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Estimating Behavioural Heterogeneity under Regime Switching*

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Abstract

Financial markets are typically characterized by high (low) price level and low (high) volatility during boom (bust) periods, suggesting that price and volatility tend to move together with different market conditions/states. By proposing a simple heterogeneous agent model of fundamentalists and chartists with Markov chain regime-dependent expectations and applying S&P 500 data from January 2000 to June 2010, we show that the estimation of the model matches well with the boom and bust periods in the US stock market. In addition, we find evidence of time-varying behavioural heterogeneity within-group and that the model exhibits good forecasting accuracy.

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JEL codes: C13, C51, G01, G10, G12

1 Introduction

Financial markets undergo episodes of dramatic changes. A striking example is the October 19, 1987 crash, when the S&P 500 Index lost more than 20% of its value in one day. Such remarkable breaks are not infrequent. In fact it is no surprise to find such breaks repeat themselves in a variety of booms and busts, if one traces back the financial market for a sufficiently long period. They include, for instance, the dot-com bubble from 1995 to 2000 and the subsequent bust from 2000 to 2003, the market euphoria from 2006-2007, and the 2007-2009 financial meltdown that followed.

Financial economists have long debated the causes of the apparent changes in the generating process of financial time series. Apart from reasons commonly referred to such as wars and government policy change, behavioural heterogeneity among investors has been shown to be a key factor[1]. The trading behaviour of heterogeneous agents can lead to various types of price changes that characterize financial crises (Huang et al 2010a). The heterogeneous agents model (HAM) literature developed over the last two decades has demonstrated that both deterministic (Day and Huang 1990; Brock and Hommes 1998; He and Westerhoff 2005; Huang et al 2010a) and stochastic HAMs (Chiarella et al 2003; Gallegati et al 2010; He and Li 2007), that address the heterogeneity in investment behaviour, are able to capture the extreme movement in the asset price without explicitly modelling it[2].

The successful experience of HAMs in capturing typical financial phenomena has inspired recent literature to estimate such models. This strand of the HAM literature first models behavioural heterogeneity as a default assumption and then estimates the parameters that capture the heterogeneity by applying financial time series techniques. Statistically significant estimations of such parameters indicate the presence of heterogeneity. Gilli and Winker (2003) and de Jong et al (2011) document the existence of behavioural heterogeneity, namely the simultaneous presence of fundamentalists and chartists, in foreign exchange markets. Boswijk et al (2007) and Frijns et al (2010) find similar evidence in the stock market and the options market respectively. De

[1] See for example Day and Huang (1990), Brock and Hommes (1998), Chiarella et al (2003), Huang et al (2010a), among many others.

[2] These models (see recent survey papers by Hommes 2006 and Chiarella et al 2009) construct dynamic mechanisms that are able to generate price series with bubbles and crashes. They do so without assuming that the price experiences a jump or break in some periods.

Jong et al (2009) even show that behavioural heterogeneity contributes to the contagion effect during crises[3].

Most estimations referred above assume that heterogeneous agents expect the prices or returns to evolve as a linear (or linearly auto-regressive) process, even though the price or return is modeled as a non-linear dynamical system. This assumption however is not well supported by the prevalent empirical evidence that stock returns follow a complicated process with multiple regimes (Ang and Bekaert, 2002) and that such a non-linear process affects investment decisions (Cooper et al. 2004). Guidolin and Timmermann (2007, 2008) show that the asset returns switch among four states, namely, crash, slow growth, bull and recovery states, and that investors' behaviour varies with these states. Explicitly incorporating such regime-dependent properties of price into investors' beliefs could improve the model fitness both theoretically and empirically. Huang et al (2010b) show that a HAM that allows investors' beliefs to be regime-dependent better matches with financial market data than those without. Specifically such a model is capable of duplicating most of the stylized facts (excessive volatility, volatility clustering, asymmetric return, long-range dependence, gradual bubbles and sudden crashes), both quantitatively and statistically. By explicitly modeling chartists' sentiment as regime-dependent, Manzan and Westerhoff (2007) find such a model has good in-sample explanatory power. They also show that in some cases, such a model exhibits more accurate forecasting ability than the simple random walk model that beats many structural exchange rate models (Meese and Rogoff 1983). These findings suggest that it is of value to account for the regime-dependent properties explicitly in modeling investors' beliefs and its impact on the price dynamic.

A natural question is how to model the regime? The few papers in the HAM literature that account for the regime-dependent property explicitly tend to pre-specify the regime according to a directly observable variable, such as price. For example, Huang et al (2010a) determine the regime according to the support and resistance price level in technical trading. Manzan and Westerhoff (2007) assume chartist sentiment to be in one regime when the absolute return is larger than a threshold value and in another regime when the opposite scenario happens. In these studies, it is from the pre-

[3]Studies that estimate behavioural heterogeneity include Baak (1999), Chavas (2000), Amilon (2008) and Franke (2009). Other studies that estimate HAMs but do not focus on behavioural heterogeneity includes Alfarano et al (2005), Goldbaum and Mizrach (2008) and Westerhoff and Reitz (2005).

specified regimes that one understands the price generating process. But what if the regime cannot be pre-determined? In most cases, the change in regime is a random variable rather than the outcome of a perfectly deterministic event. This suggests that one considers the price to be influenced by a random variable s_t describing the state or regime of the process at t (Hamilton, 1994). The regime switch is then governed by a description of the probability law. This way, we are able to let the data generating process determine in which regime it is, instead of the other way around.

Following Uhlig (2010), we assume that the market is governed by two aggregate states, a boom state and a bust state. In the 'boom' state, the market is prosperous - the price is increasing and the volatility is typically low. In the 'bust' state however, the market is depressed, characterized by declining price and high volatility. This can be illustrated in Fig. 1 by plotting the S&P 500 index and VIX, the Chicago Board Options Exchange Market Volatility Index and a popular measure of the implied volatility of S&P 500 index. It is worth pointing out that, during the recent credit crunch, the S&P 500 drops from 1521 in October 2007, when the market reaches an historical high, to 757 in March 2009, when the market hits the regional bottom, losing more than half of its value. Meanwhile, the VIX surges substantially from 18 in October 2007 to 46 in March 2009. During this period, VIX reaches its peak 79 on October 2008, soon after Lehman Brothers collapses. This value is more than 4 times of its value in October 2007. It is further observed from the figure that, the price and volatility tend to move together (in the opposite direction) with different market conditions corresponding to boom and bust states. In fact, Jacod and Todorov (2010) have documented such observation by providing empirical evidence that most stock market jumps are associated with volatility jumps.

Under the assumption that there are two states in the market, the random variable s_t takes the value of 0 when the market is in the boom state and 1 in the bust state. The value of s_t changes as the market transits from one state to the other over time. By further assuming s_t to be a discrete-valued random variable in a Markov chain, the HAM is essentially a Markov-regime-switching model. Currently, there is a lack of literature in estimating HAMs under Markov regime switching. The few exceptions are Vigfusson (1997) and Ahrens and Reitz (2005), who apply a Markov regime switching model to estimate a HAM in the foreign exchange market. These papers assume that the market is fully populated by either fundamentalists or chartists in each period. Although they find significant switching between fundamentalists

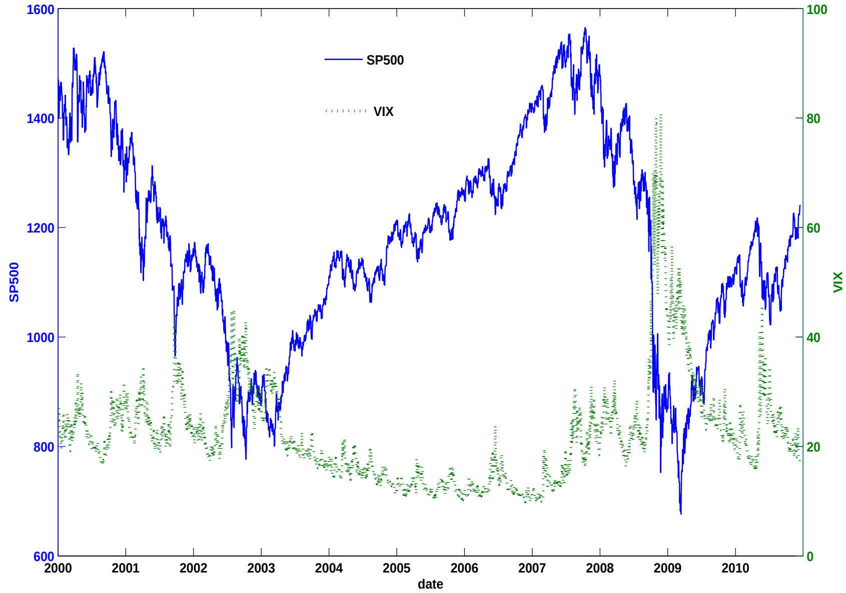

Figure 1: The time series of the S&P 500 and the VIX.

and chartists, they can not answer the question of whether there is cross sectional behavioural heterogeneity in the market, as investors in any given period are homogeneous. Holding the belief that various types of investors present in the market simultaneously, we are interested in finding out how heterogeneous investors update their beliefs under Markov regime switching, rather than looking for how investors universally switch from one strategy to another.

Apart from the standard assumption on the behaviour of fundamentalists who trade based on the fundamental value of the risky asset, this paper adds to the current literature in two respects. First, by assuming that the market is governed by a two-state Markov chain process, both chartists and noise traders update their beliefs in different ways, leading to a Markov-switching HAM. Such a set up captures the time-varying heterogeneity within the same type of investor group, apart from the between-group heterogeneity arising from different trading groups. Second, the unobserved state captures the underlying financial and economic mechanism throughout the observed time period. Its transition probability is estimated through the Markov-regime switching HAM where fundamentalists, chartists and noise traders participate in the market simultaneously. By estimating the Markov-switching HAM on S&P 500 monthly data from January 2000 to June 2010, we show that the estimation of the model matches well with the high (low) price level and low (high) volatility during boom (bust) state of the market. In addition, we find evidence of time-varying within-group behavioural heterogeneity and the model exhibits better forecasting accuracy compared to that of Boswijk et al (2007) for the sample period.

The rest of the paper is organized as following. Section 2 develops a Markov-regime switching HAM consisting of fundamentalists, chartists, and noise traders. Section 3 estimates the two-state Markov regime switching model by using S&P 500 data and discusses the estimation results. Section 4 compares the forecasting ability of our model with that of Boswijk et al (2007). Section 5 concludes. Some related issues are discussed in the Appendix.

2 The Model

Following the standard HAM literature (see for example Day and Huang (1990), Lux (1995), Brock and Hommes (1998), Farmer and Joshi (2002) and Chiarella and He (2003)), we consider a market with a risky asset and three types of traders, fundamentalists, chartists and noise traders (or liquidity providers) who are differentiated by their information and beliefs about the asset value as well as their trading strategies, together with a market maker who adjusts the market price in response to the aggregate excess demand of the three types of traders. The fundamentalists are assumed to have information about the long-term fundamental value, which is determined by real economic activity, and to trade based on their belief that the expected future market price is given by the fundamental value. However, both the chartists and noise traders trade based on market conditions and historical prices, instead of the fundamental value[4]. To characterize the high price level and low volatility during boom periods and the low price level and high volatility during bust periods, the market conditions are assumed to follow a two-state Markov regime switching process. We assume that chartists and noise traders share the same perception on the market conditions. Chartists

[4]This is either motivated by the fact that the collection of the information about the fundamental value can be expensive or due to the nature of the chartists and noise traders.

are concerned with the short-term market value, v_t. A boom (bust) market condition is then characterized by optimistic (pessimistic) investment sentiment and increasing (decreasing) risk appetite is associated with a relatively high (low) value of v_t. The noise traders place their orders e_t, which are drawn from an $N(0,\, \sigma_{n,t}^2)$ distribution. The conditional volatility $\sigma_{n,t}^2$ is regime-dependent and is relatively lower (higher) in a boom (bust) market condition. We shall discuss the regime-dependent properties in detail later. In the following, we first describe the three types of traders and their trading strategies, and then consider the price impact function that relates the price to the excess demand.

2.1 Fundamentalists

The fundamentalists are assumed to have information of the fundamental value, u_t, and believe that the expected asset price p_t is given by the fundamental value, that is $E_{f,t-1}(p_t) = u_t$. Therefore they buy (sell) the risky asset when the current p_{t-1} is below (above) u_t. Their demand function is given by

$$D_{f,t} = \alpha_f[E_{f,t-1}(p_t) - p_{t-1}] = \alpha_f\,(u_t - p_{t-1})\,, \tag{1}$$

where $\alpha_f > 0$ measures the speed of mean-reversion of the market price to the fundamental value.

For estimation purpose, the long-term fundamental value u_t can be derived from the static Gordon growth model (Gordon, 1959), so that $u_t = d_t(1+g)/(r-g)$, where d_t is the dividend flow, g is the average growth rate of dividends and r is the average required return (or the discount rate). Following Fama and French (2002), we assume that r equals to the sum of the average dividend yield $\overline{y}$ and the average rate of capital gain $\overline{x}$, that is $r = \overline{y}+\overline{x}$. The Gordon model then implies that $\overline{x}$ is equal to g. Consequently,

$$u_t = d_t\,(1+g)\,/\overline{y}. \tag{2}$$

Hence u_t is equal to the current dividend times a constant multiplier $(1+g)\,/\overline{y}$, which is also called the fundamental price to cash flow ratio.

The calculation of the static fundamental value assumes the growth rate and the rate of capital gain to be constants at their average values, g and r. Alternatively, one can obtain a dynamic fundamental value that accounts for the time-variation of the growth rate g_t and the rate of capital gain r_t as[5]

$$\begin{aligned} u_t &= E_t\left\{\sum_{i=1}^{\infty}\left[\prod_{j=1}^{j=i}(1+g_{t+j})\,/\,(1+r_{t+j})\right]\right\} \\ &\approx E_t\left[\sum_{i=1}^{\infty}\left(\frac{1+g}{1+r}\right)^i+\left.\frac{\partial u_t}{\partial r_{t+i}}\right|_r(r_{t+i}-r)+\left.\frac{\partial u_t}{\partial g_{t+i}}\right|_r(g_{t+i}-r)\right]d_t \\ &= \left[\frac{1+g}{r-g}-\frac{\rho\,(1+g)\,(r_t-r)}{(r-g)\,(1+r-\rho\,(1+g))}+\frac{\phi\,(1+r)\,(g_t-g)}{(r-g)\,(1+r-\phi\,(1+g))}\right]d_t. \end{aligned}$$

2.2 Chartists

While the fundamentalists refer to the fundamental value for trading decisions, the chartists rely on the forward-looking short-term market value of the risky asset, v_t, which is extrapolated or estimated from historical prices[6]. They extrapolate the current market price p_{t-1} by the market price deviation from the short-term market value, $p_{t-1}-v_{t-1}$, that is

$$E_{c,t-1}(p_t)=p_{t-1}+\beta\,(p_{t-1}-v_{t-1})\,,$$

where $\beta\neq 0$ is the extrapolation rate of the chartists. The demand function of chartists is assumed to be given by

$$D_{c,t}=\eta[E_{c,t-1}(p_t)-p_{t-1}]=\eta\beta\,(p_{t-1}-v_{t-1})\,,$$

where $\eta>0$ is a constant.

All the chartists hold the same beliefs about the short-term market value, v_t, which is updated in every period so as to incorporate the market condition. They however engage in two different trading strategies, based on which we classify them into two types, namely chartists A and chartists B. *Chartists A* believe that $\beta=\beta_1>0$, that is, the market price p_t will deviate further away from the current price p_{t-1} by the price trend represented by the price

[5]See Boswijk et al (2007) for a detailed derivation of the dynamic Gordon growth model, in which the approximation in the second line is obtained by a first-order Taylor expansion around the mean of r, and the third line is derived by assuming the expectations of future ex-ante required returns and cash flow growth rate follow an AR(1) process such that $E_t\,(r_{t+1}-r)=\rho\,(r_t-r)$ and $E_t\,(g_{t+1}-g)=\phi\,(g_t-g)$.

[6]It could also be interpreted as a price trend, benchmark price or target price, against which traders compare their most recent price observation.

deviation from the short-term market value, $p_{t-1} - v_{t-1}$. They behave like trend followers or momentum traders when the short-term market value v_t is estimated based on some price trend. Hence the excess demand function of chartists A is given by

$$D_{c_A,t} = \beta_A \left(p_{t-1} - v_{t-1}\right), \tag{3}$$

where $\beta_A = \eta\beta_1 > 0$. Such a chartist behaves like the investor-β in Day and Huang (1990). *Chartists B* believe $\beta = -\beta_2 < 0$ with $\beta_2 > 0$, and bet on the reversal of the price trend. Instead of expecting that the price will follow the price trend, they believe the price trend will reverse in the next period. They behave like contrarians (see Chiarella and He (2002) for a HAM with both trend followers and contrarians). Hence, their excess demand function is given by

$$D_{c_B,t} = \beta_B \left(v_{t-1} - p_{t-1}\right), \tag{4}$$

where $\beta_B = \eta\beta_2 > 0$.

What is innovative in the set up is the specification of the short-term market value v_t. Instead of estimating it from some (weighted) moving averages as in the current HAM literature, we assume it to be state-dependent and allow it to switch among two different states, as discussed earlier. Specifically, v_t is assumed to be contingent on the state s_t, which takes discrete value of 0 or 1 so that $s_t \in S = \{0, 1\}$. The dynamics of the state aim at capturing the changes in market conditions through the observed prices. The state s_t is modelled as a stationary ergodic two-state Markov chain on S with transition probabilities given by

$$P\left(s_t = j | s_{t-1} = i, s_{t-2} = k, ...\right) = P\left(s_t = j | s_{t-1} = i\right) = P_{i,j} \tag{5}$$

for $i, j, k \in S$, where $P_{i,j}$ indicates the probability that state (regime) i transits to state j for $i, j \in \{0, 1\}$. The transition probabilities are constants and satisfy the condition $\sum_{j=0}^{1} P_{i,j} = 1$ and $0 \leq P_{i,j} \leq 1$ for $i = 0, 1$. The state s_t is a random variable that is not directly observable. However, a filter estimate can be computed from the time series of the price. Some filters such as sequential filter are capable of performing accurate inferences of the state s_t (see the algorithm in Carvalho and Lopes, 2007). It is therefore reasonable to assume that the chartists, who are supposed to be strong in quantitative analysis, can estimate the state with high precision. The regime-dependent v_t is then given by

$$v_t = \begin{cases} v_0, & s_t = 0, \\ v_1, & s_t = 1. \end{cases} \tag{6}$$

2.3 Noise Traders

The noise traders do not gauge the fundamental value or extrapolate the price trend. They do however develop a general understanding of the market condition, say, from the market report generated by chartists, who are typically enthusiastic in promoting their analysis. The demand function of noise traders can be written as

$$D_{n,t} = e_t,$$

where e_t is a noise term drawn from an $N(0,\, \sigma^2_{n,t})$ distribution and $\sigma^2_{n,t}$ is regime-dependent, that is,

$$e_t \sim \begin{cases} N(0, \sigma^2_{n,0}), & s_t = 0, \\ N(0, \sigma^2_{n,1}), & s_t = 1. \end{cases} \tag{7}$$

2.4 Market Maker

The market maker collects orders from the fundamentalists, chartists and noise traders, then subsequently adjusts the price according to the excess demand with a speed of $\gamma > 0$. Using ω_1, ω_2, ω_3 and ω_4 to denote the market weights (or fractions) of fundamentalists, chartists A, chartists B, and noise traders, the price impact function can be written as

$$\begin{aligned} \Delta p_t &= p_t - p_{t-1} \\ &= \gamma\{[\alpha_f \omega_1 (u_t - p_{t-1}) + \beta_A \omega_2 (p_{t-1} - v_{t-1}) + \beta_B \omega_3 (v_{t-1} - p_{t-1}) + \omega_4 e_t]\} \\ &= \alpha (u_t - p_{t-1}) + \beta (p_{t-1} - v_{t-1}) + \varepsilon_t, \end{aligned} \tag{8}$$

where the third line is obtained by setting $\alpha = \gamma \alpha_f \omega_1$, $\beta = \gamma (\beta_A \omega_2 - \beta_B \omega_3)$ and $\varepsilon_t = \gamma \omega_4 e_t$. A positive β suggests that the trading activities of chartists A dominate that of chartists B, while a negative β indicates that the market power of chartists A is subordinate to chartists B. The distribution of ε_t follows directly from Eq.(7)

$$\varepsilon_t \sim \begin{cases} N(0, \sigma^2_0), & s_t = 0, \\ N(0, \sigma^2_1), & s_t = 1, \end{cases} \tag{9}$$

where $\sigma_0 = \gamma\omega_4\sigma_{n,0}$ and $\sigma_1 = \gamma\omega_4\sigma_{n,1}$. As v_t and ε_t are regime-dependent, this model is essentially a Markov switching model with regime-dependent means and variances.

3 Model Estimation

3.1 Data

We use data from Shiller (2005) that consists of monthly data of S&P 500 index, dividends and the consumer price index (CPI) from January 2000 to June 2010. To obtain a fair evaluation of the price level over time, the nominal value of both stock price and dividends are discounted by the CPI and converted to real value. Unless otherwise specified, all the data discussed hereafter are in real term.

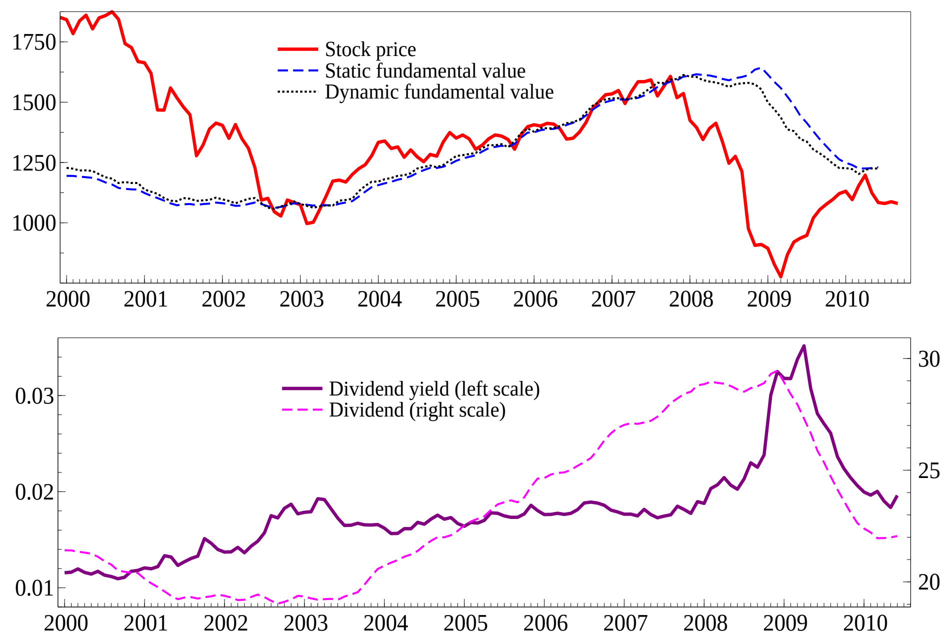

Figure 2: Stock price, fundamental value, dividend, and dividend yield. The top panel plots the S&P 500 discounted by CPI, as well as dynamic and static fundamental values from January 2000 to June 2010. The bottom panel graphs the dividend and dividend yield over the same period.

Table 1: Summary Statistics. Sample period is from January 2000 to June 2010.

Variable	Mean	Std. Dev.	Min	Max
p_t	1328.93	243.42	776.33	1874.90
Static u_t	1291.43	190.80	1061.51	1643.21
dynamic u_t	1289.16	176.55	1057.96	1612.96
d_t	23.15	3.42	19.03	29.45
d_t/p_{t-1}	1.79	.48	1.09	3.52
g_t	.03	.99	-2.95	1.95
r_t	1.82	.96	$-.15$	4.41
i_t	.04	.01	.02	.06

d_t/p_{t-1}, g_t , r_t and i_t are expressed as percent. $N = 126$.

As shown in Fig. 2, the static and dynamic fundamental values track each other closely. Both series increase gradually before reversing in 2009. Table 1 shows that the average static (dynamic) fundamental value is 1291 (1289), with a standard deviation of 191 (177). It is not surprising to see that the divided (bottom panel) exhibits the same pattern as the fundamental value, given that the static fundamental value is essentially the product of a constant multiplier and the dividend (see Eq.(2)). Fig.2 suggests that there is not a simple pattern of how price relates to fundamental value. The stock price deviates from its fundamental value between 2000 and 2003 during which time the dot-com bubble burst, and between 2008 to 2009 when the credit crunch unfolded. The price co-moves with the fundamental value broadly during 2003 to 2007 when the financial market is booming. Over all, the price ranges between 776 and 1875. The standard deviation of price is 243, which is greater than that of the fundamental value.

What is worth pointing out is that the dividend yield remains quite stable when the price and its fundamental value move in tandem and increase substantially during crisis episodes, especially during the recent credit crunch. Specifically, the dividend yield reaches its peak at 3.5% soon after the price hits its bottom in March 2010. Also presented in Table 1 are the the average growth rate of dividend, g_t, which is 0.03% and the average required return r_t, which equals 1.82%. The average monthly interest rate of the 10 year treasury bill i_t, the risk free interest rate, is 0.04 percent. As in Fama and French (2002), the risk premium equals the average required return minus the average risk free interest rate. Our sample yields an equity premium of

Table 2: Estimation results. Sample period is from January 2000 to June 2010.

Variable	α	β	v_0	v_1	σ_0	σ_1	$P_{0,0}$	$P_{0,1}$
Coefficient	$-.002$	$-.045$	665.711	1607.064	63.561	31.588	.945	.049
p-value	.948	.107	.000	.000	.000	.000	.000	.120
log-likelihood=-663.101, AIC=10.652								

1.79% per month[7].

3.2 The Estimation Results

We estimate the Markov regime switching model defined in (8), (6), and (9) by using the maximum likelihood method (Hamilton, 1994, Chapter 22). As demonstrated in Fig. 3, the fitted value based on our model matches well the real price series and the 1-step ahead prediction captures the ups and downs of the price movements even when they are extreme. Table 2 presents the detailed estimation results, based on which we have the following observations.

The estimate of α is not statistically significant. The estimate of β is negative, suggesting that the trading activities of chartists B dominate those of chartists A[8]. The regime-dependent short-term market values estimated

[7]This value is much higher than that in Boswijk et al (2007) and Fama and French (2002), both of which use annual data. Boswijk et al. (2007) obtain a risk premium of 4.84% per year in the period 1871-1950 and 2.16% for the period 1951-2003. Fama and French (2002) estimate the expected real equity premium to be 4.17% per year for 1872 - 1950 and 7.43% for 1950 - 2000. The large differences in equity premium between our estimation and that of current literature could arise from data frequency and sample period. Fama and French (2002) find that the risk premium for 1872 to 2000 is 3.54%, which is much lower than that obtained from the two sub-samples. Their finding suggests that a longer sample period might yield a relatively lower equity premium. Given the short sample period in this paper, one would expect a higher equity premium in our sample. Perhaps more importantly, in our sample, the dividend as well as the dividend yield grows substantially as shown in Fig 2 while the interest rate exhibits a declining trend, all of which contribute to the increase in the equity premium.

[8]The estimate of β does not seem to be very significant. The lack of significance is caused by the multicollinearity problem, which leads to relatively large estimates of standard errors of the correlated variables. We can easily reject the null hypothesis that $\alpha = \beta = 0$, suggesting that at least one of the coefficients are significant in explaining the regressand. The multicollinearity however only affects the estimates of the correlated

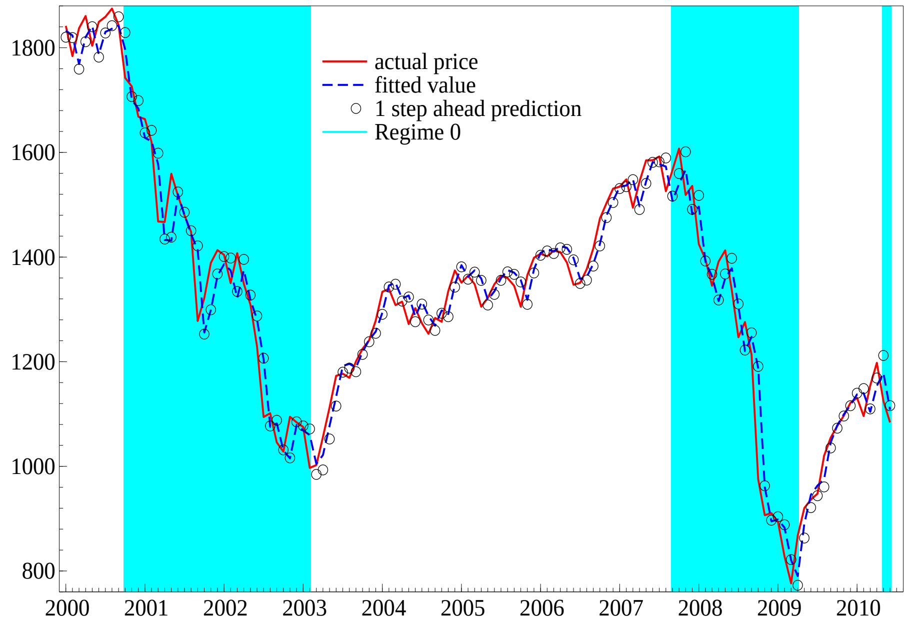

Figure 3: The index, fitted value, and 1-step-ahead prediction. The solid line represents the S&P 500 Index discounted by the CPI, the dashed line plots the fitted value from the estimation of Eq. (8), the circles correspond to the 1-step ahead forecasts, and the shaded areas indicate the regime 0.

from our model are $v_0 = 666$ and $v_1 = 1607$ in states 0 and 1. respectively. It is no surprise to see from Fig. 3 that, in general, the price moves towards v_0 (v_1) in states 0 (state 1), as the dominant chartists B bet on the reversal of the price trend. As the short-term market value is updated in correspondence to the unobserved state over time, The trading behaviour of chartists B varies accordingly. It indicates the existence of behavioural heterogeneity within the chartist group.

The regime-dependent standard deviations are found to be $\sigma_0 = 64$ and $\sigma_1 = 32$ in states 0 and 1, respectively. Note that the volatility in state 0 is twice as much as that in state 1, suggesting that noise traders are much more sensitive to external news in state 0 than in state 1. The different behaviour of noise traders in different states provides evidence of within-

variables but not the overall goodness-of-fit of the model. We provide a detailed discussion on this issue in Appendix A.

group heterogeneity over time.

The switching of the beliefs of the chartists and noise traders between the two states are indicated by the transition probabilities. The chartists do not fix their beliefs for short-term market value over time. Instead, they switch their forecast from v_0 to v_1 or vice versa with a time-varying probability. Similarly, noise traders adjust their degree of reaction according to the market condition differentiated by the state variable s_t. The results in Table 2 show that the probability of remaining in state 0 is 94.5% ($P_{0,0}$), suggesting that the bust state is persistent on average for $1/(1-P_{0,0}) \approx 18$ months. It means that the probability for chartists to stay in the relatively low short-term market value v_0 and for the noise trader to submit orders that follow an $N(0, \sigma_0^2)$ distribution is 94.5%. Given the two states identified, this also means that the probability for the chartists and noise traders to switch their forecast from state 0 to 1 is 5.5% ($P_{1,0}$). Similarly, the chartists maintain their original estimates of v_1 and σ_1^2 in state 1 with a probability of 95.1% ($P_{1,1}$) and switch to state 0 with a probability of 4.9% ($P_{0,1}$).

Overall, we find statistically significant results on the regime-dependence of v_t and σ_t, which indicate the existence of time-varying within-group heterogeneity. This means that traders in both the group of chartists and noise traders adapt their trading strategy according to the state variable s_t, which gives rise to the within-group behavioural heterogeneity.

Most interestingly, the estimated states match very well to the two states the model proposed in Section 2. State 0 is associated with relatively high volatility (high risk) and low short-term market value, which characterizes the bust (turbulent or crisis) periods. State 1 is accompanied by a relatively low volatility (low risk) and high short-term market value, which captures the salient feature of boom (tranquil) periods. What is perhaps more interesting is how chartists update their forecast for the short-term market value v_t and how noise traders adapt their degree of reaction over time, especially during periods of financial crisis. We have obtained estimates of the coefficients as well as the transition probabilities. Conditional on the whole price series $\{p_1, ..., p_N\}$, we now calculate the smoothed probability $P(s_t = j|p_1, ..., p_N)$ for each date (for details of the algorithm, see Kim and Nelson, 1999). Fig.4 shows the smoothed probability for v_t and σ_t to fall into the two states over the sample period of January 2000 to June 2010. The bust periods corresponding to state 0 cover the two latest episodes of financial crisis - the 2000-2003 dot-com crash and the 2007-2009 credit crunch. During these periods, chartists extrapolated the short-term market value to be $v_0 = 666$ and

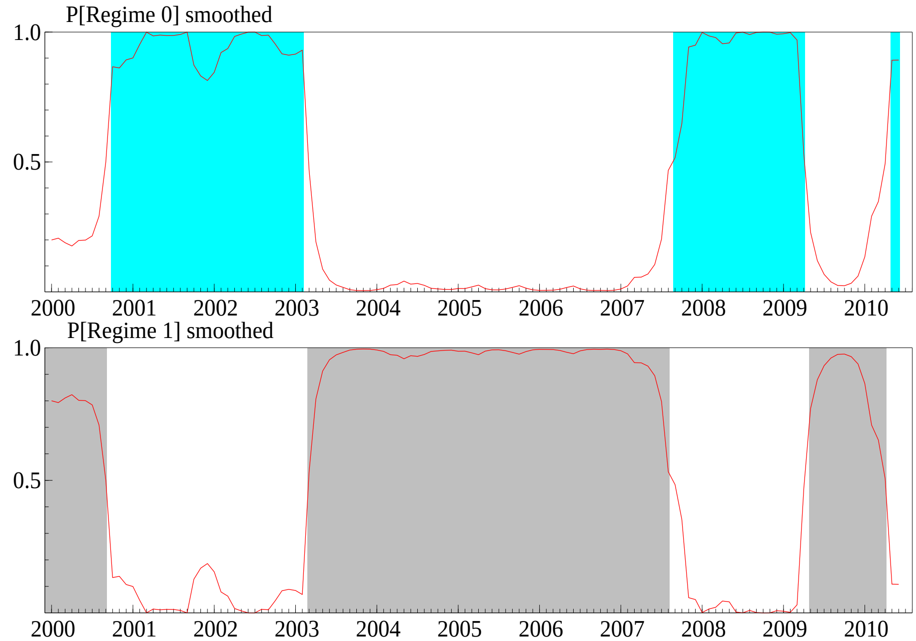

Figure 4: Smoothed transition probability. The upper panel plots the smoothed probabilities for the price to be in regime 0. The bottom panel plots the smoothed probability of being in regime 1. Periods that are classified under the specified category are shaded accordingly.

noise traders expected the volatility to be $\sigma_0 = 64$. The boom periods corresponding to state 1 include the time from late 2003 to early 2007, when the stock market was prospering. During these periods, most chartists upgraded the short-term market value to be $v_1 = 1607$ while noise traders downgraded their volatility forecasts to $\sigma_1 = 32$.

Recently, after the market hit the bottom in late March 2009, chartists and noise traders seem to have become optimistic and switched their estimates for v_t and σ_t from regime 0 to regime 1. Such a market mood however only lasts for a few month before gloom takes over again; both chartists and noise traders gradually switched from regime 1 to regime 0 in early 2010. Both types of traders eventually cluster their estimate to regime 0 from May 2010 onwards, which may hint at the possibility that the market seems to be running the risk of encountering a double dip depression. Overall, our estimates suggest that chartists expect low (high) short-term market value while

Table 3: Regime classification based on smoothed probability

Regime	Start	End	Probability*	Months	Total Months
Regime 0	2000(10)	2003(2)	0.938	29	51
(Busting)	2007(9)	2009(4)	0.920	20	
	2010(5)	2010(6)	0.892	2	
Regime 1	2000(1)	2000(9)	0.758	9	75
(Booming)	2003(3)	2007(8)	0.956	54	
	2009(5)	2010(4)	0.844	12	

* the average probability for $v_{s,t}$ to be in regime s between the start and end period.

noise traders forecast high (low) volatility during bust (boom) periods. In other words, they are bearish when the market undergoes a depressed period and bullish when the market booms. State 0 (state 1) not only captures the characterizations of the financially bust (boom) periods, but also matches well with the historical span of turbulent (tranquil) financial episodes. Although the dates of financial crises are not used in any way to estimate the parameters or form inference about transition probabilities, it is interesting that the turbulent and tranquil episodes correspond closely to the states 0 and 1 in our model.

Table 3 further classifies the sample periods according to the regime switching probability. Among the 126 sample periods we study, there are a total of 51 months (40.48%) in regime 0 when the market is depressed and a total of 75 months (59.52%) in regime 1 when the market is booming. The average duration of remaining in regime 0 consecutively is 17 months, which is shorter than that of staying in regime 1, which is 25 months. However, given the incomplete episodes at the beginning and at the end of our sample, it does not necessary mean that tranquil episodes are more persistent than turbulent episodes.

In summary, the estimation results provide evidence of between-group behavioural heterogeneity and within-group heterogeneity. Moreover, the classified regimes match well with the market booms and busts in actual financial episodes. Under the current set up, the short-term market value is state dependent, but not the extrapolation parameter β of the chartists. Intuitively, chartists may react differently when facing different market conditions. This implies that the parameter β can also be state dependent. In Appendix B, we estimate the model to allow the parameter β to be state dependent as well. The estimation results show that this model has similar explanatory power. However the classification of the regimes in this model is less accurate. Also, it exhibits poorer out-of-sample forecasting accuracy, see Appendix B for the detailed discussion.

4 Out-of-Sample Predictability

In order to evaluate the model specification under regime switching, we compare the out-of-sample predictability of the model with Boswijk et al (2007). However, before comparing the predictive power, we review the model specification in Boswijk et al (2007).

4.1 Review of the Boswijk et al (2007) Model

Boswijk et al (2007) assume two type of investors, who predict future price by extrapolating past deviation of the price dividend ratio from its fundamental ratio such that

$$E_{h,t}\left[u_{t+1}/d_{t+1} - p_{t+1}/d_{t+1}\right] = \theta_h\left(u_{t-1}/d_{t-1} - p_{t-1}/d_{t-1}\right), \ h = 1, 2,$$

where $\theta_1 \neq \theta_2$. Let $x_t = u_t/d_t - p_t/d_t$ so that the last equation can be simplified to

$$E_{h,t}(x_{t+1}) = \theta_h x_{t-1}, \qquad h = 1, 2.$$

Investors update their market weight every period according to the most recent realized profits. The proportion of investor type 1 at period t, $n_{1,t}$, evolves according to a discrete choice model with multinomial logit probabilities[9]

$$n_{1,t} = \frac{1}{1 + \exp\left[-\rho\left(\theta_1 - \theta_2\right)x_{t-3}\left(x_{t-1} - \frac{1+r}{1+g}x_{t-2}\right)\right]}, \tag{10}$$

where ρ is a scaled intensity of choice, and the term $\left(\theta_1 - \theta_2\right)x_{t-3}$ $\left(x_{t-1} - \frac{1+r}{1+g}x_{t-2}\right)$ measures the difference in realized profits of investors of type 1 compared to type 2. The equilibrium price under the assumption of zero external supply satisfies

$$x_t = n_{1,t}\theta_1 x_{t-1} + \left(1 - n_{1,t}\right)\theta_2 x_{t-1} + \epsilon_t, \tag{11}$$

[9]See Boswijk et al (2007) for a detailed derivation of the market weight $n_{1,t}$ and $n_{2,t}$.

Table 4: Nonlinear least square estimation results for the Boswijk et al (2007) model

Variable	ρ	θ_1	θ_2
Coefficient	0.768	.944	1.051
p-value	.579	.000	.000
log-likelihood $=-649.946$, AIC=10.783			

where ϵ_t is a disturbance term. Equation (11) means that the current deviation of the price dividend ratio from its fundamental ratio is a market proportion weighted average of the investor expected deviations plus a noise term.

Following Boswijk et al (2007), we estimate Eq.(11) with nonlinear least square using the data sample from January 2000 to June 2010. The estimation results shown in Table 4 are in line with Boswijk et al (2007). The coefficient of θ_1 and θ_2 are statistically significant, suggesting the presence of between-group behavioural heterogeneity. The intensity of choice ρ is not statistically significant, so there is no evidence that investors of one type switch to a different type. In terms of in-sample estimation, the log-likelihood and AIC suggests that the BHM model shares similar explanatory power to our model over the same sample period.

4.2 Forecasting Accuracy

To compare the out-of-sample forecasting accuracy of our model with Boswijk et al (2007), we divide the data sample into two segments; one for the in-sample estimation, the other for the out-of-sample comparison. Specifically, we run regression using data from January 2000 to May 2009, on the basis of which we forecast the price from June 2009 to June 2010. We then compare the forecast series with the actual price. As can be seen in Fig. 5, the model with regime-dependent beliefs exhibits better forecasting accuracy. Such outperformance is consistent for up to 13 month forecasting horizons.

Since the separation of the sample is somewhat arbitrary, a natural question is whether the result is sensitive to the sample selection. As a robustness check, we compare the forecasting performance using the root mean square error (RMSE) and mean absolute error (MAE) for different forecasting horizons. Specifically, we calculate the h-months-ahead forecasting error, for $h = 1, 2, ..., 12$. The forecasting recursion is based on a rolling estimation win-

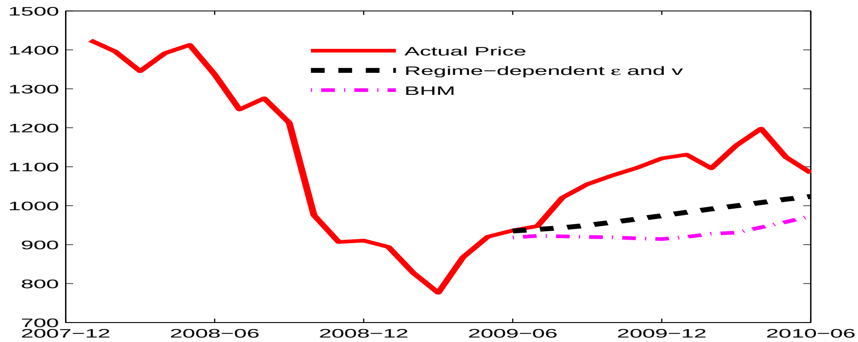

Figure 5: Comparison of out-of-sample predicability in three HAMs. The solid line plots the actual price series discounted by the CPI. the dashed and dash-dotted lines correspond to the forecast prices from Eq. (8) and Boswijk et al (2007) (BHM) model, respectively.

dow with a fixed sample size, which is 96 (8 years of data) in our practice. We estimate the model applying the first 96 observations $(p_1, p_2, ..., p_{96})$, and then obtain the first h-months-ahead forecasts $(\widetilde{p}_{96+1}, ..., \widetilde{p}_{96+h})$. Similarly, we obtain the second h-months-ahead forecasts $(\widetilde{p}_{97+1}, ..., \widetilde{p}_{97+h})$ from observations $(p_2, p_3, ..., p_{97})$. Such a process is repeated until we obtain the last h-months-ahead forecasts $(\widetilde{p}_{126-h+1}, ..., \widetilde{p}_{126})$ from observations $(p_{31-h}, p_{32-h}, ..., p_{126-h})$ (126 is the total number of observations in the full sample). This recursion leads to to a sequence of $k_h = 126 - 96 + 1 - h = 31 - h$ density forecasts. Based on recursive forecasting, we calculated h-steps-ahead RMSE and MAE, namely

$$RMSE_h = \sqrt{\frac{\sum_{t=96}^{t=126-h} (\widetilde{p}_{t+h} - p_{t+h})^2}{k_h}}, \qquad MAE_h = \frac{\sum_{t=96}^{t=126-h} |\widetilde{p}_{t+h} - p_{t+h}|}{k_h}.$$

Note that the smaller the value of RMSE and MAE, the better the forecasting accuracy. The results from RMSE and MAE are broadly in line with the simple segmentation discussed above. The second and third columns of Table 5 present the RMSE from our model estimated under Markov regime switching and the Boswijk et al (2007) model estimated using nonlinear least square estimation. The fourth and fifth columns show the MAE for the two models. For forecasting horizons 1 to 12 months, both RMSE and MAE are smaller in our model than in the Boswijk et al (2007) model. These results

Table 5: Forecasting Error

Steps	RMSE		MAE	
	BHM	RD* ε_t and v_t	BHM	RD ε_t and v_t
1	272	71	105	50
2	302	120	155	86
3	332	153	191	111
4	362	189	227	136
5	405	240	277	173
6	454	288	332	211
7	498	324	380	246
8	535	354	416	284
9	572	391	449	321
10	614	428	484	353
11	645	445	512	369
12	672	456	529	384

RD stands for regime-dependent

suggest that models estimated with regime-dependent beliefs possess better forecasting accuracy than the nonlinear model in Boswijk et al (2007).

5 Conclusion

We develop a HAM within the framework of Markov-switching by allowing investor beliefs (trading strategies) to be regime dependent. Such a set up improves the forecasting accuracy compared to the nonlinear least square estimation that is frequently applied in estimating HAMs. This may be attributed to the unobservable state variable that captures the underlying switch of different market condition through the data.

Our estimation results show that chartists and noise traders exist in the market simultaneously, which is consistent with the results in current literature that document between-group heterogeneity (Boswijk et al (2007); Frijns et al 2010). Perhaps more interestingly, we also find that behavioural heteogeneity could arise within the group, as investors adapt their strategies over time based on the state variable. Moreover, the regime-dependent property of this model captures a smooth transition from boom to bust periods and vice versa. Our estimation classifies the sample periods into two regimes, which match well the crisis and boom periods in real financial episodes.

Due to the relatively large number of parameters to be estimated, we try to keep our model as simple as possible. This raises several caveats on our results. First, we estimate a two-state Markov switching model. The market conditions however cannot be captured by only two states, which simply divide the sample into boom and bust periods. One future research direction would be to extend the model to N-states so that it captures the business cycle at different stages of development. This may require higher frequency data, which makes it easier to obtain stable estimates. Second, the demand function in our model is highly aggregated, rendering it difficult to shed light on the behaviour of investors whose beliefs are offset by the other type within the same group. Extending the model to account for more disaggregated trading strategies would be a valuable contribution to the literature.

Appendix

A. Multicollinearity

Note that the demand by fundamentalists and chartists might be correlated, especially when the short-term market value v_t is close to the fundamental value u_t over certain time periods. If this is the case, the estimated standard errors of the affected coefficients tend to be large as a result of multicollinearity, which could give rise to the misleading conclusion that the coefficient is zero. We check for such possibility by testing the null hypothesis that $\alpha = \beta = 0$. Our test rejects the hypothesis, which indicates the presence of multicollinearity. We then drop the demand function of the fundamentalists from Eq.(8) and run the estimation again. The results are shown in Table 6. As expected, the estimate of β becomes highly significant. The results indicate that the market power of chartists B, who believe the price trend $(p_t - v_t)$ will reverse, is greater than that of chartists A, who expect the price trend to persist. If we drop the demand function of chartists from Eq.(8) instead, the estimation of α becomes statistically significant at the 10% level, as shown in Table 7. It is difficult to rule out the presence of fundamentalists. Note that the multicollinearity problem affects the estimation of the standard error but not the entire model goodness of fit, neither does it bias the results. If we simply drop the demand by fundamentalists, the omission of a possibly relevant variable would result in biased estimation for the remaining

Table 6: Estimation results excluding fundamentalists. The sample period is from January 2000 to June 2010.

Variable	β	v_0	v_1	σ_0	σ_1	$P_{0,0}$	$P_{0,1}$
Coefficient	$-.044$	644.680	1617.007	63.631	31.590	.945	.049
p-value	.01	.000	.000	.000	.000	.000	.120
log-likelihood=-663.103, AIC=10.637							

Table 7: Estimation results excluding chartists. The sample period is from January 2000 to June 2010.

Variable	α	σ_0	σ_1	$P_{0,0}$	$P_{0,1}$
Coefficient	.027	70.034	35.413	.954	.033
p-value	.091	.000	.000	.000	.236
log-likelihood=-671.483, AIC=10.738					

explanatory variables. To avoid this problem, we focus on the estimation of the model that incorporates the trading activities of fundamentalists, despite the multicolinearity.

B. Regime-dependent β

In estimating the model in the main test, the parameter β, the response factor of the price to the estimation bias, was assumed to be constant. In this appendix, we allow β to be state-dependent such that

$$\beta = \beta_t = \begin{cases} \beta_0, & s_t = 0, \\ \beta_1, & s_t = 1. \end{cases} \tag{12}$$

This further allows chartists to update their market weight as well as their sensitivity to the estimation bias given that $\beta = \gamma\,(\beta_A\omega_2 - \beta_B\omega_3)$. Note however that there is not enough information to differentiate the origins of the switch in β.

Table 8 shows the estimation results for Eq.(12). Consistent with the previous estimation, the volatility is higher in regime 0 than in regime 1. As demonstrated in Fig. 6, the fitted value based on our model matches well the real price series and the 1-step ahead prediction captures the ups and downs of the price movements even when they are extreme.

Table 8: The 2-state Markov regime switching in β and v. The sample period is from January 2000 to June 2010.

Variable	β_0	β_1	α	v_0	v_1
Coefficient	0.023	-0.060	0.014	2532.370	1515.315
p-value	0.607	0.038	0.571	.000	.000
	σ_0	σ_1	$P_{0,0}$	$P_{0,1}$	
	62.160	29.221	0.958	0.043	
	0.000	0.000	0.000	0.162	
log-likelihood $=-662.303$, AIC=10.656					

The classification of regimes are broadly in line with previous estimation, as shown in Fig. 7, the regime 0 is centered around crisis periods while regime 1 corresponds to tranquil periods. Similarly, α is positive but insignificant. The estimation of β is positive in regime 0 and negative in regime 1. However it is only significant in regime 1. The expected short-term fundamental is higher in regime 0 than in regime 1 with $v_0 = 1910$ and $v_1 = 1685$. The estimation results in regime 0 is quite different from the previous estimation, which is however not significant. This may due to the under-identification of our model, making it difficult to distinguish where the switching comes from.

The in-sample estimation of the current model has similar explanatory power to the previous model with a regime dependent price benchmark and noise term. However, the classification of the regimes in this model is less accurate. It also exhibits poorer out-of-sample forecasting accuracy, as shown by Fig.8.

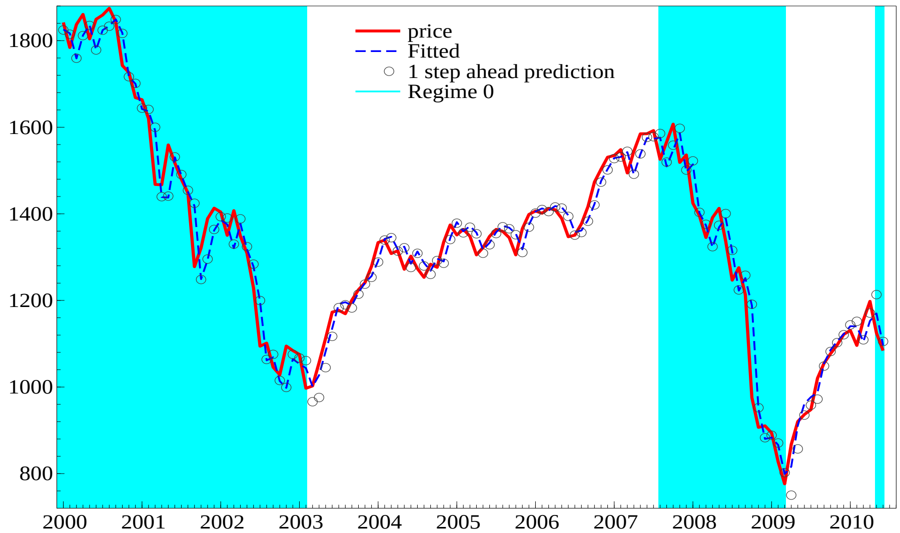

Figure 6: The index, fitted value, and 1-step-ahead prediction when β is allowed to switch. The solid line represents the S&P 500 Index discounted by the CPI, the dashed line plots the fitted value from the estimation of Eq. (12), the circles reflect the 1-step ahead forecast, and the shaded areas indicate regime 0.

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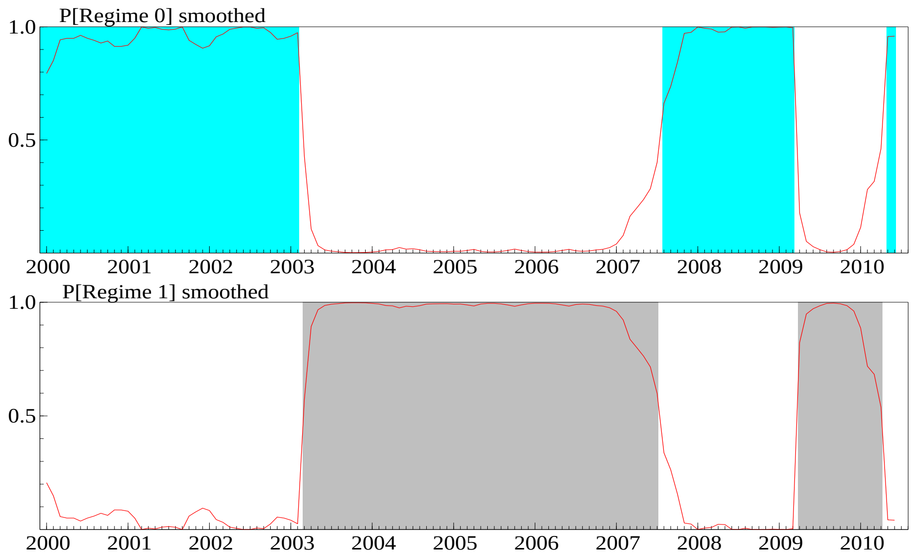

Figure 7: Smooth transitioned probability when β is regime-dependent.

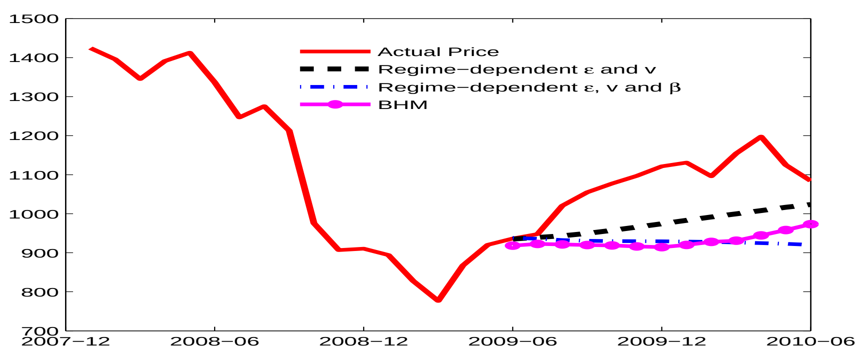

Figure 8: Comparison of out-of-sample forecasting performance. The solid line plots the actual price, the dashed, dash-dotted and the circle-marked lines plot the forecast prices from Eq. (8), Eq.(12 and Boswijk et al (2007) model.

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