t,T)$ is obtained by fitting a [39] curve each trading day to LIBOR and swap data consisting of 1-, 3-, 6-, 9- and 12-month LIBOR rates and the 2-year swap rate, similar to [47].

Let $f(t,T)$ denote the time$-t$ instantaneous forward interest rate to time T. [39] parameterize the forward interest rate curve as

$$f(t,T) = \beta_0 + \beta_1 e^{-\theta(T-t)} + \beta_2 \theta (T-t) e^{-\theta(T-t)} \tag{18}$$

from which we can price LIBOR and swap rates. This also yields for zero-coupon bond prices the expression

$$P(t,T) = \exp\left\{\beta_0(T-t) + (\beta_1+\beta_2)\frac{1}{\theta}\left(1 - e^{-\theta(T-t)}\right) + \beta_2(T-t)e^{-\theta(T-t)}\right\}. \tag{19}$$

On each observation date, we recalibrate the parameters $\beta_0, \beta_1, \beta_2$ and θ, by minimizing the mean squared percentage differences between the model implied forward rates (18) and the observed LIBOR and swap curve consisting of the 1-, 3-, 6-, 9- and 12-month LIBOR rates and the 2-year swap rate on that date.

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[9] For details see for example the discussion on invariant transformations in [17].

3.4. Testing the return–volatility relation

[23] introduced the News Impact Function (NIF) as an empirical tool to test the return–volatility relation. Typically, ARCH type models have been employed in the literature to study this effect, see for example the asymmetric ARCH model discussed in [26]. In line with [50], we employ the NIFs which are suited for continuous time models, see Appendix A for details. Let us denote by $R_F(t,T)$ the return of the futures prices of contracts with maturity T, then our model can be expressed as

$$R_F(t,T) \approx \frac{dF(t,T,\mathbf{V_t})}{F(t,T,\mathbf{V_t})} = \sum_{i=1}^{n} \sigma_i(t,T) dW_i^1(t), \tag{20}$$

where

$$\sigma_i(t,T) = \varphi_i(t,T)\sqrt{\mathbf{V_t}^i} = (\kappa_{0i} + \kappa_i(T-t))e^{-\eta_i(T-t)}\sqrt{\mathbf{V_t}^i}. \tag{21}$$

From the stochastic volatility process (2) we have the approximation

$$\mathbf{V_{t+1}}^i \approx \mu_i^V \nu_i^V \Delta t + (1-\mu_i^V \Delta t)\mathbf{V_t}^i + \varepsilon_i^V \sqrt{\mathbf{V_t}^i}\epsilon_{it}, \tag{22}$$

where ϵ_{it} follows a standard normal distribution. By fixing information at time t or earlier at a constant, the NIF can be expressed as

$$\begin{aligned} &\mathrm{E}[\sigma_i^2(t+1,T)|\epsilon_t, \sigma_i^2(t,T) = \overline{\sigma}_{it}^2, \sigma_i^2(t-1,T) = \overline{\sigma}_{it}^2, \ldots] = \\ &\phi^2(t+1,T)\mathrm{E}[\mathbf{V_{t+1}}^i|\epsilon_t, \mathbf{V_t}^i = \bar{V_t}^i, \mathbf{V_{t-1}}^i = \bar{V_t}^i, \ldots] = \\ &\phi^2(t+1,T)\Big[\mu_i^V \nu_i^V \Delta t + (1-\mu_i^V \Delta t)\bar{V_t}^i + \rho_i \varepsilon_i^V \sqrt{\bar{V_t}^i}\epsilon_{it}\Big], \end{aligned} \tag{23}$$

where $\bar{V_t}^i = \overline{\sigma}_t^2/\phi^2(t,T)$. Thus in our model, the return–volatility relation can be assessed through ρ. A negative ρ means that when there is a negative shock in the stochastic volatility factor, there will be a positive shock to the futures return, leading to the *asymmetric* volatility phenomenon. On the other hand, a positive ρ implies a positive relationship between return and conditional volatility, leading to the *inverted asymmetric* volatility phenomenon. We can also identify which volatility factors are asymmetric and which ones are inverted asymmetric, as well as to compute a weighted average effect.

4. Commodity derivatives data

By using an extended database of gold futures and options, as well as crude oil futures and options traded on the NYMEX[10], we estimate the stochastic volatility model of Section 3 for these two commodities. The gold database spans about 31 years from 4 October 1982 to 16 April 2012. Our crude oil database spans only about 21 years from 2 January 1990 to December 2010, as option price data were available only from 1990. While the database of the gold derivatives market is one of the longest derivatives databases, crude oil is one of the richest as crude oil is the most active commodity derivatives market.

[10] The database has been purchased from CME.

As the number of available futures contracts over the sample period is very large, for our estimation exercise we select contracts based on their liquidity (more specific information follows in the next section). From the options data, we consider the options on the selected futures contracts, but we intentionally avoid maturities that are more than two years. The proposed model assumes non-stochastic interest rates, therefore the option pricing formula (13) for long maturities is not precise. The option prices provided by the CME are American options which we convert to European prices as required for our model, by using the same approach proposed by [10] for equity options and by [47] for commodity options.[11]

4.1. Gold data

In the 31-year gold database, the number of available gold futures contracts with positive open interest per day has increased from 11 on 4^{th} of October 1982 to 22 on 16^{th} of April 2012. In the same period the maximum maturity of futures contracts with positive open interest has also increased from 690 (calendar) days to $2,078$ days. The contract prices change significantly throughout the sample period. The maximum futures price was US$470 per 100 troy ounces in 1982 reaching US$1800 per 100 troy ounces in 2012 (and has subsequently fallen again).

For our analysis, we select the most liquid futures contracts. Thus, we begin with the first three monthly contracts, near to the trade date. As for contracts with maturity less than 14 days liquidity is very low, the first contract should have more than 14 days to maturity. After that liquidity is mostly concentrated in the contracts expiring in February, April, June, August, October and December. Thus the first three monthly contracts are followed by the four contracts which have either February, April, June, August, October or December expiration months. Beyond that, liquidity is concentrated in June and December contracts only, hence these four contracts are followed by four semi-annual contracts which have either June or December expiration months. As a result, the total number of futures contracts to be used in our analysis is $69,684$, with the number of contracts to be used on a daily basis varying between a maximum of 10 and a minimum of 8. Figure 1.a, Figure 1.b and Figure 1.c plot selected futures prices on Wednesdays during the sample period.

From the options data, we consider the options on the first six selected futures contracts, namely the first three monthly contracts and the next three contracts which have either February, April, June, August, October or December expiration months. For each option maturity, we consider six moneyness[12] intervals, $0.86-0.90, 0.91-0.95, 0.96-1.00, 1.01-1.05, 1.06-1.10, 1.11-1.15$. In each moneyness interval, we use only the out-of-the-money (OTM) and at-the-money (ATM) options that are closest to the interval mean. Based on this selection criteria, we consider 367,412 option contracts over the 31 years, with the daily range varying between 19 and 72 contracts (per trading day). Note that the total number of trading days where both futures and options data are available is 7,427. Figure 1.g, Figure 1.h and Figure 1.i displays the ATM lognormal implied volatilities of options on the first six gold futures contracts on Wednesdays.

[11] We invert the [1] formula for American option prices to obtain lognormal implied volatilities that we subsequently use to compute European [7] prices.

[12] We define as moneyness the ratio of the option strike and the price of the underlying futures contract.

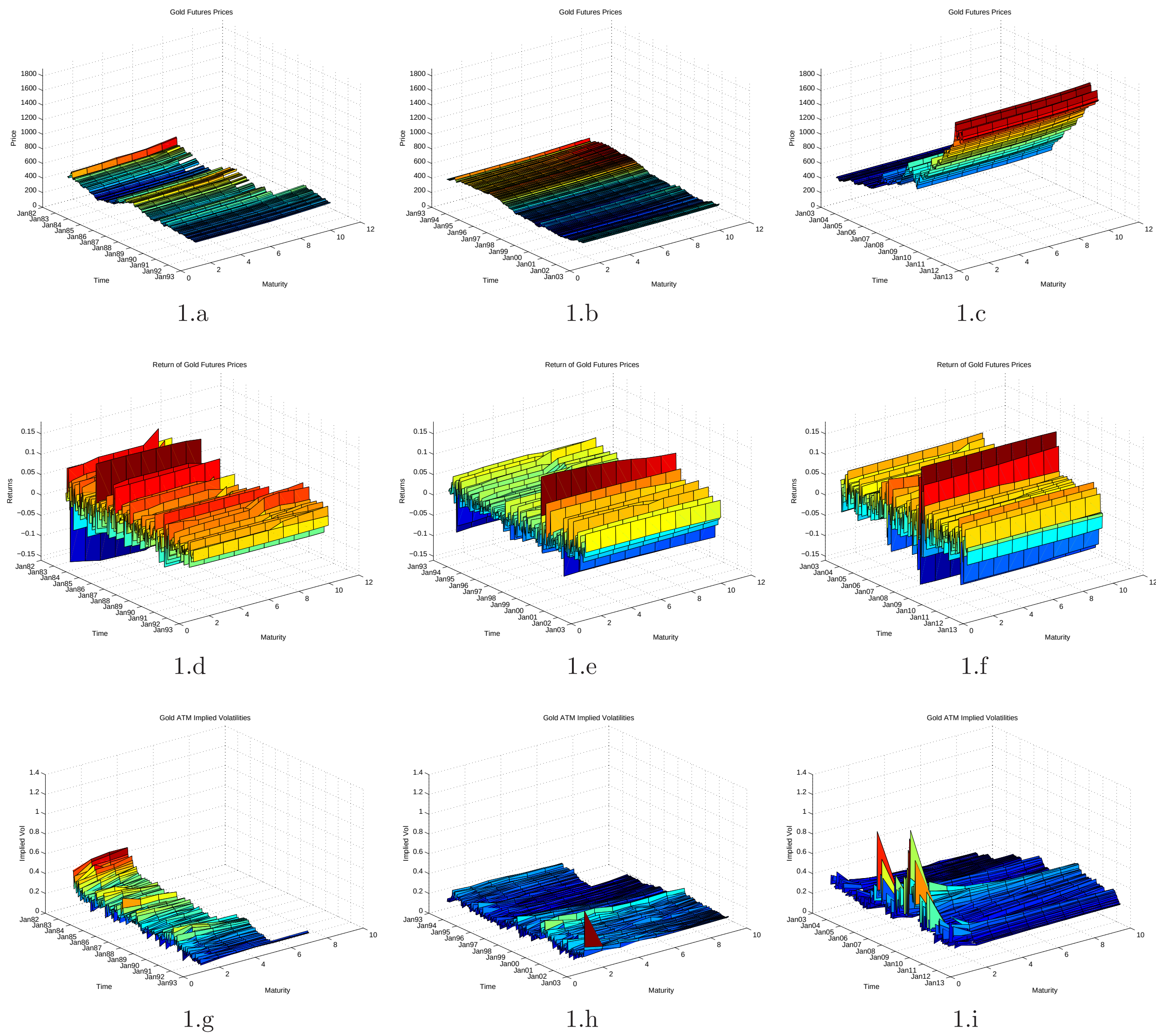


Figure 1: **Gold Futures**

The figure presents prices, returns and implied volatilities of selected gold futures contracts in three different periods, namely, October 1982 to December 1992, January 1993 to December 2002 and January 2003 to April 2012. The selected contracts are: the first three monthly contracts near to the trade date (with the first contract having more than 14 days to maturity); the next four contracts which have either February, April, June, August, October or December expiration; and the next four semi-annual contracts (June or December). Thus, the number of contracts to be used on a daily basis varies between a maximum of 10 and a minimum of 8. The ATM implied volatilities of options on the first six selected gold futures contracts (namely the first three monthly contracts and the next three contracts which have either February, April, June, August, October or December expiration), were computed by using the [1] option pricing formula. The data are displayed only on Wednesdays.

4.2. Crude Oil Data

In the 21-year crude oil database, the liquidity has consistently increased for all maturities. Based on their liquidity, we begin with the first seven monthly contracts, near to the trade date. Similarly to the gold futures contracts, the first contract should have more than 14 days to maturity. After that liquidity is mostly concentrated in the contracts expiring in March, June, September and December. Thus the first seven monthly contracts are followed by the three contracts which have either March, June, September or December expiration months. Beyond that, liquidity is concentrated in December contracts only, therefore the next five December contracts are included. As a result, the total number of futures contracts to be used in our analysis is 70,735, with the number of contracts to be used on a daily basis varying between a maximum of 15 and a minimum of 8. Figure 2.a and Figure 2.b plots the selected futures prices on Wednesdays during the sample period.

From the crude oil options database, we consider the options on the first ten futures contracts only, namely the first seven monthly contracts and the next three quarterly contracts. For each option maturity, we consider six moneyness intervals, $0.86 - 0.90, 0.91 - 0.95, 0.96 - 1.00, 1.01 - 1.05, 1.06 - 1.10, 1.11 - 1.15$. In each moneyness interval, we use only the out-of-the-money (OTM) and at-the-money (ATM) options that are closest to the interval mean. Overall, we consider 433,137 option contracts, with the daily range varying between 29 and 100 contracts (per trading day). Note that the total number of trading days where both futures and options data are available is 5,272. Figure 2.e and Figure 2.f displays the ATM lognormal implied volatilities of options on the first ten oil futures contracts on Wednesdays.

4.3. Volatility and subsample selection

We have selected to run the analysis by fitting the model to the entire sample, as well as, to subperiods of an approximate length of a decade. As it can be visually detected in Figure 1 and Figure 2, that over these decades, marked differences in futures prices and in volatility have occurred in both commodity markets, and we intend to gauge the features of commodity futures market volatility over different volatility environments and market conditions. Over this period, noteworthy events such as the gold price crisis in 1990, the Gulf War in 1991 and the GFC in 2008 have influenced the commodity markets and led to extreme market price changes. In addition, to study the volatility of the futures commodity markets, it is more informative to concentrate on subperiods as trading of futures contracts is used to relatively short-lived strategies undertaken by hedgers and speculators.[13]

For the crude oil market, [5] point out that the market changes significantly before and after 1999. Given the reduction in OPEC spare capacity and the increase in the US and China's oil consumption and imports, there was an increase in the oil price as well as its volatility. We therefore also break our sample into two subsamples, one from 1990-1999, and one from 2000-2010.

Table 5 displays the summary statistics for the gold futures market. Over the three decades, we also observe an increase in the price level. However, the middle period is quite

[13]Nevertheless, for a study on volatility of the spot commodity markets, it might be more effective to look at the whole period as many investors tend to hold positions on the underlying commodity, especially investment commodities such as gold, over longer periods of time.

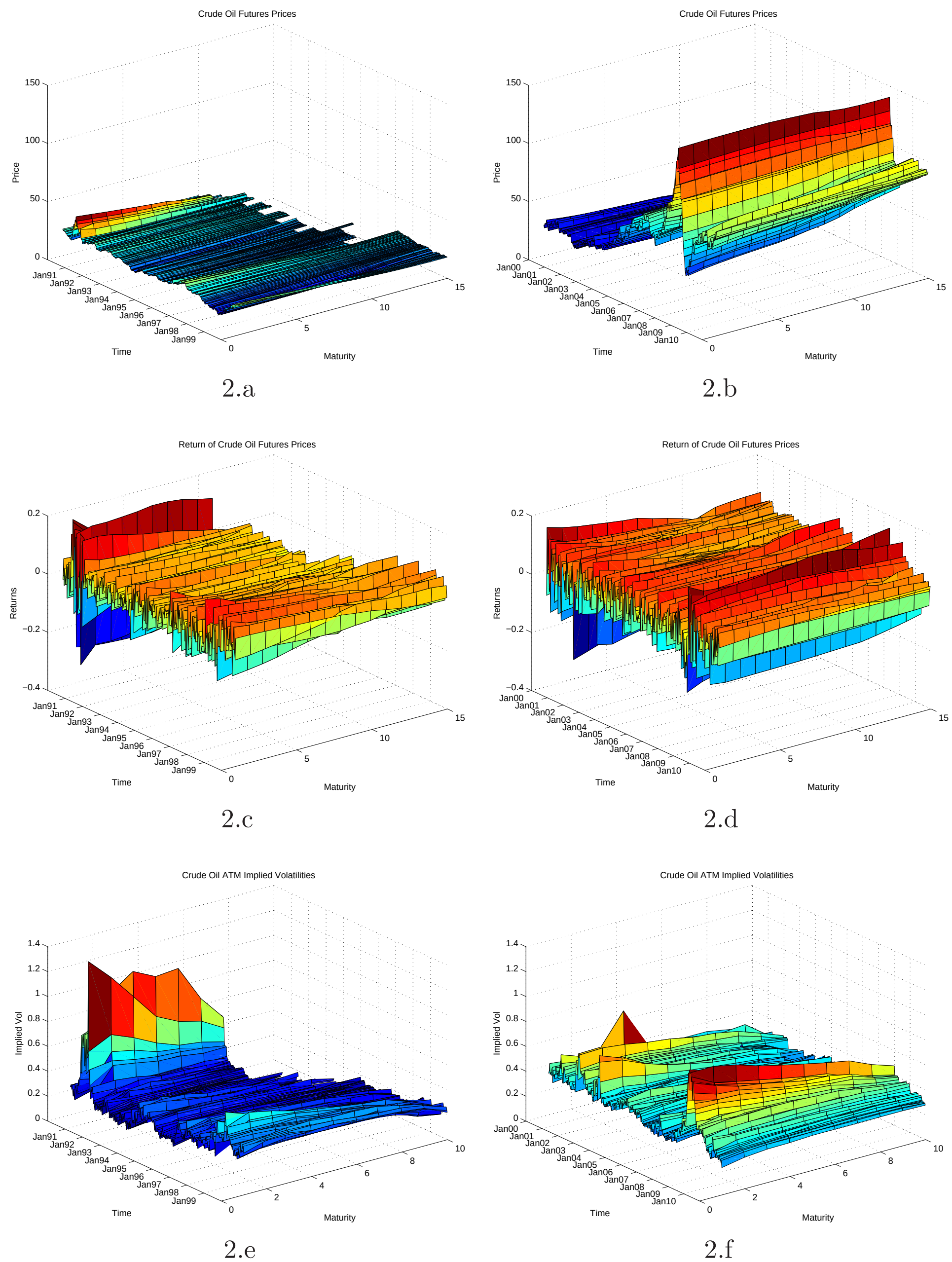


Figure 2: **Crude Oil Futures**

The figure presents prices, returns and implied volatilities of selected crude oil futures contracts in two different periods, namely, January 1990 to December 1999 and January 2000 to December 2010. The selected contracts are: the first seven monthly contracts near to the trade date (with the first contract having more than 14 days to maturity); the next three contracts which have either March, June, September or December expiration months; and the next five December contracts. Thus, the number of contracts to be used on a daily basis varies between a maximum of 15 and a minimum of 8. The ATM implied volatilities of options on the first ten selected gold futures contracts, namely the first seven monthly contracts and the next three quarterly contracts, were computed by using the [1] option pricing formula. The data are displayed only on Wednesdays.

different from the first and last periods, where prices experience much lower changes, there is also positive skewness and excess kurtosis. We therefore also break the gold data into three sample periods.

The number of driving stochastic volatility factors affecting the evolution of the futures curve are initially assessed by performing a principal component analysis (PCA) of futures price returns. The results of the PCA are displayed in Table 1 and Figure 3. The left hand side of Figure 3 reveals that for the gold futures market, only one eigenvalue is significant, while the two first eigenvalues for the crude oil market are significant. Table 1 confirms that with a one factor model, on average 98% of the variations can be explained in the gold futures market, whereas for the crude oil market, we will need a two-factor model (two factors explain 98% between 1990 and 1999 and 94% between 2000 and 2010). To choose the shape of the volatility function, we inspect the right panel of Figure 3. For the gold market, an exponential function will be suited, whereas for the crude oil futures market, a hump-shaped volatility specification seems more appropriate. We therefore estimate a one-factor stochastic volatility model with exponential decay volatility functions for gold and a two-factor stochastic volatility model with hump-shaped volatility functions for crude oil, see also [12].

Table 1: **Accumulated percentage of factor contribution**

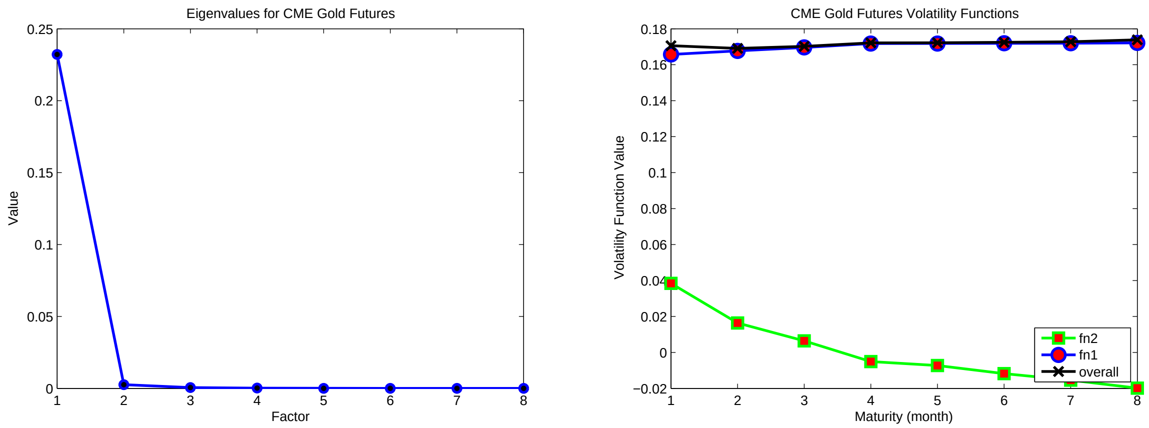
		Gold	
Time Period	One factor	Two factors	Three factors
1983 - 1992	0.9815	0.9967	0.9983
1993 - 2002	0.9786	0.9946	0.9977
2003 - 2012	0.9922	0.9988	0.9998

		Crude Oil	
Time Period	One factor	Two factors	Three factors
1990 - 1999	0.9042	0.9822	0.9972
2000 - 2010	0.8761	0.9402	0.9719

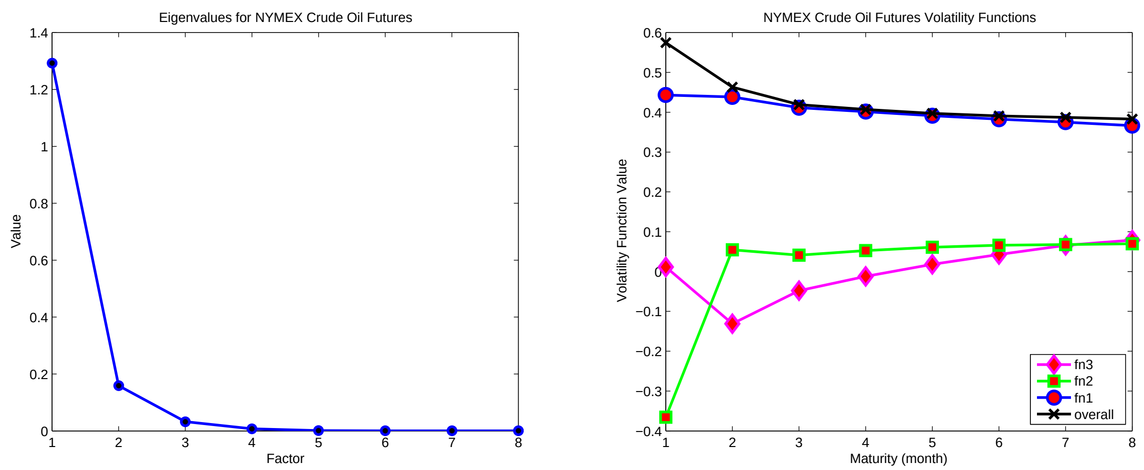
The table displays the accumulated percentage of PCA factor contribution towards gold futures return variation and crude oil futures return variation. We found that one factor is able to explain most of the variations of the gold futures returns, while at least two factors are required for the crude oil futures returns, during each of the subperiods.

5. Estimation results

We present next the parameter estimates of the proposed model when fitted to the gold market and the crude oil market and we analyze the prevailing return–volatility relation for these two markets in terms of possible explanatory factors.



3.a: Gold



3.b: Crude Oil

Figure 3: **PCA Analysis.**
3.a presents the PCA analysis for gold futures prices; 3.b presents PCA analysis for crude oil futures prices.

5.1. *Gold futures market*

The parameter estimates of our one-factor stochastic volatility model with exponential decay specifications for gold futures are displayed in Table 2.[14] We estimated the model over the whole sample period of approximately thirty-one years, as well as, over three ten-year subperiods.

Table 2: **Parameter estimates - Gold futures market**

	Period 1 1982 – 1992	Period 2 1993 – 2002	Period 3 2003 – 2012	Whole sample 1982 – 2012
κ_0	0.2737	0.5683	0.8842	0.4925
	(0.0161)	(0.0420)	(0.0344)	(0.0116)
η	0.0010	0.0063	0.0015	0.0010
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
μ^V	0.0876	0.3171	0.0817	0.1027
	(0.0013)	(0.0035)	(0.0005)	(0.0045)
ε^V	1.9898	2.0000	0.6933	0.5200
	(0.1422)	(0.0691)	(0.0466)	(0.0033)
ρ	0.1820	−0.1423	0.2288	0.6670
	(0.0063)	(0.0136)	(0.0173)	(0.0087)
λ^V	1.0211	1.0211	0.9948	0.5106
	(0.0511)	(0.0431)	(0.0521)	(0.0096)
λ	0.3641	-0.5620	0.1676	0.1821
	(0.0218)	(0.0391)	(0.0154)	(0.0037)
f_0	5.8986	2.6578	1.8157	1.8349
	(0.0231)	(0.0101)	(0.0032)	(0.0621)
σ_f	0.0012	0.001	0.001	0.001
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
σ_o	0.06	0.08	0.05	0.07
	(0.0023)	(0.0012)	(0.0013)	(0.0009)

The table displays the maximum likelihood estimates for the one-factor model specifications and the standard errors in parenthesis over thirty years, in addition to three ten-year subperiods, namely; October, 1982 to December, 1992, January 1993 to December 2002 and January 2003 to April 2012. Here f_0 is the homogenous futures price at time 0, namely $F(0,t) = f_0, \forall t$. The quantities σ_f and σ_o are the standard deviations of the log futures prices measurements errors and the option price measurement errors, respectively. We normalized the long run mean of the volatility process, ν_i^V, to one to achieve identification.

The performance of the model is adequate with low root mean squared errors (RMSEs) of the percentage differences between actual and fitted gold futures prices as well as of the

[14] As a robustness check, we also estimated a two-factor stochastic volatility model for gold futures but the additional factor did not improve the model fit to futures and option prices. In addition, we investigated both hump-shaped and exponential decay ($\kappa_i = 0$) specifications and we found that the exponential decay provides a better fit.

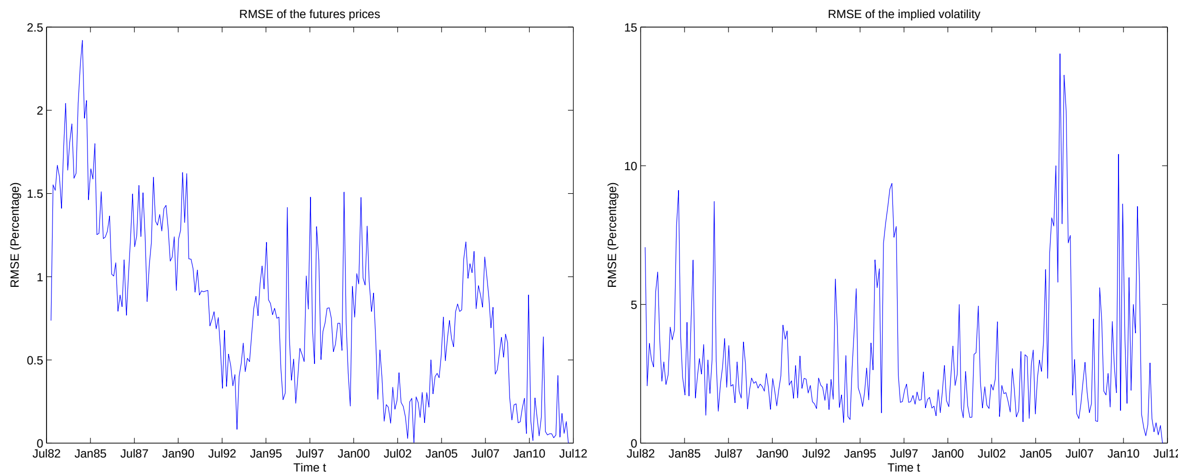


Figure 4: **Model goodness of fit - gold futures market**

The figure shows the RMSEs of the percentage differences between actual and fitted gold futures prices (the left-hand panel) as well as of the difference between actual and fitted implied option volatilities (the right-hand panel) for the one-factor model with exponentially decaying volatility. The model is estimated for the whole period from October 1982 to April 2012.

difference between actual and fitted implied gold option volatilities. Figure 4 displays the results.

We detect substantial differences in the three subperiod estimates. The underlying stochastic volatility factors V_i has a higher mean reversion coefficient μ_i^V in the middle period (1993-2002) than in the other two periods. The impact of each stochastic volatility factor on the volatility of the gold futures market is scaled by the function $\varphi_i(t,T) = (\kappa_{0i} + \kappa_i(T-t))e^{-\eta_i(T-t)}$. For all periods, the estimates of attenuation parameter η are small, allowing volatility shocks to impact relatively equally along different maturities of the futures contracts.

We note that the correlation coefficient ρ between shocks to the stochastic volatility factors and shocks to the gold futures returns are small (in the magnitude of 14-23%) but non-zero. The gold market volatility is therefore not completely spanned by futures contracts.

5.1.1. Return–volatility relation in the gold futures market

According to our empirical results, the correlation estimate when we fit the model to the whole sample is comparatively large (0.667) and positive, implying that in general the gold futures market possesses an inverted asymmetric volatility. Thus we found that the gold futures volatility has a similar response to the spot gold volatility as it has been shown in existing literature, including [2].

However when we estimate the model over the three ten-year subperiods, we provide evidence that the responses vary, depending on the source and the level of the volatility. In the less volatile Period 2 where commodity–specific effects would become more influential, the estimated correlation is negative and equal to -0.1423. In contrast, in Period 1 and Period 3, a positive correlation is observed in both periods. The results of the estimation procedure confirm our hypotheses regarding the return–volatility relation in investment commodity futures markets such as gold. Period 2 is characterized by low volatility and commodity futures prices are more likely to be principally determined by commodity specific effects leading to an asymmetric gold futures volatility response. Conversely, in Period 1 and Period 3, the volatility was considerably higher at times. In the equity markets, there is empirical evidence that during high volatility regimes, systematic risk factors become more influential, see [18]. This is also prevalent in the gold futures markets. The positive correlation of 0.182 in the period 1982-1992 and of 0.228 in the period 2003-2012 signify an inverted asymmetric volatility reaction that signals that the safe haven effect was dominant over these periods. As the safe haven property is typically instigated by market-wide shock effects, this reaction is consistent with the reaction reported in the equity markets by [18].

One noteworthy observation is that the results in Period 2 and 3 are also consistent with the convenience yield effect. Gold futures market is a market that is typically in contango thus should be characterised by negative return-volatility relation as prevailed from our analysis in Period 2. However, during Period 3 and more specifically on 2 December 2008, gold futures market went into backwardation for the first time in history, augmenting the empirically observed positive return-volatility relation.

Figure 5 displays the News Impact Function over the three subperiods used in our analysis. We additionally depict the NIF for four different times to maturity; 0.1, 0.5, 1 and 5 years. First, the NIF is a monotonically decreasing (asymmetric volatility) or an increasing (inverted asymmetric volatility) function. Second, as the time to maturity increases the slope of the NIF remains essentially the same due to the low value of η_i that eliminates the impact of the time to maturity. However, shocks of the same magnitude in the futures returns have a marginally stronger impact to near maturity futures prices compared to the longer maturity futures prices. Furthermore, this effect is stronger over the less volatile periods such as the period between January 1993 to January 2003 as shown in panel 5.b.

5.2. Crude oil futures market

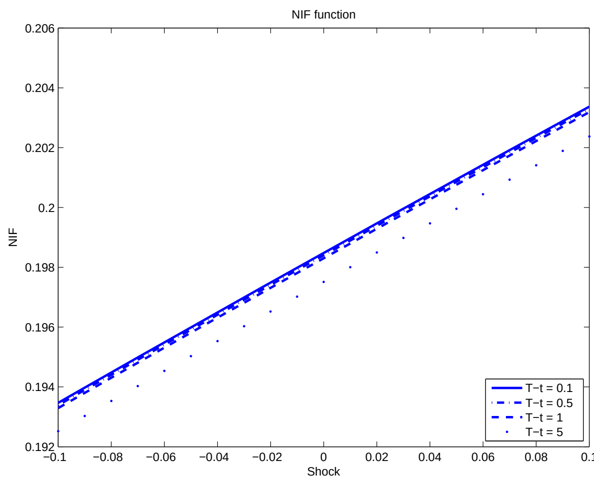
The parameter estimates of our two-factor stochastic volatility model with hump-shaped specifications for crude oil futures are displayed in Table 3.[15] Estimation is carried out over the whole sample period of approximately twenty years as well as, over two ten-year subperiods (approximately the same decades that are used in the gold futures market analysis).

Crude oil market differs from the gold market in the way volatility changes as time to maturity of contracts change. In the gold market, the effect dies out (though very slowly) as the time to maturity increases. In the crude oil market, the significant estimates of κ_i indicates the existence of hump–shaped volatility. However, as can be seen from Figure 6, for three out of four cases, the hump is beyond 4 years to maturity. We term this as "hump(e)" volatility.

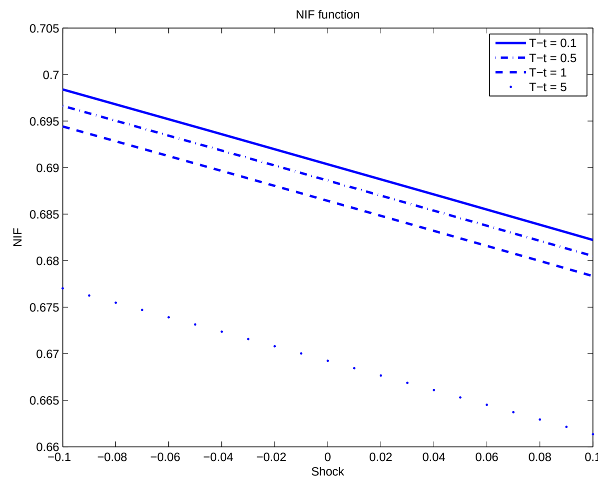
Similar to the gold market, the volatility of the crude oil market is not completely spanned by futures contracts. The correlation coefficient ρ between shocks to the stochastic volatility factors and shocks to the crude oil futures are small (in the magnitude of 3-15%).

Figure 7 shows the RMSEs of the percentage differences between actual and fitted crude oil futures prices as well as of the difference between actual and fitted implied crude oil

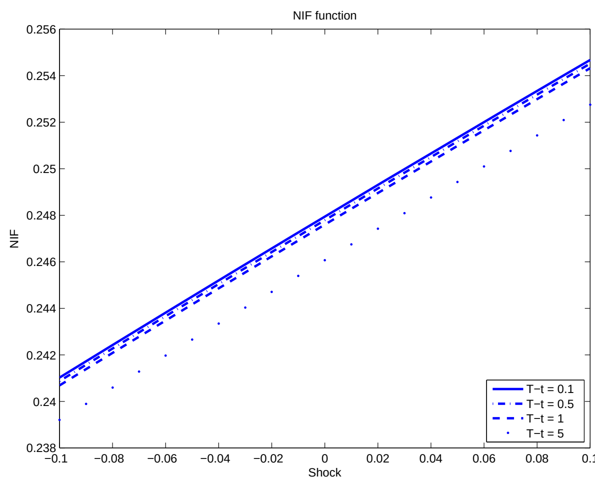
[15]We investigated both hump-shaped and exponential decay ($\kappa_i = 0$) specifications and we found that hump-shaped provides a better fit to both futures and option prices.



5.a



5.b



5.c

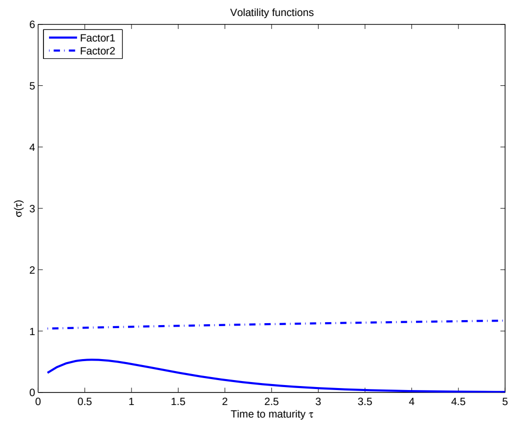
Figure 5: **NIF for gold futures market**

The figure compares the NIF for gold futures. Panels 5.a, 5.b, and 5.c present the NIF functions for the three periods, namely, October 1982 to December 1992, January 1993 to December 2002 and January 2003 to April 2012 respectively. For each period, we show the NIF for four different time to maturities; 0.1, 0.5, 1 and 5 years.

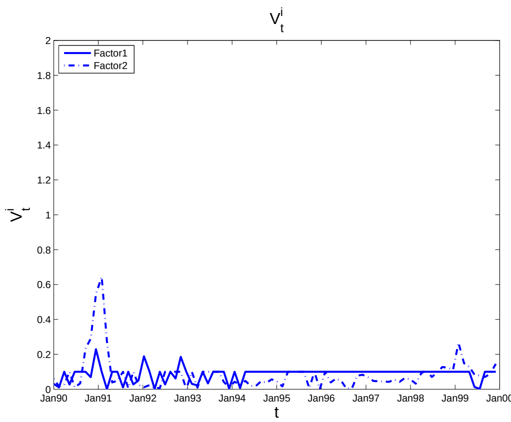
Table 3: **Parameter estimates - Crude oil futures market**

	Period 1: 1990 – 1999		Period 2: 2000 – 2010		Whole sample: 1990 – 2010	
	$i=1$	$i=2$	$i=1$	$i=2$	$i=1$	$i=2$
κ_{0i}	0.1852	1.0374	0.0677	0.7459	0.0010	0.7077
	(0.0116)	(0.0768)	(0.0072)	(0.0316)	(0.0100)	(0.0436)
κ_i	1.8370	0.0677	1.9167	0.4010	1.8635	0.2037
	(0.0743)	(0.0065)	(0.0684)	(0.0152)	(0.0376)	(0.0082)
η_i	1.4754	0.0325	0.0931	0.0890	0.5881	0.0010
	(0.0846)	(0.0024)	(0.0050)	(0.0062)	(0.0137)	(0.0022)
μ_i^V	0.0010	0.1490	0.0010	0.0010	0.0019	0.2379
	(0.0001)	(0.0057)	(0.0004)	(0.0005)	(0.0003)	(0.0133)
ε_i^V	0.7543	1.2690	0.5880	1.2806	0.8513	1.4707
	(0.0352)	(0.0689)	(0.0473)	(0.0871)	(0.0639)	(0.0791)
ρ_i	-0.0668	0.1225	-0.0290	-0.1442	-0.1606	-0.1681
	(0.0042)	(0.0168)	(0.0041)	(0.0109)	(0.0105)	(0.0096)
λ_i^V	-0.9521	1.0619	0.9974	1.0423	-0.9574	1.1939
	(0.0721)	(0.0678)	(0.0351)	(0.0645)	(0.0532)	(0.0825)
λ_i	0.6349	1.0227	0.7424	1.0227	0.1844	1.0227
	(0.0328)	(0.0637)	(0.0449)	(0.0623)	(0.0050)	(0.0661)
f_0	2.0099		1.9530		1.9513	
	(0.0132)		(0.0211)		(0.0257)	
σ_f	0.0010		0.0010		0.0010	
	(0.0000)		(0.0000)		(0.0000)	
σ_o	0.0100		0.1070		0.0377	
	(0.0021)		(0.0053)		(0.0015)	

The table displays the maximum-likelihood estimates for the hump-shaped two-factor stochastic volatility model specifications and the standard errors in parenthesis for twenty years as well as for two ten-year subperiods, namely, January, 1990 to December, 1999, January 2000 to December 2010. Here f_0 is the homogenous futures price at time 0, namely $F(0,t) = f_0, \forall t$. The quantities σ_f and σ_o are the standard deviations of the log futures prices measurements errors and the option price measurement errors, respectively. We normalized the long run mean of the volatility process, ν_i^V, to one to achieve identification.



6.a



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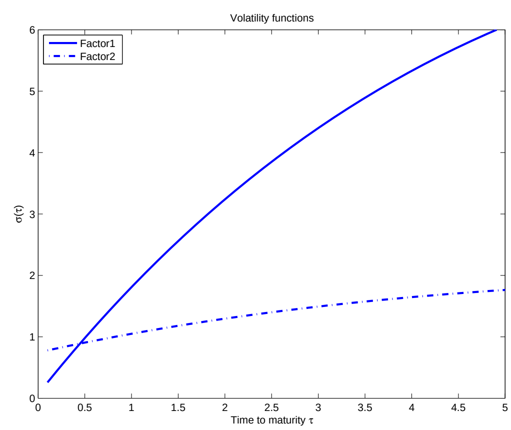

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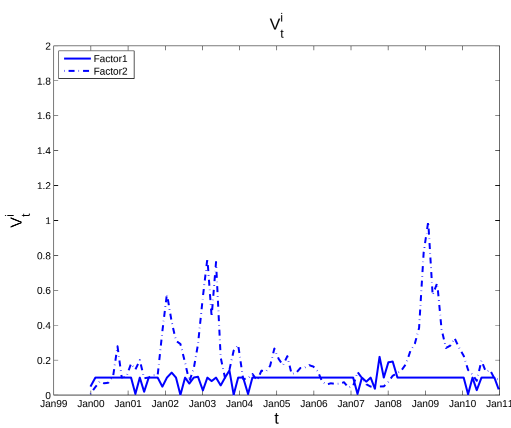


6.d

Figure 6: **Estimated crude oil futures price volatility processes**

The panels on the left-hand side show the futures price volatility function $\varphi_i(t,T) = (\kappa_{0i} + \kappa_i(T-t))e^{-\eta_i(T-t)}$ of each futures price volatility process. The panels on the right-hand side show the estimated time-series of each volatility state factor $\mathbf{V_t}^i$ for the two-factor model used for crude oil. Figure 6.a and Figure 6.b refer to January 1990 to December 1999; Figure 6.c and Figure 6.d refer to January 2000 to December 2010. Time to maturity is expressed in months.

option volatilities for the two–factor model with hump-shaped volatility. Generally, the model performs well, except a short period of the post-GFC period.

5.2.1. Return–volatility relation in the oil futures market

When the model is fitted to the whole sample, the estimated correlations between futures returns and innovations in the volatility for the two factors capturing the evolution of the volatility structure are −0.1606 and −0.1681, respectively. These negative correlations suggest that the crude oil volatility is characterized by asymmetric responses to news. Thus we found that the nature of the return-volatility relation in the crude oil futures market is similar to the spot crude oil market, as documented in the empirical studies of [30] and [43]. This is also the response in the equity markets, thus essentially this implies that the volatility feedback effect and/or the leverage effect impose the perception that bad news in the crude oil futures market would potentially increase volatility more than good news (leverage effect). It has been demonstrated, see for instance [18], that in the equity markets these effects become generally more pronounced during volatile market conditions.

A more thorough analysis though reveals that there is a lot more to the crude oil futures return–volatility relation. We assess the return–volatility relation over two ten-year subperiods. A visual inspection of the implied volatilities displayed in Figure 2.e and Figure 2.f suggests that Period 2 (2000–2010) is far more volatile than Period 1 (1990–1999), especially around the 2003 Gulf War and the GFC in 2008. The same evidence also emerges from Figure 8 that depicts the estimated volatility from the two volatility factors in the crude oil market. In Period 1, the connection between inventories, convenience yield and volatility, as explained by the Theory of Storage, justifies a positive return-volatility relation. These commodity-specific effects also led the market to be in normal backwardation. Our positive correlation estimate (0.1225) for one of the volatility factors verifies the argument that commodity markets in normal backwardation imply inverted asymmetric volatility as the convenience yield effect suggests. Table 4 displays the relative contribution of each volatility factor to the total volatility, and the type of volatility reaction. Furthermore, by inspecting the contribution of each volatility factor to the total variance from Table 4, we realise that this is the dominant factor as it accounts for 65% of the market variation. This finding supports again the [18] notion asserting that when the market is quiet then commodity-specific shock effects dominate. Since in this period, the market did not experience influential market-wide shocks, the second (less-contributing) factor is likely to capture the impact of the standard well-known effects, such as the volatility feedback effect, with a correlation of −0.0668 that indicate an asymmetric volatility reaction.

For the more volatile Period 2, our correlation estimates for the two volatility factors are both negative implying that the crude oil futures market retains an asymmetric volatility. Given that the market was in contango for over four years, these correlations support our claim that commodity markets in contango should entail asymmetric volatility (the convenience yield effect). Additionally, it is clear that this extended period of contango was triggered primarily by the significant market-wide shock effects that the crude oil market experienced over this period (rather than commodity-specific effects). Taking into account also the contribution of each factor to the total variance, see Table 4, we notice that the more dominant factor with a contribution of 64.48% displays now a correlation of −0.0290, while the other factor has a correlation of −0.1442. Thus the dominant volatility factor

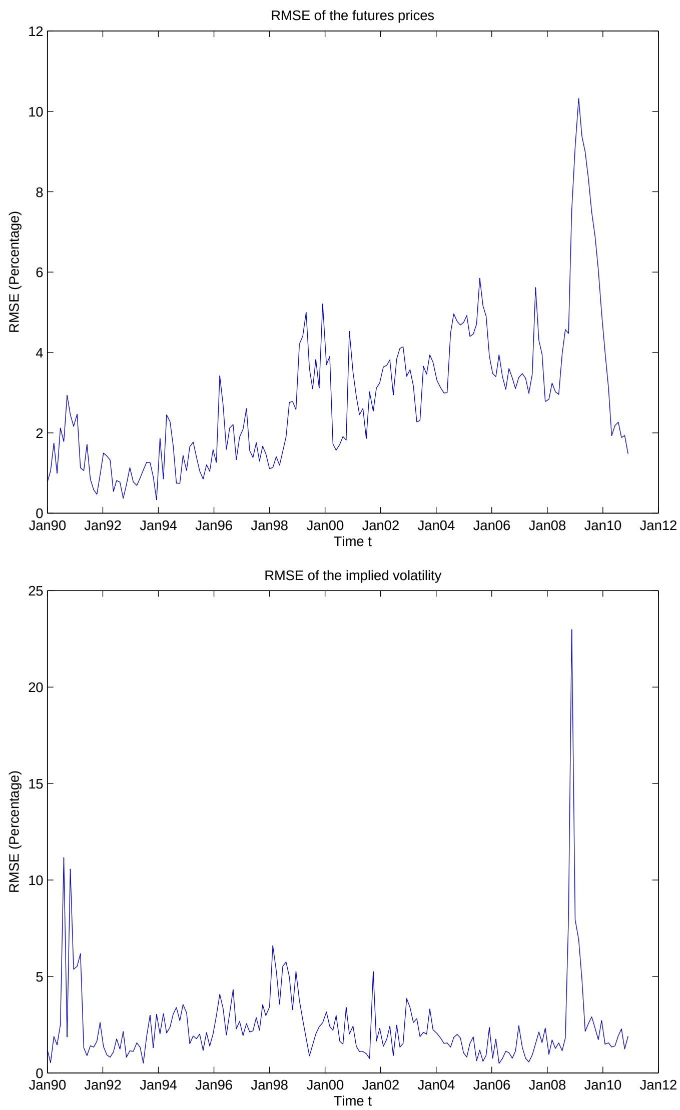


Figure 7: **Model goodness of fit - crude oil futures market**

The figure shows the RMSEs of the percentage differences between actual and fitted crude oil futures prices (the upper panel) as well as of the difference between actual and fitted implied option volatilities (the lower panel) for the one-factor model with exponentially decaying volatility. The model is estimated for the whole period from January 1990 to December 2010.

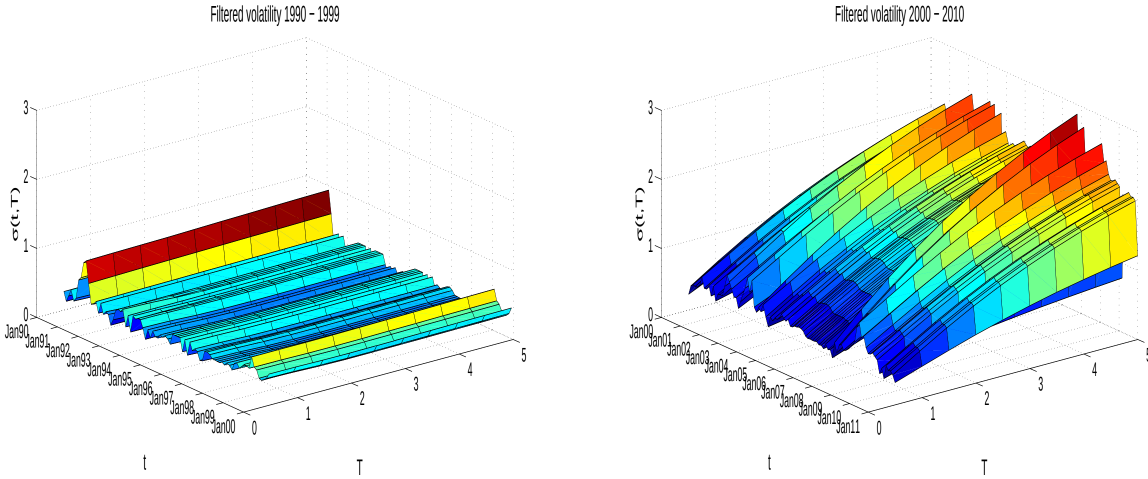


Figure 8: **Volatility of the crude oil futures market**
The graph shows the filtered volatility of the crude oil futures return $\sqrt{\sum_i \sigma_i^2}$. The left panel is for the first period 1990 – 1999. The right panel is for the second period 2000 – 2010.

has an inverse reaction compared to the Period 1. This is probably due to the possibility that the volatility in Period 2 was heavily influenced by market–wide shocks, that lead the market being into contango and thus involving asymmetric volatility.[16] Thus when market uncertainty is high, then market–wide shock effects dominate. The second volatility factor displays a strong asymmetric reaction (-0.1442) justifying the [18] conclusion that asymmetric reaction is more pronounced over volatile market conditions.

Table 4: **Contribution of volatility factors**

	1990 – 1999		2000 – 2010	
	Contribution	Vol. reaction	Contribution	Vol. reaction
σ_1	34.97 %	Asymmetric	64.48 %	Asymmetric
σ_2	65.03 %	Inverted asymmetric	35.52%	Asymmetric

The table reports the contribution of each volatility factor to the total variance of the crude oil futures price returns for the two-factor model. Asymmetric volatility is observed when ρ is negative, whereas inverted asymmetric volatility presents when ρ is positive.

The NIF for the crude oil futures market is a surface, rather than a line as in the gold futures market, due to the existence of two volatility factors, see Figure 9. In the second period of 2000–2010, both volatility factors have negative correlation with shocks to the futures returns, therefore the NIF surface clearly tilts down as volatility shocks increase.

[16]The magnitude of the correlation of this volatility factor is marginally negative probably due to the fact that we fit the model into a ten-year dataset and the market was in contango for only four years. When the model was fitted into the 5-year dataset from 2006 to 2010 where the market was mostly in contango then the correlations are -0.2130 and -0.0614 respectively, see Table 4.3 of the working paper version of [12] available at the QFRC research paper series.

Even though the volatility factors are hump-shaped, the hump for both factors is beyond five years time to maturity. As a result, in the practical range of futures contract volatility, we observe an monotonic increase in volatility impact (when there is a shock in the futures market) as the time to maturity increases.

In the first period of 1990–1999, on the other hand, only the hump for the first volatility is beyond 5 years (we call it hump(e)-shaped), whereas the hump for the second volatility is around 5 months (we call it hump-shaped). The two volatility factors have an opposite relationship with the shocks to futures return (negative ρ_1 and positive ρ_2), however, the second volatility factor is a lot stronger than the first one (see Table 9), resulting in the upward-tilting NIF.

5.3. Skewness of futures returns

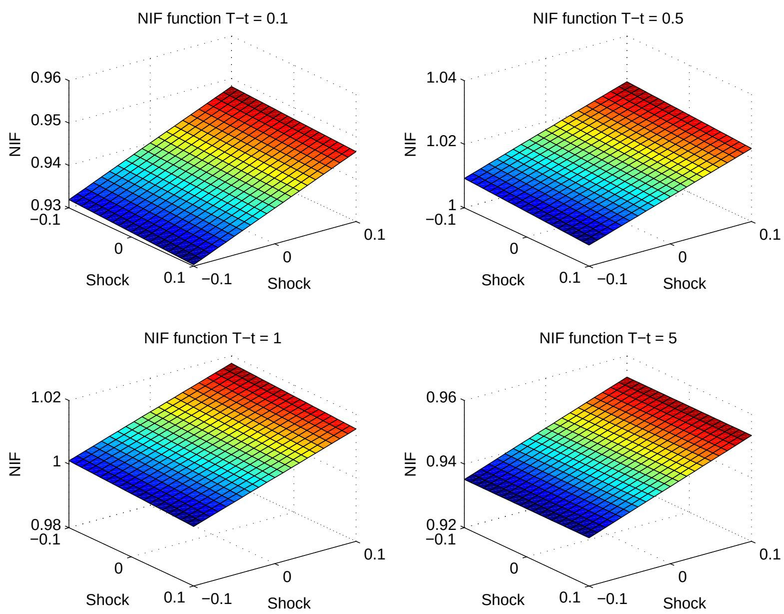
We assess the ability of the futures price volatility skewness to explain the return-volatility relation in the two commodity futures markets. Table 5 presents the descriptive statistics for one-month and thirteen-month gold futures prices and crude oil futures prices. Generally over volatile market conditions, a negative skewness has been detected in the commodity (spot and futures) markets. This negative skewness is also evident in equity markets and has been linked to their asymmetric volatility feature, [29]. Two noteworthy observations can be made in the commodity futures markets; (a) high volatility in the gold futures market is related to a negative skewness of the gold futures returns, while low volatility is related to a positive skewness of the gold futures returns and (b) high volatility in the crude oil futures market is related to a negative skewness of the crude oil futures returns, while low volatility is related to a more negative skewness. During volatile periods, the convenience yield effect linked to contango is critical in explaining the negative return–volatility relation in the crude oil futures market, while the safe haven property of gold prices can explain the positive return–volatility relation in the gold futures market. However, under low volatility regimes, their importance becomes secondary as does their link to the asymmetric behavior of volatility.[17] Under stable market conditions, gold traders, acting as rational investors, tend to prefer positive return feedbacks (more than the negative ones) while crude oil traders respond with more negative return feedbacks than positive ones. A tentative explanation is that under stable market conditions, the contribution of the convenience yield increases, causing crude oil futures prices to drop, thus more negative futures return feedbacks are more likely to occur rather than positive ones.

6. Conclusion

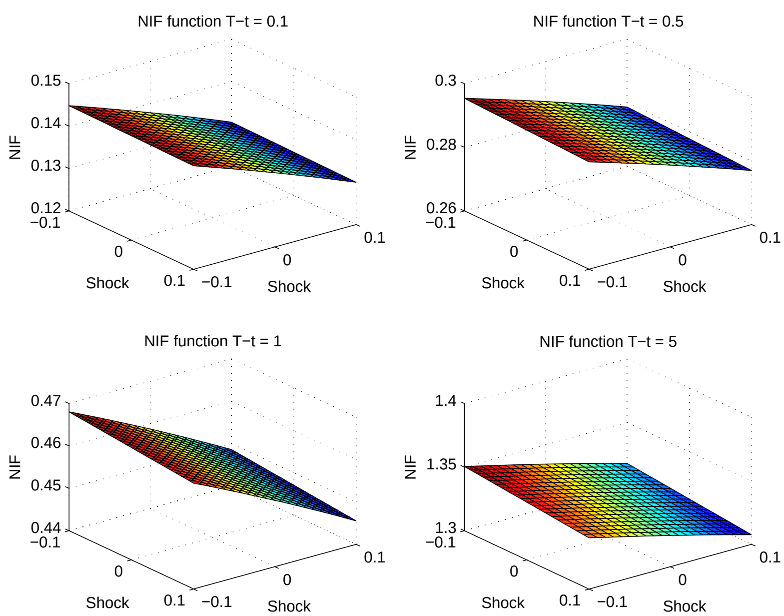
We examine the relation between futures returns and volatility changes in two major commodity futures markets, gold and crude oil. We demonstrate that, in these two commodity futures markets, there is more to the return-volatility relation than suggested by the leverage effect and/or the volatility feedback effect.

By using an extensive database on futures and options, we estimate a continuous time stochastic volatility model that assumes that the futures returns follows a multi-dimensional

[17] [49] empirically demonstrated that over stable market conditions the volatility feedback effect has a secondary role on explaining volatility asymmetry in equity markets.



9.a



9.b

Figure 9: **NIF for crude oil futures market**

Panel 9.a is for the first period 1990 – 1999, while Panel 9.b is for the second period 2000 – 2010

Table 5: **Descriptive statistics**

Gold Futures

	1982 – 1992		1993 – 2002		2003 – 2012		1982 – 2012	
Maturity	1M	13M	1M	13M	1M	13M	1M	13M
Mean	-0.000061	-0.000091	0.000027	0.000020	0.000661	0.00066	0.000194	0.000181
St. Dev.	0.010674	0.010662	0.008064	0.007884	0.012754	0.012852	0.010623	0.010578
Kurtosis	9.364871	9.285527	17.842600	16.5688	6.692487	6.682154	9.668339	9.312701
Skewness	-0.283403	-0.294973	1.064952	0.925083	-0.2806	-0.30895	-0.07089	-0.14321

Crude Oil Futures

	1990 – 1999		2000 – 2010		1990 – 2010	
Maturity	1M	13M	1M	13M	1M	13M
Mean	0.000044	-0.000002	0.000471	-0.000569	0.000263	0.000294
St. Dev.	0.024396	0.012993	0.025635	0.017173	0.025051	0.015327
Kurtosis	36.4799	12.0566	7.073648	5.603809	19.69962	7.593329
Skewness	-1.792799	-0.66896	-0.203111	-0.204234	-0.90038	-0.32904

The table displays the descriptive statistics for daily log returns of gold futures prices between October 4, 1982 and December 28, 2012 and for daily log returns of crude oil futures prices between January 2, 1990 and April 16, 2010.

diffusion process, with each term associated to a multi-dimensional [32] type process that captures the evolution of the volatility in futures markets. The correlation structure between the futures returns and the innovations of its volatility gauges the nature of the futures return-volatility relation.

Our empirical investigations have led to four main findings regarding the return-volatility relation in futures markets. First, we find that our model is consistent with results from existing literature that claim that the gold volatility is inverted asymmetric, mostly due to safe haven property and the crude oil futures volatility is asymmetric, due to the volatility feedback effect and/or the leverage effect. Nevertheless, our approach is different to prior research in two respects, a) we use a continuous time stochastic volatility model while most literature employs GARCH type models and b) we demonstrate that this relation holds similarly in the commodity futures markets, rather than the spot commodity markets where most literature is dedicated.

Second, we find that, at times, the volatility reacts differently. To explain this reaction, we assume the convenience yield effect, that links the state of the commodity futures market (contango or backwardation) to the volatility behavior (asymmetric or inverted asymmetric). More specifically, we argue that, as convenience yield increases (decreases) implying that the commodity futures market goes into backwardation (contango) mode, then the increasing volatility associated with the market-wide effects that impact on the convenience yield will lead to increasing (decreasing) futures returns. Thus commodity futures markets in normal backwardation relate to inverted asymmetric volatility, while commodity futures markets in contango relate to asymmetric volatility. This effect becomes an important factor in explaining the futures return-volatility relation, especially for crude oil.

Third, we find that the intensity of the volatility also plays an important role as the reaction over periods of low volatility is typically different to the reaction over periods of high volatility in both the gold and the crude oil futures markets. Gold futures volatility is asymmetric over quiet periods, while it has an inverted asymmetric reaction during volatile periods when market-wide effects dominate. Thus, tail events are very important determinants of the return–volatility relation in the commodity futures markets.

Fourth, our model allows for several volatility factors, each one of which can potentially capture different volatility effects. For instance, one can be driven by market-wide effects and the other from commodity specific effects. We find that indeed for crude oil the two driving volatility factors have different reactions. In a low volatility regime, the most contributing volatility factor is inverted asymmetric due to the convenience yield effect (crude oil futures market was in normal backwardation mode), while the less contributing volatility factor is asymmetric due to the volatility feedback effect. However in a high volatility regime, the most contributing volatility factor becomes asymmetric due to the convenience yield effect (crude oil futures market was in contango mode).

This evidence suggests that futures market participants should consider both market-wide factors as well as commodity-specific factors when they seek to explain and/or understand the return-volatility relation in futures markets. Also volatility conditions are very critical in appreciating this relation. However, we do not provide any indication on the nature of these factors. [38] provides a thorough investigation of the factors that impact crude oil dynamics but it remains an open question as to which factors and to what extent these drive volatility in the crude oil futures market and the gold futures market.

Appendix A. News Impact Functions

According to [50], we denote by $R_F(t,T)$ the return of the futures prices of contracts with maturity T and by $\sigma^2(t,T)$ its conditional variance and we consider the Euler approximation of a [32] type stochastic volatility model, expressed as,

$$\begin{aligned} R_F(t,T) &= \sigma(t,T)\epsilon_t, \\ \sigma^2(t+1,T) &= \alpha + \beta\sigma^2(t,T) + \sigma_v\sigma(t,T)\theta_t, \end{aligned} \tag{A.1}$$

with ϵ_t and θ_t to be iid $N(0,1)$ and $corr(\epsilon_t,\theta_t) = \rho$. Then by fixing information at time t or earlier at a constant, we evaluate the lagged σ^2_{t+1} at the long run mean of σ^2_t, denoted as $\overline{\sigma}^2$. Then the generalised NIF relates expected future volatility and return shocks and is defined as

$$\mathrm{E}[\sigma^2_{t+1}|\epsilon_t, \sigma^2_t = \overline{\sigma}^2, \sigma^2_{t-1} = \overline{\sigma}^2, \ldots]. \tag{A.2}$$

For the model (A.1) the NIF is expressed as

$$\mathrm{E}[\sigma^2_{t+1}|\epsilon_t, \sigma^2_t = \overline{\sigma}^2_t, \sigma^2_{t-1} = \overline{\sigma}^2_t, \ldots] = \alpha + \beta\overline{\sigma}^2 + \rho\sigma_v\overline{\sigma}\epsilon_t. \tag{A.3}$$

If the correlation is positive then the NIF has a positive slope thus positive shocks will increase volatility and negative shocks will decrease volatility. Thus positive shocks increase volatility by more than negative shocks and an *inverted asymmetric* reaction is revealed. On the other hand, if the correlation is negative then NIF has a negative slope thus positive shocks will decrease volatility and negative shocks will increase volatility. Thus negative shocks increase volatility by more than positive shocks and an *asymmetric* reaction is revealed.

If the model includes multiple factors, say n volatility factors, so that we may write

$$\begin{aligned} R_F(t,T) &= \sum_{i=1}^{n} \sigma_i(t,T)\epsilon_{it}, \\ \sigma^2_i(t+1,T) &= \alpha_i + \beta_i\sigma^2_{it} + \sigma_{iv}\sigma_{it}\theta_{it}, \end{aligned} \tag{A.4}$$

with ϵ_{it} and θ_{it} to be iid $N(0,1)$ and $corr(\epsilon_{it},\theta_{it}) = \rho_i$, for $i = 1, 2, \ldots, n$, then we extend the NIF to the multi-factor case as follows

$$NIF(\epsilon_{1t}, \ldots, \epsilon_{nt}) = \sum_{i=1}^{n} \Big\{ \alpha_i + \beta_i\overline{\sigma}^2_i + \rho_i\sigma_{iv}\overline{\sigma}_i\epsilon_{it} \Big\}. \tag{A.5}$$

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