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**Research Paper 204**

**October 2007**

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### A Causal Framework for Credit Default Theory

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ISSN 1441-8010

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# A causal framework for credit default theory

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(5 June 2007)

(Last revised 20 August 2007)

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## Abstract

Most existing credit default theories do not link causes directly to the effect of default and are unable to evaluate credit risk in a rapidly changing market environment, as experienced in the recent mortgage and credit market crisis. Causal theories of credit default are needed to understand lending risk systematically and ultimately to measure and manage credit risk dynamically for financial system stability. Unlike existing theories, credit default is treated in this paper by a joint model with dual causal processes of delinquency and insolvency. A framework for developing causal credit default theories is introduced through the example of a new residential mortgage default theory. This theory overcomes many limitations of existing theories, solves several outstanding puzzles and integrates both micro and macroeconomic factors in a unified financial economic theory for mortgage default.

JEL classification: B41, C81, D14, E44, G21, G32, G33.

Keywords: Causal framework, credit default risk, delinquency, insolvency, mortgage default.

# 1. Introduction

A credit default represents the financial failure of an entity (a person or a company). A theory of credit default should therefore represent a systematic understanding of the causes which directly lead to the effects which are associated with credit defaults. Such a theory is required to provide direct causal connections between macroeconomic causes of changing financial environment and their microeconomic effects on changing personal or corporate financial conditions, leading to possible credit defaults. Most existing theories<sup>1</sup> of credit default do not meet this causal requirement.

The purpose of this paper is to set down a framework from which causal credit default theories can be developed through a new structure for incorporating risk factors. The framework and its application to develop a causal credit default theory include several other contributions.

Firstly, a definition for credit default is given<sup>2</sup>. Surprisingly credit default has rarely been defined in research, even though in practice the term is used with several different meanings depending on the situation. The credit default definition given here is used to develop the framework which is described in the next section.

Secondly, the framework provides a new method<sup>3</sup> for developing causal credit default theories which are not heavily dependent on empirical data. Many existing approaches<sup>4</sup> face difficult problems of statistical estimation due to the lack of abundant and high quality data and their models suffer from historical sampling bias. Our approach can be used to price credit risk in changing environments which may be without historical precedence whereas most existing approaches cannot, as we witnessed in the mortgage and credit crisis in 2007.

Thirdly, the causal framework leads to unified credit default theories where the probability of default (PD) and loss given default (LGD) are determined<sup>5</sup> endogenously and consistently. Existing approaches treat PD and LGD as separate estimations<sup>6</sup> and therefore face additional problems of consistency: such as those in relation to assumptions about how PD and LGD are correlated.

Fourthly, as a concrete example, a causal credit default theory for residential mortgages is developed to illustrate the above benefits of the framework. Surprisingly mortgage default theory has not advanced beyond credit default theories available generally for corporate securities even though the causes of mortgage defaults are much simpler<sup>7</sup> than the causes of corporate debt defaults. Several sections<sup>8</sup> below are devoted to describing the details of the new theory of mortgage default.

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<sup>1</sup> See Section 7 for a discussion of related work including a review of existing structural and reduced form approaches. Some structural approaches may be considered causal, but most have only indirect causality when including additional risk factors, such as interest rates. Reduced form approaches generally make no claims to be causal, see Jarrow and Protter (2004).

<sup>2</sup> See Section 2.

<sup>3</sup> This is illustrated by the concrete example of a new mortgage default theory.

<sup>4</sup> See Section 7 particularly for the needs of reduced form approaches.

<sup>5</sup> See Section 5 for the case of mortgage defaults.

<sup>6</sup> See Section 7 for recent debates on their relationships.

<sup>7</sup> Wallace (2005) made this observation for mortgages, while Descartes (1637) made this observation generally for all analytical knowledge in his third rule of the "Method". Other examples of use of mortgage default theories include Elul (2006), Capozza, Kazarian and Thomson (1998) and Deng, Quigley and Van Order (2000).

<sup>8</sup> See Sections 3-5.

Fifthly, the causal mortgage default theory provides a valid basis for stress testing of loan portfolios where macroeconomic causes are directly and nonlinearly linked<sup>9</sup> to microeconomic effects of default. Existing non-causal theories suffer from linearization assumptions and from the uncertainty about whether the observed correlations between dependent and independent variables are not merely coincidences. The theory developed here shows examples<sup>10</sup> of the drawbacks of linearization and how causal uncertainties can occur.

Sixthly, with the new mortgage default theory, several outstanding puzzles in the credit default literature are resolved. For example, the puzzles resolved include explanations of how falling mortgage delinquency rates and falling property prices can occur simultaneously in Hong Kong SAR<sup>11</sup> 1998-2003 and how positive and negative correlations between PD and LGD are possible simultaneously in a portfolio.

After the development of a causal credit default theory<sup>12</sup>, we define at the end of Section 5 the difference between direct and indirect causation and then discuss macroeconomic causes to credit default in Section 6. In Section 7, we discuss related work by providing a limited review of existing approaches compared with the new approach described in this paper. The final section is given over a conclusion and suggestions for further research.

## 2. Definition of Credit Default

The term “credit default” has been used with many different meanings in practice. It can mean<sup>13</sup> something as minor as a late payment of a debt obligation, so that a bank can apply a penalty “default” interest rate between the due date and the actual payment date. It can also mean something as serious as a bankruptcy or insolvency where the lender initiates a recovery process to limit loss from a collateralised loan. In spite of an extensive literature on credit defaults there are few clearly stated definitions of credit default.

Such varied use of terminology in practice raises doubt about what information is really contained in the empirical default frequency data collected by lending institutions and it is indicative of the absence of an accepted credit default paradigm which would have assisted in a more systematic collection of default data across the industry.

Two concepts: delinquency and insolvency underlie most definitions of credit default. Delinquency is defined as a failure to meet a loan payment by a due date, whereas insolvency is defined as a situation where assets are less than liabilities. Most usages of the term credit default really revolve around the concept of delinquency. The current Basel II definition of default is essentially a 90-day delinquency definition, although there are issues of materiality, which relates to the dollar substance of the payments in arrears. On the other hand, the structural approach<sup>14</sup> to credit default theory uses implicitly the insolvency definition of credit default.

The clearest definition of credit default is given by Moody's<sup>15</sup> where a credit default<sup>16</sup> involves both delinquency and the notion of expected loss to the lender. This definition of

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<sup>9</sup> See Section 6.

<sup>10</sup> See Section 5 for a discussion.

<sup>11</sup> See Yam (2003) and Fan & Peng (2005). A resolution of the puzzle is given in Section 6.

<sup>12</sup> See Sections 3-5.

<sup>13</sup> In strict legal terms, a default is any failure to meet the terms of a credit agreement and therefore this meaning is a legal definition of default. Clearly this definition is not useful for financial economic applications, because many such defaults would have little or no economic consequence.

<sup>14</sup> See Section 4 and Section 7 for a discussion.

<sup>15</sup> See Keenan (1999) for a description of their collection of default data.

default comes closest to why one is interested<sup>17</sup> in credit defaults in the first place: it is to estimate expected losses from lending. Even the delinquency definition of default with a specified time lag such as currently adopted by Basel II can be interpreted as a convenient indicator of potential loss for secured loans.

The underlying logic is: if a corporation or an individual is unable to meet a debt payment obligation by the due date, then the assumption is either a debt-restructure or bridging finance will be sought by the borrower. If the implied cash flow problem is not solved within a fixed period such as 90 calendar days for Basel II, then it is assumed that the entity may be insolvent, the loan is considered formally in default and the creditor would then initiate proceedings to recover what remains of the secured assets.

In this paper, it is assumed that a credit default theory is mainly intended for use to estimate expected losses through an understanding of the causes of credit default. In practice a loss from a given default often involves lengthy delays (of months or even years) either in a sale of the collateralised asset or in a sale of that asset to a debt collector for loan value recovery or in making a claim from an insurer. The practical definition of a default as a delinquency with a time lag is therefore merely to provide an early recognition of potential loss and the time lag may vary from country to country due to cultural and legal differences.

There is no compelling research to suggest a particular delinquency period: 30-days, 90-days or 180-days which will optimise the trade-off between timeliness in the warning of a loss and the likelihood of an actual loss from default. Therefore there is a need to make a distinction between the current practical definitions of default and a theoretical definition, which is necessary to create a credit default theory. A successful credit default theory should be able to estimate the optimal delinquency time lag which is likely to indicate significant expected loss in any given jurisdiction. Therefore a specific time lag should not be used in a theoretical definition of credit default.

With this preamble, simple working theoretical definitions of credit defaults are given as follows.

*For an unsecured loan, a credit default is defined as a delinquency. For a secured loan, a credit default is defined as the occurrence of both delinquency and insolvency.*

For an unsecured loan such as a credit card loan, counting a delinquency as a credit default seems a little harsh at first sight. But many such loans have very low payment obligations so that delinquency rates and therefore default rates are substantially less than what one would expect. Indeed lenders of unsecured loans seek to obtain substantial gains from charging high interest rates on outstanding balances after the minimum payment obligations have been made. The loss given default when the borrower is unable to even make the minimum payment obligation will depend on the debt collection process and other cultural and legal factors. It is outside the scope of this paper to provide a theory of expected loss for unsecured loan defaults.

Our definition of credit default for a secured loan is consistent with Moody's definition<sup>18</sup> mentioned above, as it includes the notion of an expected loss. In this definition, neither

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<sup>16</sup>Moody's defines a bond default as any missed or delayed disbursement of interest and/or principal, bankruptcy, receivership, or distressed exchange where (i) the issuer offered bondholders a new security or package of securities that amount to a diminished financial obligation (such as preferred or common stock, or debt with a lower coupon or par amount) or (ii) the exchange had the apparent purpose of helping the borrower avoid default.

<sup>17</sup>A good credit default theory provides more than expected loss estimates, as it should provide insights into what determines the soundness of financial positions of entities.

<sup>18</sup>See Keenan (1999).

delinquency nor insolvency alone is sufficient to cause a credit default. Both delinquency and insolvency are necessary and sufficient for credit default.

In reality a credit default event for a secured loan is a sequence of two temporally separated events: a delinquency event followed by an insolvency event. If a borrower always make full loan payments by due dates, insolvency is irrelevant. Insolvency alone cannot cause a credit default, because the assets of an entity is not easily observable at any given time, even to insiders, particularly if the assets include intangibles such as franchise value, trademarks and intellectual property. Only a delinquency is assumed to cause a net equity position to be evaluated, leading to a possible conclusion of insolvency and therefore credit default.

If a corporation or an individual has strong and steady cash flows, such as companies in some service industries or utilities, then very high debt levels relative to assets can be carried without the risk of the entity's solvency condition being evaluated. Another example<sup>19</sup> is the Hong Kong property market 1998-2003 where home owners had sufficient liquidity to forestall default even though many had negative equity. In such cases, structural models which use insolvency as condition of default would overestimate the risk of default.

On the other hand, during the dot.com era, many hi-tech companies were continually delinquent on their debt payments due negative operating cash flows ("burning cash") but were able to survive through capital raisings as shareholders and lenders continually raised the valuations of their corporate assets. Inability to meet a debt payment obligation by the due date in such cases does not necessarily lead to credit default.

Another typical example is sovereign debt, which may be delinquent but often does not lead to credit default by our definition. On 2 July 1998 the Republic of Venezuela missed a debt payment<sup>20</sup> on treasury bonds but made up for it on 9 July 1998. This was not a credit default event by our definition but was briefly a delinquency event.

The causal framework therefore consists of two parts: delinquency and insolvency. Loan delinquency is a condition of liquidity failure or having insufficient cash flow to service the loan. Delinquency triggers a solvency assessment which may lead to a conclusion of negative equity position causing loan termination and an expectation of loss by the lender. This framework is expected to have wide applications where the main tasks are to build specific causal models for delinquency and models for insolvency within the general framework.

The causal framework also includes a formal theory of how a model of delinquency and a model of insolvency can be joined together to create a theory of credit default. An investigation into causes of credit default is a joint investigation into causes of delinquency or liquidity failure and causes of insolvency or negative equity. The formal theory is best introduced by a specific example, which is a new theory for mortgage default developed below.

### 3. Model of Delinquency

Delinquency occurs when a borrower is unable to make a loan payment by the due date, caused by liquidity failure. Negative cash flow is considered one of the main causes of liquidity failure. For a corporation, debt payments are usually made from operating cash flows. Liquidity failure tends to occur when there is insufficient income from operating a

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<sup>19</sup> This example is discussed in detail below in Section 6.

<sup>20</sup> See Keenan (1999).

business which is running at a loss. As a simplification<sup>21</sup> liquidity failure is modelled by a situation of negative cash flow.

In the analogous situation of a household with a home loan liquidity failure occurs when total disposable income after allowing for cost of living and other expenses is insufficient to meet debt payments. For an investor of a rental property with an investment loan cash flow before debt payment is determined by rental income plus the tax benefit from possible negative gearing. For simplicity of illustration, we restrict ourselves to owner-occupier loans or home loans for brevity.

### 3.1 Loan Serviceability Ratio

For application to residential mortgages, we introduce a loan serviceability rate  $r_s$ , which is defined as the maximum loan interest rate a owner-occupier borrower can service a loan amount  $L$  from net disposable income after living expenses,

$$r_s = \frac{\tau(W) - D - X}{L} \quad (1)$$

where  $W$  represents gross income from wages, salaries and other sources,  $\tau(\cdot)$  is a nonlinear function to calculate after-tax income from pre-tax income,  $D$  is other debt repayment and  $X$  is the cost of living which may be represented by some minimum or acceptable standard of living cost for the given number of persons. The numerator can be considered the net disposable income available to service a housing loan.

Fiscal policy would affect the nonlinear tax function  $\tau(\cdot)$  directly, employment conditions, divorce rates etc would affect the gross income  $W$  and inflation would affect the cost of living and also wages and salaries. For brevity, we will refer to an owner occupier loan simply as a home loan in this paper.

A typical home loan in Australia has a variable or adjustable interest rate and a 25-year term at origination. The mortgage payment rate for home loan is therefore<sup>22</sup>

$$r + \delta = r + \frac{r}{(1+r)^n - 1} \quad (2)$$

where  $\delta$  is the loan amortization rate and  $n = 25$  is the typical term to maturity at origination. In this case, the loan size is reduced by the factor  $(1 - \delta)$  each year.

Given a loan serviceability rate we can define a Loan Serviceability Ratio (LSR) which is the loan serviceability rate divided by the loan payment rate. For home loans, LSR is denoted mathematically by

$$x_s = \frac{r_s}{(r + \delta)} = \frac{\tau(W) - D - X}{(r + \delta)L} \quad (3)$$

Note that other serviceability ratios such as Debt Service Ratio (DSR) or Debt Cover Ratio (DCR) typically does not include the cost of living as we have done. The advantage of LSR as a serviceability risk measure is that because LSR is normalised to loan size,

<sup>21</sup> Liquidity failure could also be caused by business interruptions due to natural catastrophes like earthquakes or floods.

<sup>22</sup> This is equivalent to the more familiar expression:  $r / \{1 - (1+r)^{-n}\}$



serviceability of loans of different sizes can be compared.

### 3.2 LSR Evolution

The risk in loan serviceability comes from the fact that serviceability changes over time due changes in individual circumstances and changes in the economic environment. A loan which may have started off as being easily serviceable loan may become a struggle for the borrower due to unanticipated adverse developments.

The LSR variable given an initial value  $x_s$  can evolve a priori in any stochastic manner since the underlying individual variables on which the LSR variable depends can evolve according a number of stochastic processes. The finance literature provides studies on many possibly relevant stochastic processes, including some which are described by probability density functions with fat tails. It is premature for this paper to examine the best candidate for the stochastic process which determines the evolution of the LSR variable. As we have no strong reason at this stage to reject a simple Gaussian stochastic process, we will make this assumption in this paper for simplicity, leaving other stochastic processes for possible future investigation.

With the Gaussian assumption, LSR evolves according to the well-known parabolic heat diffusion equation used by Black and Scholes (1973) and Samuelson (1973) for option pricing. As we are not concerned with traded markets, the appropriate discounting rates or drift rate<sup>23</sup> to avoid arbitrage opportunity rising in option pricing of securities is unimportant here. The free boundary solution is given by a lognormal distribution for LSR with a drift rate  $\mu_s$  and volatility  $\sigma_s$ . Another way of expressing this result is that after a time  $t$  the evolution of LSR can be described a the random variable  $z_s$  defined by

$$z_s = \frac{\ln(x_s) + (\mu_s - \frac{1}{2}\sigma_s^2)t}{\sigma_s\sqrt{t}} \quad (4)$$

That is  $z_s$  is a standard Normal variate with zero mean and unit standard deviation. The probability of liquidity failure  $P_s$  when  $LSR < 1$  is given by

$$P_s = N(-z_s) \quad (5)$$

where  $N(\cdot)$  is the standard Normal (cumulative) probability function.  $P_s$  is the probability that the borrower will face a cash flow problem in meeting a loan payment at time  $t$ . This provides a simple model for loan delinquency<sup>24</sup> and may be used to explain some empirical "default" data which are actually delinquency data.

A borrower may purchase insurance to avoid potential situations of liquidity failure. A manufacturing company may buy business interruption insurance and a wage earner may buy income protection insurance. In such cases, the borrower will face a cash flow problem only when the insurance company rejects the claim and the probability of liquidity failure including counter-party risk is then given by  $P_s P_{CD}$  where  $P_{CD}$  denotes the probability of claim denial. Under usual circumstances the probability of claim denial is assumed to be very small. Unless there is specific information on insurance, we will generally make the simple assumption that the borrowers are uninsured against liquidity failure.

<sup>23</sup> See Samuelson (1973).

<sup>24</sup> Commins, Esho, Pattenden, Sy, and Thavabalan (2007) have used this model to forecast delinquency rates for a portfolio of newly approved residential mortgages.

Note that delinquency is modelled by a single composite variable LSR which is a nonlinear function of micro and macroeconomic causal variables. In general the causal variables interact nonlinearly to determine LSR and cannot be made linear without making further compromising assumptions. The LSR variable is random and uncertain over time because the causal variables are uncertain over time due to changing borrower circumstances and unexpected developments in the macroeconomic environment. Risk factors such as interest rates cause changes in the probability of delinquency through the LSR variable in combination with all other risk factors.

In concluding this section, we emphasize this is merely one of many potential models for delinquency or liquidity of the borrower. We have chosen the above simple one because empirical data needed for the model have been collected by lending institutions and are available to be used to estimate loan serviceability. The LSR variable which is the key stochastic variable to model liquidity in our framework can depend on an arbitrary number of risk factors each of which may have its own peculiar statistical properties. Our main contribution to credit default theory is to suggest how the relevant risk factors should be combined in a single causal variable in a parsimonious model. The alternative is to have a large number of ad-hoc independent risk variables each with its own stochastic dynamic in an unstructured model devoid of any causality or insight.

#### 4. Model of Insolvency

In the original Merton model<sup>25</sup> for insolvency the random variable which determines credit default risk is the assets to liabilities ratio, which defines a situation of negative equity if it is less than one. The Gaussian model for this random variable has been generalised<sup>26</sup> to include non-Gaussian models in the KMV modifications.

Here we follow the simple approach but will not repeating the derivations and arguments of the original Merton paper, except for summarizing the main results in our notation. In our application to residential mortgages the corresponding random variable is the property Value-to-Loan ratio or the reciprocal of the Loan-to-Value ratio (LVR) which we designate as  $x_V$  and defined simply by

$$x_V = V / L \quad (6)$$

If we model the evolution of  $x_V$  as a Gaussian stochastic process, as did Merton, then the evolution of the LVR variable after time  $t$  can be described by a random variable  $z_V$  defined by

$$z_V = \frac{\ln(x_V) + (\mu_V - \frac{1}{2}\sigma_V^2)t}{\sigma_V \sqrt{t}} \quad (7)$$

where the random inverse LVR variable has a drift rate  $\mu_V$  and volatility  $\sigma_V$ . The variable  $z_V$  sometime called the “distance to default” in the Merton model. It is a standard Normal variate with zero mean and unit standard deviation. In our model we may call this “distance to negative equity”.

The probability of negative equity  $P_V$  when  $LVR > 1$  is given by

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<sup>25</sup> See Merton (1974).

<sup>26</sup> See Bohn (2006).

$$P_V = N(-z_V) \quad (8)$$

where  $N(\cdot)$  is the standard Normal (cumulative) probability function. The random value-to-loan ratio  $x$  evolves subsequently with a probability density function given by

$$p(y) = \frac{1}{\sqrt{2\pi\sigma_V^2 t}} \exp \left\{ -\frac{(y - \mu_V t + \frac{1}{2}\sigma_V^2 t)^2}{2\sigma_V^2 t} \right\} \quad (9)$$

where  $y = \ln(x)$  and the distribution depends on the drift, volatility parameters and time. The initial value-to-loan ratio  $x_V$  evolves to  $x_V e^y$  at time  $t$  where  $y$  is random variable described by the above probability density function.

The borrower can be viewed as a holder of a perpetual American put option, which can be exercised at any time  $t$  provided the option has a positive value. The put option is “in-the-money” if the property value is less than the loan value: a condition of negative equity. The expected value of the put option is given by the expected value of the “pay-off” per unit of the loan value:

$$\text{Max}(0, 1 - x_V e^y) \quad (10)$$

Integration of the pay-off function over the probability density function gives the expected pay-off for the borrower in exercising the option. Since the expected gain to the borrower equals the expected loss to the lender  $V_{EL}$ , the value of the expected loss per unit loan value is given by

$$V_{EL} = N(-z_V) - x_V \exp(\mu_V t) N(-z'_V) \quad (11)$$

where we have introduced a second argument for the cumulative Normal probability function  $N(\cdot)$  with  $z'_V = z_V + \sigma_V \sqrt{t}$ .

We have an expression for the expected loss and an expression for the probability of negative equity. If negative equity happens to cause a default then the expected value of loss-given-default  $V_{LGD}$  is given by

$$V_{LGD} = 1 - x_V \exp(\mu_V t) N(-z'_V) / N(-z_V) \quad (12)$$

We consider this to be an adequate model for estimating expected loss-given-default (LGD) for residential mortgages in this paper. Note that we do not discuss discounting to obtain the present value, as we are not concerned with trading LGD in market equilibrium, but we are only concerned with its expected value at time  $t$ .

The above expression assumes the lender has no mortgage insurance. If the lender does have mortgage insurance then the expected loss-given-default must be adjusted for likely insurance recoveries from claims. The amount of realised protection falling short of the full insured value is itself a random variable, which may be modelled in a similar way to the above random variables.

Introducing an analogous random variable  $z_I$  for insurance shortfall, at a later time, the insured loss-given-default is modified to

$$V_{LGD} = N(-z_I) \{1 - x_V \exp(\mu_V t) N(-z'_V) / N(-z_V)\} \quad (13)$$

where the modifying term on the right hands side denotes the probability of shortfall  $z_I$ , which is a random variable modelling the position of the insurer. In most situations the probability of insurance shortfall would be zero or very small, according to recent experiences in a benign financial environment. However under highly stressed market conditions where insurers are making substantial losses the insurance shortfalls from claim denials or loss adjustments may increase significantly. In most cases, one would use models where insurance shortfall becomes significant only under large and sustained falls in property prices. Equation (13) provides an approach to incorporate counter-party insurer risk for the lender.

We have considered expected loss in relation to insolvency of an entity. The model can also be modified to consider expected loss by a debt holder of various levels of seniority in relation to corporate debt securities. The recovery rates<sup>27</sup> from loss due defaults are areas of current research.

In concluding this section, we note that our contribution restores the simplicity of Merton's original idea of insolvency as a cause of default. In doing this we are suggesting<sup>28</sup> that subsequent developments of structural approach mostly attempt to extend the basic solvency model by introducing risk factors which do not sit comfortably in that context. Instead, we suggest risk factors such as interest rate for example belong more appropriately in the model for liquidity.

## 5. Model of credit default

Given our hypothesis that credit defaults are determined jointly by two random variables  $z_S$  for liquidity failure and  $z_V$  for negative equity, we can model credit default itself as a random variable  $z_D$  which is some function  $f$  of the independent variables:

$$z_D = f(z_S, z_V, t). \quad (14)$$

### 5.1 Model of Default Probability

Let  $\mathbb{D}$  denote the set of default events at time  $t$ . The default events are determined solely by liquidity failure events and negative equity events, denoted here by the sets  $\mathbb{S}$  and  $\mathbb{V}$  respectively where

$$\mathbb{S} = \{z \mid z < -z_S : t - t_g\} \quad (15)$$

$$\mathbb{V} = \{z \mid z < -z_V : t\} \quad (16)$$

Note that  $\mathbb{S}$  and  $\mathbb{V}$  may be temporally separated<sup>29</sup> by a time gap  $t_g$  and the probability  $P(\mathbb{D})$  for  $\mathbb{D}$  is given by Bayes' rules for conditioning

$$P(\mathbb{D}) = P(\mathbb{D} \mid \mathbb{S} \cap \mathbb{V}) P(\mathbb{S} \cap \mathbb{V}) \quad (17)$$

<sup>27</sup> For example, see Dullmann and Trapp (2004).

<sup>28</sup> For a more detail discussion see Section 7 below.

<sup>29</sup> In most applications we can ignore this time gap as a first approximation, even though a finite value is more realistic.

Our assumption that  $\mathbb{S} \cap \mathbb{V}$  is necessary and sufficient for  $\mathbb{D}$  means

$$P(\mathbb{D} | \mathbb{S} \cap \mathbb{V}) = 1. \quad (18)$$

Hence the probability of default  $P(\mathbb{D})$  is simply given by the joint probability  $P(\mathbb{S} \cap \mathbb{V})$ . In general the random variables  $z_s$  for liquidity failure and  $z_v$  for negative equity may be correlated. Under our Gaussian assumptions, the probability of default at time  $t$  is described by a bivariate Normal probability density function:

$$p(z, z', t) = \frac{\text{Exp}(-\frac{1}{2}Q)}{2\pi\sqrt{1-\rho^2}} \quad (19)$$

where  $\rho$  is the correlation coefficient between the two random variables and  $Q$  is defined by

$$Q \equiv \frac{z^2 - 2\rho zz' + z'^2}{1 - \rho^2}. \quad (20)$$

The joint probability of default is then given by

$$P_D \equiv P(\mathbb{D}) = \int_{-\infty}^{-z_s} \int_{-\infty}^{-z_v} p(z, z', t) dz dz' \quad (21)$$

In the special case where the twin causes of default are independent and uncorrelated then  $\rho = 0$  then

$$P(\mathbb{D}) = P(\mathbb{S} \cap \mathbb{V}) = P(\mathbb{S})P(\mathbb{V}) \quad (22)$$

or,

$$P_D = P_S P_V = N(-z_s)N(-z_v). \quad (23)$$

This result provides immediately an important insight. Since by necessity  $P_S \leq 1$  the probability of default in our theory is always less than or equal to that from Merton-type models, all else being equal. This may explain why the expected default frequency (EDF) predicted from the KMV approach<sup>30</sup> tends to over-estimate the actual default rates. In case of the Hong Kong property crash 1998-2003 described by Fan and Peng (2005), we expect  $P_S \ll 1$  as the mortgage holders' ability to service their debt remained largely unimpaired<sup>31</sup> even though many loans have had negative equity.

## 5.2 Model of Loss Given Default

Regardless of how the default probabilities of delinquency and insolvency are correlated, the LGD is given by the expected equity shortfall:

$$V_{LGD} = N(-z_I) \{1 - x_v \exp(\mu_v t) N(-z'_v) / N(-z_v)\}, \quad (24)$$

for the general case with mortgage insurance. The corresponding expected loss for the case of uncorrelated causes is given by the analytic expression:

<sup>30</sup> See Kealhofer (2003).

<sup>31</sup> The improved mortgage serviceability is explained in detail in Section 6.

$$V_{EL} = N(-z_S)N(-z_I)\{N(-z_V) - x_V \exp(\mu_V t)N(-z'_V)\}. \quad (25)$$

Note that evaluation of the random variables may be temporally separated but in the following order:  $z_S$ ,  $z_V$  and  $z'_V$ , then  $z_I$ .

Unlike the reduced form approach<sup>32</sup> which requires a separate and possibly inconsistent procedure to calculate expected losses through recovery rate estimation, in our approach all relevant quantities are determined endogenously and consistently. Vexing questions such as how PD and LGD are correlated, what are acceptable levels for LGD or whether microeconomic or macroeconomic factors are more important simply do not arise in our approach.

Note that the current approach links the four drivers of credit risk identified by the Basel II Accord: exposure at default, probability of default, loss given default and maturity into a single unified model, where questions about how the drivers are correlated through the economic cycle being implicitly addressed through the model parameters.

### 5.3 Term structure of PD and LGD

Given initial values of LSR and LVR, our model can be used to assess how the loan characteristics will change in time under various scenario assumptions. For a home loan with a LSR given by  $x_S = 1.4$  and a range of LVR levels we consider two illustrative example scenarios of steadily rising and steadily falling property prices.

In the examples chosen, we have assumed independent causation because over a period of a few years there is no compelling reason to assume risk factors such as interest rates and property prices must necessarily be positive or negatively correlated. However, we leave open the possibility of future refinements in the treatment of correlation through the use of equation (21).

We summarise in Table 1 two sets of the model parameters which will be discussed more thoroughly in the next section. In a property boom prices rise rapidly and employment and interest rate environments are at least mildly positive, as is generally the case in the past few years. Judging by relevant historical experiences<sup>33</sup>, a property bust may have moderate and steady property price declines for a few years, with a net-neutral employment and interest rate environment, but marked increases in volatilities of the risk factors.

**Table 1: Model Parameters Representing Boom and Bust Conditions**

<i>Parameter</i>	<i>Boom (%)</i>	<i>Bust (%)</i>
$\mu_V$	20	-10
$\sigma_V$	20	25
$\mu_S$	5	0
$\sigma_S$	15	25

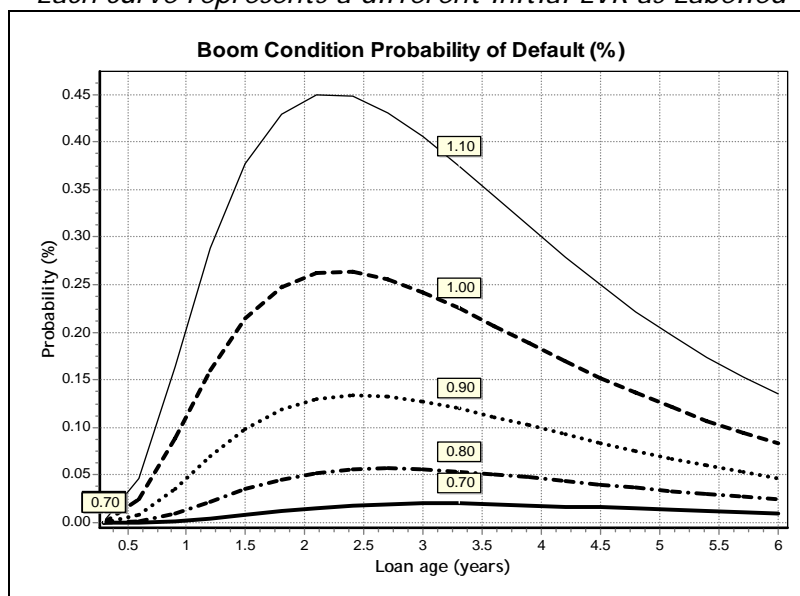
Figure 1 shows the term structure of the probability of default (PD) for boom conditions

<sup>32</sup> See Section 7 for a discussion.

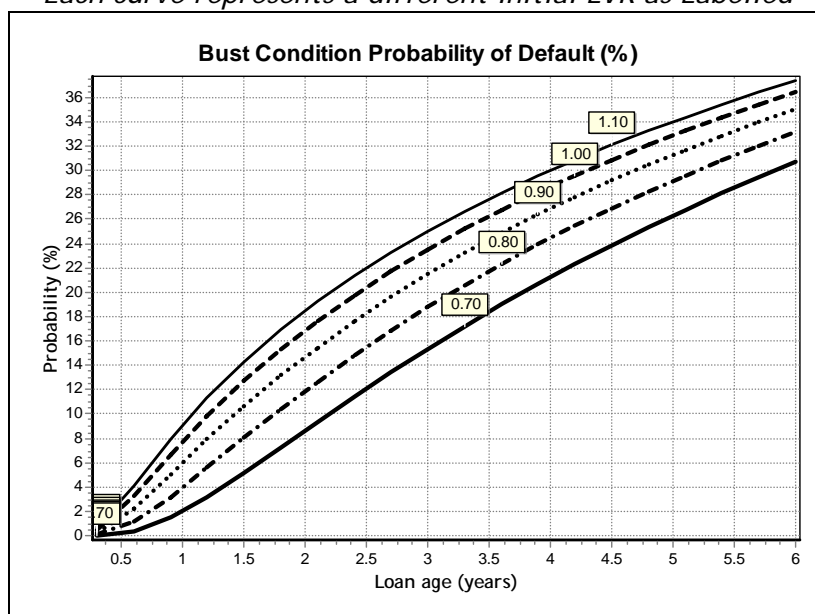
<sup>33</sup> See Helbling (2005) and Schnure (2005).

defined in Table 1 with initial LSR value of 1.4. Figure 2 shows the corresponding term structure for bust conditions defined in Table 1.

**Figure 1: Boom Condition Probability of Default**  
*Each curve represents a different initial LVR as Labelled*



**Figure 2: Bust Condition Probability of Default**  
*Each curve represents a different initial LVR as Labelled*



The first stylised observation is that the term structure of default probabilities or its shape depends on market conditions. In rising property markets, the probability of default for owner occupier or home loans peaks in the two or three years. This is what has been observed empirically in the property boom of the last several years. This term structure is inappropriate for falling property markets.

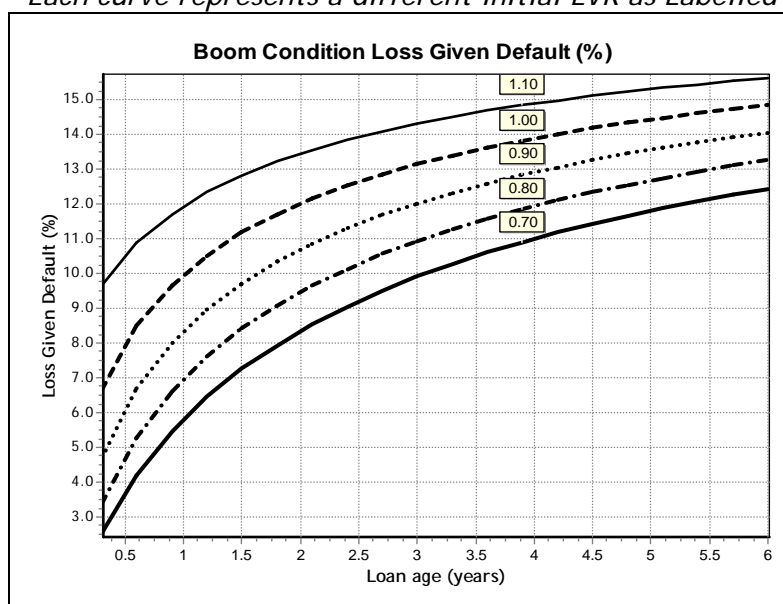
The second qualitative remark is that loan age is not an exogenous variable, as is sometimes assumed to be. If one estimates statistical regression models using recent data of rising property markets, then one would get highest probability of default correlations with loan ages of two or three years. The peaking of default probability in the first two or three years of a loan is not an intrinsic property of home loans but is reflective of the

particular economic environment of the empirical data. Such regression coefficients are therefore biased by the historical data and would be inappropriate in a falling property market, where probability of default will continue to rise with loan age.

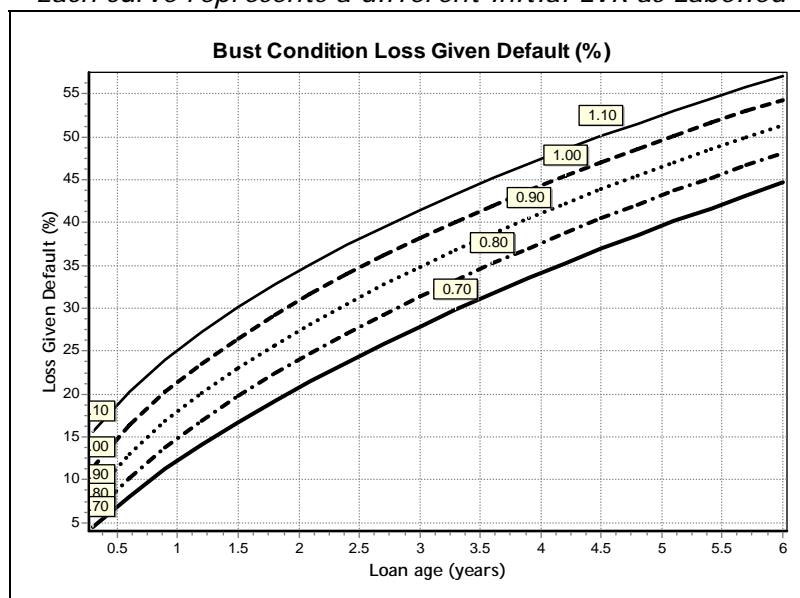
Thirdly, comparing Figure 1 and Figure 2, it is seen that the probability of default can increase dramatically when a strongly rising property market changes to a falling property market. Statistical regression models will most likely suffer from myopic historical sampling bias after a long period of boom conditions. From our model, a probability of default for high LVR loans can change from at a small fraction of 1% in a strongly rising market to more than 20% after a few years of a falling market. The turnaround can be highly nonlinear and dramatic.

The term structures of loss-given-defaults (LGD) for boom and bust conditions defined in Table 1 corresponding Figure 1 and Figure 2, with initial LSR value of 1.4 are shown in Figure 3 and Figure 4.

**Figure 3: Boom Condition Loss Given Default**  
*Each curve represents a different initial LVR as Labelled*



**Figure 4: Bust Condition Loss Given Default**  
*Each curve represents a different initial LVR as Labelled*





The fourth observation is LGD is not constant and it rises with time, mainly because uncertainty increases with time. If the loans are uninsured, the LGD can quickly rise above 10% in a couple of years for high LVR loans even in a strongly rising market. In a falling property market the corresponding values can be a few times higher and the LGD increases significantly over time. These characteristics differ substantially from the constant LGD assumptions used in some of the current credit default models, which may be reflective of the statistics of recent boom conditions where LGD tends to plateau after several years, as Figure 3 suggests.

How PD and LGD are correlated depends on market conditions and the loan portfolio. For example, in strongly rising property market, a loan portfolio with high average loan age, then PD and LGD would be negatively correlated, whereas in a falling market, PD and LGD would be positively correlated. Also, in a rising market, because of the shape of the term structure of PD, the part of the portfolio with low loan age would have PD and LGD positively correlated, whereas the part of the portfolio with high loan age would have PD and LGD negatively correlated. Our theory suggests that current debates about whether PD and LGD should be positively or negatively correlated at different parts of the economic cycle suggest incompleteness of existing theories.

Direct causes of credit default are defined as those independent primary variables which define and model liquidity and equity variables as in equation (3) and (6). Primary variables may be modelled by dependencies on secondary variables, which are then considered indirect causes. For example, liquidity is directly dependent in equation (3) on wages or gross income, which in turn may be modelled by a dependence on industrial production, which is then a secondary variable representing an indirect cause.

Any plausible risk factor such as inflation which may be statistically correlated to the frequency of credit default cannot be considered a direct cause of credit default unless we can show that the real mortgage interest rate is fixed and constant over time, making mortgage interest rates in equation (3) vary directly with inflation. As this is not the case, inflation is not a direct cause of credit default in our theory. Rather, it is an indirect cause.

## 6. Macroeconomic Causation

In principle there should be a credit default model for each loan, because a house located in one part of the city may behave predictably differently in term of its likely price movement from that of another house located in another part of the same city. There is also information on individual circumstances of the borrower which would be relevant in estimating default probability, if known. In practice we cannot expect such fine granularity of the empirical data. Our ignorance of such details is modelled through the uncertainty implied by the cross-sectional volatility of the risk variables.

However, there may be some scope for modelling housing loans in each state or region differently if we believe that there are common factors driving default risk based on geography. For example some suburbs are known to have high unemployment or some regions may be experiencing housing shortages due localised economic booms. Such knowledge may be used to advantage in accessing credit default risk.

There are certainly macroeconomic factors which will have an impact potentially on all housing loans. Macroeconomic stresses are positively correlated on all housing loans, only the degree of correlation between individual loans may be uncertain in some cases. But this uncertainty is of no concern to us, because it is implicitly accounted for in our intended loan-by-loan calculations.

Because we have developed a causal theory, we have direct mathematical links between known risk factors such as a rise or fall in interest rates to our model parameters. It is through these linkages that we can carry out macroeconomic stress testing on the housing loan portfolios. How do changes in fiscal and monetary policy, unemployment and housing property prices affect the borrower's liquidity and equity risks?

### 6.1 Macroeconomic impact on home loans

We are principally interested in stress testing over a time horizon of a few years. For simplicity of illustration we will ignore the impact of fiscal policy changes which operate via a nonlinear tax function  $\tau(\cdot)$ . We introduce a compressed expression for LSR and write

$$\ln(x_s) = \ln(U / mL) \quad (26)$$

The symbol  $U$  denoted "uncommitted" net disposable income after tax and living expenses available to service a mortgage and  $m$  is the loan payment rate. Differentiating this equation with respect to time we obtain

$$\mu_s = \mu_U - \mu_m \quad (27)$$

and

$$\sigma_s^2 = \sigma_U^2 + \sigma_m^2. \quad (28)$$

As the most variable part of  $U$  is the gross wages and salaries ( $W$ ),  $\mu_U$  and  $\sigma_U$  can be considered the rate of change and the log volatility of average wages and salaries in the economy. Also, since the most variable part of  $m$  is the general interest rate, rather than mortgage spread,  $\mu_m$  and  $\sigma_m$  can be approximated the rate of change and the log volatility of the interest rate in the economy.

In a similar consideration for equation (6), for home loans, the rate of change of the equity random variable  $\mu_v$  is the rate of change of property prices  $\mu_p$  plus the loan amortization rate  $\delta$ , since the Value-to-Loan ratio increases with loan amortization.

### 6.2 Hong Kong Like Conditions

In a service oriented economy like Hong Kong<sup>34</sup>, property is widely used as collateral for consumer and business loans. This adds significantly to the property market exposure of the banking sector which has already more than 50% of its loans directed to residential mortgages, property construction, development and investment.

In the aftermath of the Asian financial crisis in 1997, over a period of six years from 1998 to 2003 average property prices dropped 60-70%, with office properties being the hardest hit. As much as 30% of residential mortgage loans had negative equity. While property prices were dropping continuously throughout the period, the loan delinquency ratio rose at first from slightly above 2% to a peak of more than 7% in 1999. But thereafter it dropped continuously to less than 3% by the end of 2003.

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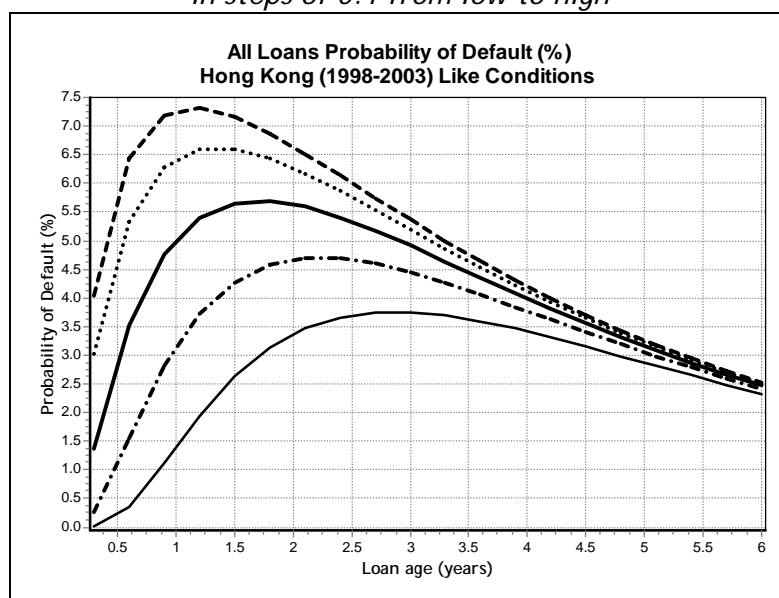
<sup>34</sup> See Fan and Peng (2005).

This would represent an unsolved puzzle<sup>35</sup> for a credit default theory based largely on negative equity alone. But we also know that interest rates largely fell on average over the period. After an initial reflexive hike in interest rates probably to defend the currency the Hong Kong Monetary Authority lowered the discount window base rate from 8% to 2.5% by the end of 2003. Base money rose from \$196.5 billion to \$292.6 billion through the period, representing significant injection of liquidity into the financial system. Even though the bank sector's asset quality and profitability fell considerably, the sector as a whole remained relatively healthy with a strong capital position.

Without data on loan details it is not possible to fully account for the development of this event with any quantitative accuracy. It is worthwhile to take this example to illustrate how our theory can be used to provide the broad outline of an explanation and how it can be applied for stress testing.

The macroeconomic data suggest that  $\mu_v = -0.15$ , representing a fall in average property prices of 15% p.a. for six years and that  $\mu_s = 0.20$ , representing a 20% p.a. improvement in loan serviceability due to a 20% p.a. fall in the change of interest rates. Using volatility parameters  $\sigma_v = 0.20$  and  $\sigma_s = 0.25$  and taking as a typical case an initial Loan Serviceability Ratio:  $x_s = 1.2$ , we have a set of probability of default curves for LVR ranging from 0.7 to 1.10 in steps of 0.1 over the six-year period. Evidently the curves in Figure 5 have the same qualitative behaviour as that actually observed<sup>36</sup> in Hong Kong between 1998 and 2003.

**Figure 5: Probability of Default under Hong Kong Like Conditions 1998-2003**  
*Each curve represents a different initial LVR from 0.7 to 1.1  
 In steps of 0.1 from low to high*

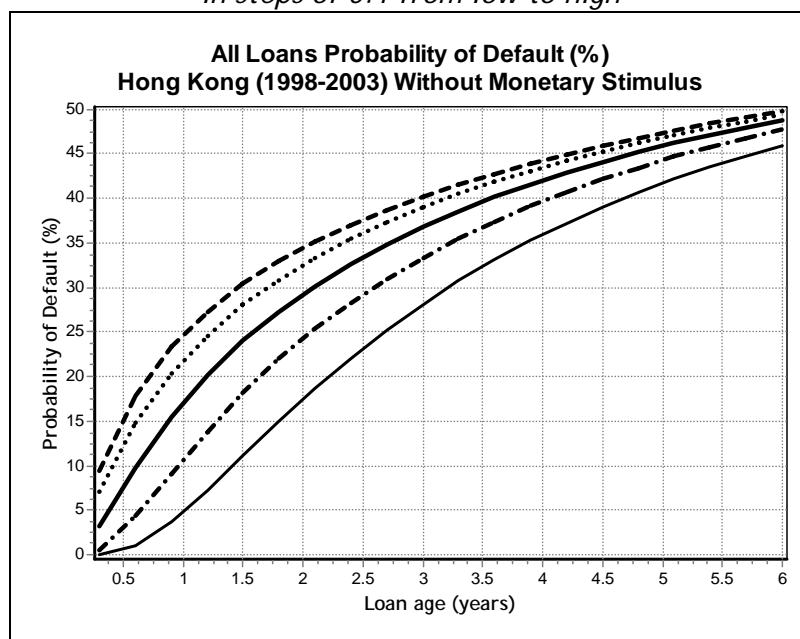


<sup>35</sup> Yam (2003) said "Quite apart from its effects on individuals and families, negative equity has broader economic implications. It has an enervating effect on spending. And it adds to the pressures on the banking system, although it should be added that the delinquency ratio on mortgages continues to be very low. This is, I think, attributable partly to the fortitude with which homeowners in negative equity have borne the problem, and partly to the willingness of most banks to restructure loans in cases of difficulty." He did not attribute any part to monetary stimulus which is the main explanation preferred in this paper.

<sup>36</sup> See Fan and Peng (2005).

The peak of Hong Kong interest rates roughly coincided with the bursting of the US stock market bubble in 2000. Had this not happened, the strong monetary stimulus in Hong Kong might not have occurred and the subsequent development for residential mortgage loans could be significantly different from what did happen, as shown in Figure 6, where all parameters remained the same except for the remove of the fall in interest rates. Instead of delinquency rates peaking at 7%, they might have continued to rise to more than 50% over six years, assuming the monetary authorities remained inert over the period (which is probably unlikely).

**Figure 6: Probability of Default under  
Hong Kong Conditions without Monetary Stimulus**  
*Each curve represents a different initial LVR from 0.7 to 1.1  
In steps of 0.1 from low to high*



## 7. Discussion on Related Work

If we date the beginning of modern credit default theory from the Merton model<sup>37</sup> in 1974, then there has been more than 30 years of significant research contributions on credit default to review. In the limited space below our remarks will not be balanced or complete and will not do full justice to past contributions. We will merely highlight major differences in our approach to the existing approaches to put the contributions of this paper in perspective. The main existing approaches are the structural approach and the reduced form approach which form the basis of a number of commercial risk management products<sup>38</sup>.

### 7.1 Structural Approach

The structural approach to credit default refers to the Merton model<sup>39</sup> and its extensions and modifications. Kealhofer (2003) rightly claimed that the structural approach can be

<sup>37</sup> See Merton (1974).

<sup>38</sup> See e.g. Crouhy, Galai and Mark (2000). CreditPortfolioView is based on a multi-period reduced form approach with macroeconomic risk factors (Wilson, 1997). CreditMetrics, CreditVaR, CreditRisk+ and KMV can be classified under the structural approach (CreditMetrics, 1997; Kealhofer, 2003). The Kamakura Corporation uses both approaches.

<sup>39</sup> See Merton (1974).

causal, unlike the reduced form approach which can only be non-causal and will be discussed below. Our indebtedness to the structural approach is evidenced by our acceptance of Merton model as a model of insolvency in Section 4 above.

Subsequent extensions and modifications have left the basic idea of insolvency in the Merton model unchanged. Extensions to the original Merton model by Vasicek and Kealhofer incorporated into the KMV model<sup>40</sup> include expanded definitions of assets and liabilities, payments of coupons and dividends, empirically determined default barriers and non-Gaussian probability density functions. The KMV model is the main Merton-type model in use today to estimate “default” probabilities. There are many other variations<sup>41</sup> (to the original Merton model), such as changing default thresholds but they all have essentially the same basic Merton model structure.

Some authors have considered risk factors which are considered important in this paper such as failure of cash flow to cover interest payment<sup>42</sup> and interest rate impact on coupon payments<sup>43</sup>. However these factors are considered in those studies as exogenous stochastic processes which modify the structure of the basic model and in doing so lose the important property of causality, making them more and more non-causal and similar to the reduced form approach discussed below.

Most of these subsequent models relinquish another important advantage of the basic Merton model which is the simultaneous determination of the probability of default and expected loss endogenously. In their attempts to improve the estimation of the probability of default, they have made expected loss an exogenous variable and thus created another theoretical problem for estimating expected loss.

In this paper, the Merton model is restored as a model of expected loss from insolvency. Since insolvency is a necessary but not a sufficient condition by our definition of credit default, the Merton model is not a model of credit default. A necessary concept for credit default is liquidity failure leading to delinquency described above. The various risk factors which later modifications sought to introduce to the Merton model should be introduced through the delinquency model of the causal framework. Our approach to new risk factors is to provide endogenous explanations through the causal framework rather than to introduce new independent variables needing additional exogenous stochastic processes.

While including a large number of risk factors in the analysis presents a challenge<sup>44</sup> for the structural approach, it is easily accommodated in the reduced form approach.

## *2.2 Reduced Form Approach*

The reduced form approach has been gaining in popularity over the structural approach recently<sup>45</sup> because they can take into account more of the observable risk factors which appear empirically to be relevant in determining credit default risk. They appear<sup>46</sup> to fit default data better than structural models, particularly when applied to credit default spreads (CDS) where there are abundant empirical data. This superior assessment has

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<sup>40</sup> See Bohn (2006).

<sup>41</sup> See Leland & Toft (1996), Longstaff and Schwartz (1995) and Kim, Ramaswamy and Sundaresan (1993), which pre-date the KMV model.

<sup>42</sup> See Nielsen, Saa-Requejo and Santa-Clara (1993).

<sup>43</sup> See Kim, Ramaswamy and Sundaresan (1993).

<sup>44</sup> See Tarashev (2005).

<sup>45</sup> See Jarrow and Protter (2004).

<sup>46</sup> See Bharath and Shumway (2005). If a model's sole purpose is to fit empirical data then it should fit data better than other models which may have other objectives apart from fitting observed data.

been disputed in true out-of-sample tests by others<sup>47</sup>.

The reduced form approach does not actually produce credit default theories. Jarrow and Protter (2004) state that the reduced form approach is a subjective approach preferred by modellers with incomplete information for the purpose of pricing and hedging in markets. The authors imply in their paper that the reduced form approach does not lead to objective theory.

Reduced form models<sup>48</sup> are empirical models of default probability which use whatever data that are available and considered relevant to the modeller. Important risk variables which are known to be significant determinants of credit default may be excluded from the model if the data are inadequate or unavailable<sup>49</sup>. The models are based on logistic regression<sup>50</sup> where statistical estimation of the linear regression coefficients is critical to the performance of the models. Despite their popularity, reduced form models have serious drawbacks.

Firstly, reduced form models are non-causal. Correlation of a perceived risk driver to frequency of default may be coincidental or only indirectly related. In the sciences<sup>51</sup>, discovery of correlation is merely a first step to investigate causation which leads to theoretical causal models.

Secondly, reduced form models are hostage to the empirical data needed for model estimation. The data determine the regression coefficients and hence the model itself. There is a different model for every set of data. The estimates of expected loss can differ arbitrarily even for exactly the same loan portfolio if different empirical data and risk factors are used. Also, it is doubtful whether models constructed with bull market data will be valid for bear markets.

Thirdly, in the reduced form approach PD and LGD are estimated separately. There are many different suggestions<sup>52</sup> for how to estimate LGD. Divergent approaches for LGD lead to much debate<sup>53</sup> over such simple issues as the level and direction of correlation between PD and LGD. These issues suggest the possibility of logical contradictions in the methods used.

Finally, reduced form models assume linear or log-linear relations between dependent and independent variables. They are limited only to small changes to default probabilities in a given equilibrium state of the world and they are invalid for large changes which are required for stress testing purposes<sup>54</sup>. They are also inadequate for changing market environments such as those encountered in the mortgage and credit crisis in 2007.

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<sup>47</sup> See Arora, Bohn and Zhu (2005).

<sup>48</sup> See Jarrow and Turnbull (1996).

<sup>49</sup> See Jarrow and Protter (2004) for the reasons to exclude the asset values of an entity.

<sup>50</sup> See Cox (1972).

<sup>51</sup> See Cox (1972) and applications to hazard models in clinical medicine.

<sup>52</sup> For example, Duffie and Singleton (1999) suggest an econometric approach, while Unal et al. (2001) suggest an approach based on the seniority of the debt holder over the claims on defaulting firm's tangible assets. Dullmann and Trapp (2004) believe systematic risk is a major factor influencing recovery rates. Hackbarth, Miao and Morellec (2006) consider capital structure and macroeconomic conditions.

<sup>53</sup> See Altman, Brooks, Resti and Sironi (2005).

<sup>54</sup> See Sorge (2004).

Consider a logit default variable  $y$  driven for simplicity by only one independent variable  $x$ . A Taylor series expansion about a given state  $x_0$  can be written as

$$y(x) = y(x_0) + \frac{dy}{dx}(x - x_0) + \frac{1}{2} \frac{d^2y}{dx^2}(x - x_0)^2 + \dots \quad (29)$$

In a linear regression typical of reduced form models, one estimates  $y(x_0)$  and  $dy/dx$  (evaluated at  $x_0$ ) of the first two terms on the right hand side from empirical data to determine the regression coefficients for the intercept and slope of a straight line. The other terms are ignored or assumed to be negligible, which is valid only if the deviations  $(x - x_0)$  are small. For stress testing large deviations are assumed and therefore higher order nonlinear terms may be important.

To make these remarks more precise in a highly simplified illustration, consider a hypothetical world where the default probability is accurately given by a simple Merton model which has only the one known exogenous variable. Without knowing this “hidden” true causal relationship imagine how one might estimate a reduced form model from the data. The logistic regression equation is

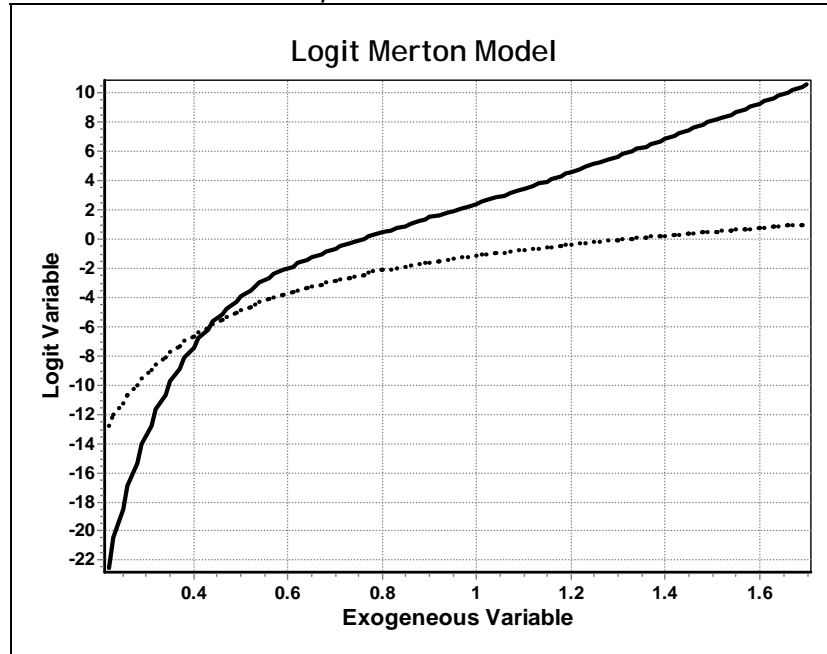
$$\log\left(\frac{1-p}{p}\right) = Y = a + bX \quad (30)$$

Here  $p$  is the probability of default,  $Y$  is the endogenous variable and  $X$  is the exogenous variable. Given a set of  $n$  empirical data points  $\{X_i, Y_i\}$ ,  $(i = 1, \dots, n)$ , one estimates the regression coefficients  $a$  and  $b$ .

Suppose the data set comes from a scenario represented by the dotted lines in Figure 7, then we would expect the data points to cluster around the dotted lines. Because extreme values are less frequent than less extreme values, the distribution of points would concentrate mostly around the exogenous variable being slightly less than one in the chart, representing small failures. The nonlinear relationship between the endogenous and exogenous variables implies that the regression coefficient of the slope  $b$  weighted by high density points would under-estimate the probability of default of high impact events, further to the left of 1 in the chart. Hence the reduced form approach can under estimate the risk of high impact events.

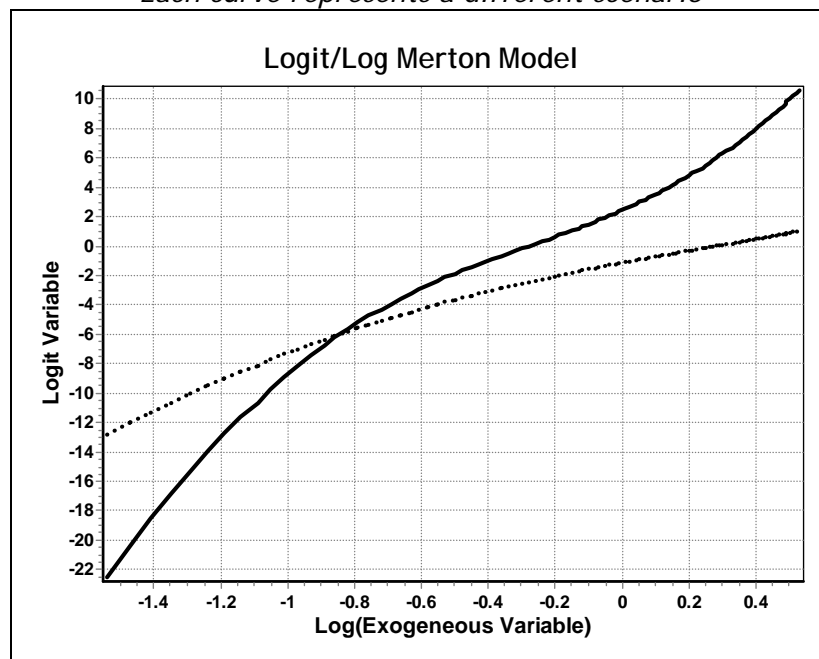
A second scenario represented by the solid lines obviously has very different functional relationship between the probability of default and the exogenous variable. If we had data only for the first scenario, then there is no method which could be used to infer the behaviour of the second scenario from knowledge of the first scenario. Hence the reduced form approach can lead to significant model errors due to the lack of empirical data to estimate future events.

Figure 7: Logit Merton Model  
Each curve represents a different scenario



The knowledgeable and observant reader may immediately suspect that it is the rate of change of the exogenous variable that is linearly related to the default variable. Figure 8 shows that even if we exponentially transform the exogenous variable, which changes the variable to a rate of change variable, nonlinearity still remains. This shows the potential limitations of the linearization assumption which is inherent in the reduced form approach.

Figure 8: Logit/Log Merton Model  
Each curve represents a different scenario





The linearization assumption is consistent with a general equilibrium assumption of the market or the economy, where observed changes are merely small deviations from an essentially static world. There may be an abstract sense in which this idea of equilibrium may be valid but there are no theoretical estimates of time scales or sizes of fluctuations, under which assumed deviations may be considered small.

In this paper we have taken a scientific rather than a market based approach. Given a set of conditions or causes our objective is to predict their effects on future observed rates of credit default. There is a substantial research literature<sup>55</sup> on the relationship between the probability of default and corporate bond prices or credit spreads. There are many reasons not to follow such market based approaches. The main one is that credit default occurs in many situations where there are no traded markets. Credit defaults should be understood in its own right, independent of trading behaviour. Therefore we have a much more modest ambition of predicting the rates of credit defaults a few years ahead from given initial conditions.

The most common approach to credit default currently is to treat delinquency as default and estimate the probability of default using a reduced form approach. Then the expected loss is estimated by separate recovery models<sup>56</sup>, which are independent of the model for default and therefore may lead to inconsistent assumptions being made in separate models. In this paper the definition of credit default which includes the potential for loss in the probability of default binds the models together and forces consistent modelling.

In the mortgage and credit market crisis of 2007, substantially changed market conditions rendered most existing empirical models of credit default inappropriate as they were estimated under different market conditions. As a result, traders were unable or unwilling to price credit risk and the market became illiquid. Our approach to credit default and credit risk pricing does not depend on large amount of empirical data or the existence of traded market. Therefore it can be used potentially to calculate credit risk prices in the volatile environment of 2007 and thus could potentially help maintain market liquidity.

## 8. Conclusion and Further Research

A causal framework has been proposed where credit default theories can be systematically developed to investigate the reasons for credit default. Our hypothesis is that a credit default is caused by both delinquency and insolvency. Any risk factor can only be considered relevant if it has a demonstrable causal impact on delinquency or insolvency. This imposes a structural discipline which is lacking in many other theories. For example, even if sunspot frequency or crime rate were to show significant correlation to credit default rates, it can only be used as a risk variable in our approach if it can be causally linked to the model in either liquidity or the equity position of a financial entity.

Implicit in this framework is a research program to investigate credit default generally and systematically. New theories of credit default are then created from new models for delinquency and/or insolvency. In this paper, we have only illustrated one of the many possible theories. Already the power of the approach has resolved a number of puzzles observed in empirical data in relation to existing credit default theory. Moreover, by changing model parameters, our approach can be used to price credit risk dynamically in changing market environments, whereas most existing approaches being empirical cannot.

The development of the causal mortgage default theory in this paper also shows other advantages in our joint probability approach. The delinquency and insolvency models can

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<sup>55</sup> See Duffie & Singleton (1999), Elton, Gruber, Agrawal & Mann (2001) and Arora, Bohn & Zhu (2005).

<sup>56</sup> See Altman, Brooks, Resti & Sironi (2005) and Dullmann & Trapp (2004).

easily be extended to include such effects as insurance on liquidity from income protection insurance and on solvency from lender mortgage insurance. There are other possible enhancements such as non-Brownian stochastic processes and time dependent dynamic model parameters for varying macroeconomic conditions. The present account of the theory has been kept deliberately as simple as possible to highlight the essential ideas.

As the new mortgage default theory developed here is fully causal and nonlinear, it provides a valid method for stress testing<sup>57</sup> home loan portfolios due to large changes in risk factors. As the theory is intended to apply to individual loans, portfolio correlation is implicit as common risk drivers simultaneously affecting each loan but in different ways. Practical application of the theory requires disaggregated data on individual loans at a particular point in time which may not be generally available due to deficiencies of past data collection and may need to be estimated. Reports from such studies are in preparation and will be presented elsewhere.

Clearly a causal theory of corporate default has significant implications for the pricing of equity and debt securities of companies and for prudential regulation of financial institutions. The model for liquidity or delinquency can be determined by the data contained in profit and loss statements, while the model for equity or solvency can be determined by the data contained in balance sheet statements. Such data are publicly available if only infrequently such as annually. The causal framework described in this paper holds substantial promise for the development of causal theories of corporate default, which will be pursued elsewhere.

## Acknowledgements

The author has benefited from comments on the above ideas by APRA staff, particularly by Patrick Meaney on Hong Kong property. The author has written this paper in a non-traditional format<sup>58</sup> for greater clarity suggested by Steve Davies. He thanks his family for support and understanding while he was working "out of hours" on this paper.

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<sup>57</sup> For global interest in stress testing, see Bank of International Settlements (2005), Basurto, Goodhart and Hoffmann (2006), Hoggarth, Logan and Zicchino (2003) and Sorge (2004).

<sup>58</sup> See Jones (2007).

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