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Valuation of Options in a Setting with Happiness-Augmented Preferences

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Valuation of Options in a Setting with Happiness-Augmented Preferences

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Abstract

We derive a pricing formula for a European call option written on equity in a framework where returns and consumption covary with external happiness. Being a non-tradable variable, happiness is regarded as an extra variable in a parameterised version of state dependent utility. We derive an extended version of the Black-Scholes (*BS*) formula and find that, in an optimistic environment (that is, where a high growth rate of happiness is expected), the standard *BS* formula may underestimate the value of the call option, and overestimate its sensitivity to changes in the underlying parameters. Under the assumption of log-normality of the happiness distribution, testable hypotheses for quality of hedging strategies can also be implemented.

1. Introduction

This paper finds a pricing formula for a European call option written on equity in a framework where returns and consumption covary with happiness. We claim that, by providing additional information to those reflected by market prices, happiness may play a primary role in the determination of the individual's optimal choice. This view is supported by our recent empirical findings on asset pricing. In

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Merella and Satchell (2005), we show that the introduction of a happiness-augmented specification of preferences eliminates the equity premium puzzle. If this preference variation is able to improve robustness of security valuation in an asset pricing model, we argue, then the same must hold for the derivatives written on those securities.

In general, the reader may think of our model as a parameterised version of state-dependent utility. The state of nature is to be considered completely exogenous to the household's control, and can be conceived as happiness, although it may certainly have numerous alternative interpretations. We warn that the concept of happiness we use is not necessarily the happiness of the individual decision-makers but rather an external air of well-being, due to non-market factors, which can influence the degree of satisfaction (utility) the representative agent experiences when consuming.

The reason why our strategy departs from standard utility optimisation follows from the external nature of happiness. Being non-tradable, happiness cannot be directly included into the household's budget constraint. Its effects on household's consumption choice are, however, widely acknowledged (see e.g. Oswald, 1997; Frey and Stutzer, 2001). We thus assume happiness affects marginal utility of consumption in a direct fashion, and derive a pricing formula for a European call written on a stock using an augmented version of the two-period power-utility option pricing model.

The paper is organised as follows. Section 2 describes our notion of happiness and discusses parallel concepts in other literatures, along with the issue of measuring happiness. Section 3 describes the model. Section 4 describes our main findings. Section 5 concludes. The appendix contains the majority of mathematical derivations.

2. The Concept of External Happiness.

The concept of external happiness we use is rather novel, and in the context of a representative agent economy refers to external factors that influence the utility of consumption. As mentioned above, one interpretation of happiness might be the state of nature in parameterised utility. In this view, a straightforward example of an environmental nature could be the weather. A second, more elusive interpretation may regard happiness as some sort of external social factor, influencing utility but not influenced by the individuals' decisions except in a larger, possibly dynamic, equilibrium framework. In our economy, there are only two dates, so dynamics are disregarded. It should be noted that a

dynamic model of happiness has been presented by Sprott (2003). Although his work is not based on optimising behaviour of agents, he models the factors influencing happiness as being external.

Returning to the idea of external forces influencing happiness, we find such an idea prevalent throughout philosophy, psychiatry, politics, and quite likely many other fields. We next list a tiny subset of examples of external social factors taken rather randomly from a much larger set, referring where possible to articles that give amenable explanations, rather than some of the challenging original texts.

In philosophy, Aristotelian ethics of the good life for humans, known as *Eudaimonia*, had a number of components, one of which was "good fortune". Adler (1991) comments on good fortune in Eudaimonia as something that "we may wish for or pray for, but we cannot choose between seeking it and not doing so." The source of good fortune may or may not be social; it may be assumed to come from a religious explanation, through a society being privileged because of either its past behaviour or its ethnicity. This is clearly a concept of external happiness.

Hegel used another related concept, translated into English as "the social world". This is described in Hardimon (1992), who refers to "the central social institutions of that era: the family, civil society, and the state."(p.165) Individuals could realise themselves by reducing the extent to which they were alienated from the social world. Presumably, the social world could move also, thereby reducing alienation in an external sense. Herbert Spencer (1851) published a book, widely read in the nineteenth century, called "Social Statics, or the Conditions Essential to Human Happiness". We have not investigated this further.

In an article that intersects psychiatry and social philosophy, Fromm (1939).uses the notion of *Weltanschauung* (a rough English translation is "look onto the world"), by which he means the prevailing social setting to evaluate the question as to whether the results of psychotherapy are objective or dependent on Weltanschauung. He concludes that the latter is the case. In the same article, Fromm also discusses the social philosophy of Freud. He describes an important tenet of Freudian social philosophy as being "that external circumstances for which man is not responsible mould him and make him into the person he is." He calls this "milieu"; and gives *Thanatos*, the death instinct, something that man cannot change, only sublimate, as an example of part of Freudian milieu.

In politics, David Cameron (2006) famously said: "It's time we admitted that there's more to life than money, and it's time we focused not just on GDP, but on GWB – General Well-Being." Without

digressing too far into party manifestos, this could be interpreted as meaning that high consumption is not beneficial when it is a consequence of inequality, which also leads to high crime. Here again, the social environment is influencing the utility of consumption. In this case, whether it is social inequality, or the criminality that follows from it, it is not really clear. Whether social inequality is something that the representative agent can manage or reduce is a question outside the framework of our model.

Many economists have used ideas that could be placed in this framework. Keynes (1936) introduced the notion of *animal spirits*, which he describes on page 161 as "a spontaneous urge to action rather than inaction", and on page 163, when describing the investment process, he writes that "…it is our innate urge to activity which makes the wheels go round, our rational selves choosing where we can, but often falling back for our motive on whim or sentiment or chance". This could be interpreted as saying that it is not just conventional utility maximisation that leads to portfolio choice: the optimal portfolio is the result of a number of other considerations. Collard (1996), in discussing the works of Pigou, refers to real, psychological and monetary impulses that influence expectations formation. In terms of our structure, this could be thought of as different conditional densities for equity payouts, given external happiness.

Although this range of concepts are not explicitly placed in an expected utility framework, not least because many of them predate expected utility, they all have in common, we would argue, some notion of a process, possibly random and/or dynamic that influences the individual's utility and is external to him.

An important issue is about measuring happiness. A number of potential indices may serve as proxy for this variable. Following the above examples, we could use some ordinal index of the weather (such as temperature, rainfall, or hours of sunlight); the crime rate or its inverse; the closed end fund discount, shares turnover, or even sales of lottery tickets! We must, however, be careful to think about happiness proxies as something external to the decision maker, but impinging upon his utility.

In this view, we may use investor sentiment, an index naturally associated with the notion of animal spirits. This index can be straightforwardly included in a consumption-based asset pricing model, as the representative agent is to be simultaneously considered as consumer and investor. For the same reason, we may also opt for consumer confidence, a well-established measure of the degree of optimism households express through their activities of consuming and saving, which can be seen as the households' analogous of investor sentiment. We may finally choose to use some suitably

assembled index. Indices of this kind are not uncommon. One example is given by Baker and Wurgler (2006), who use principal components to combine six separate measures of investor sentiment. Whether these particular choices are consistent with our definition of external happiness or are more endogenous notions of happiness remains, however, an open point.

3. The Model

Consider a two-period securities markets economy. Individuals' utility functions are time-additive extended power functions. In a securities markets economy, at time 0 an individual is endowed with consumption and shares of traded securities. Under rather restrictive constraints on individual risk parameters, a representative agent can be constructed with power utility functions:

$$u_0(C_0, X_0) + u_1(C_1, X_1) = \frac{C_0^{1-\alpha} X_0^{1-\beta}}{1-\alpha} + \rho \frac{C_1^{1-\alpha} X_1^{1-\beta}}{1-\alpha},$$

where C_t and X_t are respectively consumption and external happiness at time $t = 0,1$, α and β are preference parameters, and ρ is the time preference parameter. The parameters α and β are both assumed to be less than one so that increases in consumption or external happiness raise utility (hereafter, we shall refer to external happiness simply as happiness). As a general result, the price S_j for a security *j* is:

$$S_j = E\left[\frac{\rho C_1^{-\alpha} X_1^{1-\beta}}{C_0^{-\alpha} X_0^{1-\beta}} Z_j \right], \tag{1}$$

where Z_j is the random payoff of security *j* at time 1. This equation gives us a pricing kernel for any traded asset, and can also be used to price derivatives. The expectations employed here refer to either using objective probabilities or assuming that each investor has a subjective distribution. In which case it is known (see Rubinstein, 1974) that we can aggregate the subjective probabilities so that the probability of the representative agent, in the case of power utility, is the arithmetic average of subjective probabilities. This result has only been derived for the discrete case, although Williams (2006) considers some continuous extensions.

A European call option is a security that gives its holder the right to purchase a share of its underlying security at a fixed exercise price on the maturity date of the option. Let $Z_j(k)$ be the payoff

at time 1 from a European call on one share of the *j*th security maturing at time 1 with an exercise price of *k*, and let $p_j(S_j, k)$ be the price of this call at time 0 when its underlying stock price is S_j. Since an option gives its holder the right but not the obligation to exercise on the maturity date,

$$Z_j(k) = \begin{cases} Z_j - k & \text{if } Z_j - k > 0, \\ 0 & \text{otherwise.} \end{cases}$$

By applying equation (1) to this call, we have:

$$p_j(S_j, k) = \rho E\left[\max\left(S_0(S_j - k), 0\right) \left(\frac{C_1}{C_0}\right)^{-\alpha} \left(\frac{X_1}{X_0}\right)^{1-\beta} \right] \tag{2}$$

where $S_0 \equiv \exp(-r_f)$ is the economy discount factor, and can also be defined as the price at time 0 of a complex security that pays one unit of consumption at time 1 in all states (i.e. $Z_0 = 1$).

We assume that $Y \equiv (Z_j, C_1, X_1)$ is a trivariate log-normally distributed vector, that is, $\ln Y \equiv (\ln Z_j, \ln C_1, \ln X_1)$ is trivariate normally distributed with mean $\mu_Y \equiv (\mu_J, \mu_C, \mu_X)$ and variance-covariance matrix:

$$\Sigma_Y \equiv \begin{pmatrix} \sigma_j^2 & \kappa_{jc}\sigma_j\sigma_C & \kappa_{jx}\sigma_j\sigma_X \\ \kappa_{jc}\sigma_j\sigma_C & \sigma_C^2 & \kappa_{cx}\sigma_C\sigma_X \\ \kappa_{jx}\sigma_j\sigma_X & \kappa_{cx}\sigma_C\sigma_X & \sigma_X^2 \end{pmatrix},$$

where κ_{mn} is the correlation coefficient between the generic variables *m* and *n*. This assumption implies that $y \equiv (z, c, x)$, with $z \equiv \ln(Z_j / S_j)$, $c \equiv \ln \rho (C_1 / C_0)^{-\alpha}$ and $x \equiv \ln(X_1 / X_0)^{1-\beta}$, is a trivariate normally distributed vector with mean:

$$\mu_y \equiv (\mu_j, \mu_c, \mu_x),$$

where $\mu_j \equiv \mu_J - \ln S_j$, $\mu_c \equiv -\alpha\mu_C + \ln\rho + \alpha \ln C_0$, and $\mu_x \equiv (1-\beta)(\mu_X - \ln X_0)$; and variance-covariance matrix:

$$\Sigma_y \equiv \begin{pmatrix} \sigma_j^2 & \kappa_{jc}\sigma_j\sigma_c & \kappa_{jx}\sigma_j\sigma_x \\ \kappa_{jc}\sigma_j\sigma_c & \sigma_c^2 & \kappa_{cx}\sigma_c\sigma_x \\ \kappa_{jx}\sigma_j\sigma_x & \kappa_{cx}\sigma_c\sigma_x & \sigma_x^2 \end{pmatrix}.$$

where $\sigma_c^2 \equiv \alpha^2\sigma_C^2$, $\sigma_x^2 \equiv (1-\beta)^2\sigma_X^2$, $\kappa_{jc}\sigma_j\sigma_c \equiv -\alpha\kappa_{jc}\sigma_j\sigma_C$, $\kappa_{jx}\sigma_j\sigma_x \equiv -\alpha(1-\beta)\kappa_{jx}\sigma_j\sigma_X$, and

$\kappa_{cx}\sigma_c\sigma_x \equiv (1-\beta)\kappa_{cx}\sigma_C\sigma_X$.

Given the above distributional assumption, (2) can be written as:

$$p_j(S_j,k) = S_j \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{a}^{+\infty}\left(e^z - k/S_j\right)e^c e^x f(z,c,x)\,dzdcdx, \tag{3}$$

where $f(z,c,x)$ is the joint density function for *z*, *c*, and *x*, and $a \equiv \ln(k/S_j)$. Denoting by $N(\cdot)$ the distribution function of a standard normal random variable, whose density function is given by:

$$n(z) \equiv \frac{1}{\sqrt{2\pi}} e^{-z^2/2}, \tag{4}$$

after some algebra, this expression can be stated as in the following:

Proposition 1. *Under the above assumptions on households optimising behaviour, the time-0 price of a European call option written on a stock is given by:*

$$p_j(S_j,k) = S_j N(Z_k^+) - kS_0 e^{-\mu_x - \frac{\sigma_x^2}{2}} N(Z_k^-) \tag{5}$$

where:

$$Z_k^- = Z_k^+ - \sigma_j = \frac{\ln(S_j/k) - \ln S_0}{\sigma_j} - \frac{\sigma_j}{2} + \frac{1}{\sigma_j}\left(\mu_x + \frac{\sigma_x^2}{2}\right). \tag{6}$$

Proof. See appendix A. ■

The standard framework, as adapted to the valuation of complex securities with preference restrictions by Rubinstein (1976), delivers the pricing equation:

$$p_j^{st}(S_j,k) = S_j N(Z_k + \sigma_j) - ke^{-r_f} N(Z_k) \tag{7}$$

where:

$$Z_k = \frac{\ln(S_j/k) + r_f}{\sigma_j} - \frac{\sigma_j}{2} \tag{8}$$

Equations (7) and (8) are the well known *Black-Scholes* (*BS*) *option pricing formula* derived in a discrete time economy by making joint conditions on the distributions as well as on the individuals' preferences. Our findings, as displayed by equations (5) and (6), are obtained by adding happiness in the joint distribution and by augmenting the individual's preferences with the same variable. They depart from the standard result (7) and (8) in two respects. On the one hand, the quantity $\mu_x + \sigma_x^2/2$ appears in the exponential on the right-hand side of the pricing formula (5). On the other, the same

quantity appears as a new addend, multiplied by σ_j^{-1}, in (6).

As a result, the version we present above depends only upon the same parameters as the standard *BS* formula, plus the parameters associated with the marginal distribution of happiness. It is unaffected by the correlation parameters of the augmented joint distribution. It is, however, possible to derive alternative representations that include such parameters. One example is stated in the following:

Corollary 1. *An alternative version of the pricing formula (5) can be defined by replacing (6) with:*

$$Z_k^- = Z_k^+ - \sigma_j = \frac{\ln(S_j / k) + \mu_j + \kappa_{jc}\sigma_j\sigma_c + \kappa_{jx}\sigma_j\sigma_x}{\sigma_j}.$$

Proof. See appendix A. ■

The final version to use is a matter of choice, and the decision taken should probably depend upon estimation issues.

4. Hedging Risk

In this section, we compute partial derivatives of our new option pricing formula and, where possible, compare them with those computed from the standard *BS* formula. We show that comparisons depend upon whether expected external happiness is increasing (i.e. households experience *optimism*) or decreasing (i.e. households experience *pessimism*). To clarify our meaning of optimism, it is easily understood if we consider the environmental example given in section 2: experiencing optimism means that households expect the weather to improve. As above explained (see section 3), in this paper we allow for expectations being objective or subjective if, in the context of optimism, we assume subjective probability. This would lead to the interpretation of optimism or pessimism as a measure of investor sentiment (see section 2). Optimism can be parametrically expressed as:

$$\eta_x = \int_{-\infty}^{+\infty} e^x dn(x) = \mu_x + \frac{1}{2}\sigma_x^2 \tag{9}$$

where we recall that $x \equiv \ln(X_1 / X_0)^{1-\beta} \sim N(\mu_x, \sigma_x^2)$, whose density function is given by (4). That is, households experience optimism if $\eta_x > 0$, and pessimism if $\eta_x < 0$. This parameter is therefore positively proportional to the expected growth rate in external happiness, since by assumption $\beta < 1$. We shall refer to optimism as the expected growth rate of happiness, although its formulation is

actually a little more complicated. Our results are summarised in the following propositions.

Proposition 2. *If households experiences optimism ($\eta_x > 0$), then the price $p_j(S_j,k)$ of a European call option written on a stock is:*

1. *more valuable than under BS, i.e. $p_j(S_j,k) > p_j^{st}(S_j,k)$;*
2. *more sensitive to changes in the underlying stock price S_j, i.e. the hedging coefficient of $dp_j(S_j,k)/dS_j$ is increased compared to that of $dp_j^{st}(S_j,k)/dS_j$.*

The reverse occurs if households experience pessimism ($\eta_x < 0$).

Proof. First, notice that if $\eta_x = 0$, then $p_j(S_j,k) = p_j^{st}(S_j,k)$, $\partial p_j(S_j,k)/\partial S_j = \partial p_j^{st}(S_j,k)/\partial S_j$, $Z_k^- = Z_k$ and $Z_k^+ = Z_k + \sigma_j$. In this case, the two option pricing formulas (5) and (7), and the respective derivatives with respect to S_j, thus coincide. Then, considering that $\partial p_j(S_j,k)/\partial \eta_x > 0$ (see appendix B.1), it must hold that $p_j(S_j,k) > p_j^{st}(S_j,k)$ if $\eta_x > 0$ and $p_j(S_j,k) < p_j^{st}(S_j,k)$ if $\eta_x < 0$. Finally, considering that $\partial^2 p_j(S_j,k)/(\partial S_j \partial \eta_x) > 0$ (see appendix B.2), it must hold that $\partial p_j(S_j,k)/\partial S_j > \partial p_j^{st}(S_j,k)/\partial S_j$ if $\eta_x > 0$ and $\partial p_j(S_j,k)/\partial S_j < \partial p_j^{st}(S_j,k)/\partial S_j$ if $\eta_x < 0$. ■

Proposition 2 points out two important facts about an economy working in an optimistic (pessimistic) environment. On the one hand, the standard *BS* pricing formula may underestimate (overestimate) the value of a European call option. On the other, the effect of a variation in the time-0 underlying security price on the value of the option may also be underestimated (overestimated).

It is worth noting, however, that this effect is qualitatively unchanged. The sign of each partial derivative of the option pricing formula (5), taken with respect to those parameters also appearing in the standard *BS* formula (7), is in fact the same as the corresponding partial derivative computed from the latter. Along with $\partial p_j(S_j,k)/\partial S_j > 0$, it also holds that $\partial p_j(S_j,k)/\partial k < 0$, $\partial p_j(S_j,k)/\partial r_f > 0$ and $\partial p_j(S_j,k)/\partial \sigma_j > 0$ (see appendix B). The magnitude of such derivatives does, however, change with increasing optimism. Denoting by $\overline{\eta}_x$ the degree of optimism such that these derivatives are unaffected by a differential change in optimism itself, i.e. $\overline{\eta}_x : \sigma_j N[Z_k^-(\eta_x)] - n[Z_k^-(\eta_x)] = 0$ (see appendix B.3), the changes in magnitude can be summarised in the following proposition:

Proposition 3. *If households experience a sufficiently high degree of optimism ($\eta_x > \overline{\eta}_x$), then the price $p_j(S_j, k)$ of a European call option written on a stock becomes less sensitive to changes in 1) the exercise price k; 2) the risk-less asset rate of return r_f; 3) happiness expected value μ_x; 4) happiness volatility σ_x.*

The opposite holds if households experience a sufficiently low degree of optimism ($\eta_x < \overline{\eta}_x$).

Proof. See appendices B.3, B.4, B.6 and B.7. ■

Proposition 3 points out another important fact about an economy working in a sufficiently optimistic (pessimistic) environment. That is, the standard *BS* pricing formula may overestimate (underestimate) the sensitivity of the value of a European call option to changes in the underlying parameters. A similar result applies to the effect of a variation in mean and variance of the happiness growth rate. In a sufficiently optimistic (pessimistic) environment, the sensitivity of the option price to changes in these parameters decreases (increases).

We also compute the partial derivative of the option pricing formula with respect to stock volatility. As mentioned above, this is positive as expected. Comparing its magnitude with that of the same derivative from the standard *BS* case yields a result analogous to that of proposition 3. The difference lies in the degree of optimism such that $\partial p_j(S_j, k)/\partial \sigma_j$ is unaffected by a differential change in optimism itself, in this case denoted by $\tilde{\eta}_x = -\left[\ln(S_j / k) - \ln S_0 + \sigma_j^2 / 2\right]$ (see appendix B.5). The change in magnitude can be summarised in the following proposition:

Proposition 4. *If households experience a sufficiently high degree of optimism ($\eta_x > \tilde{\eta}_x$), then the price $p_j(S_j, k)$ of a European call option written on a stock becomes less sensitive to changes in the stock volatility σ_j.*

The opposite holds if households experience a sufficiently low degree of optimism ($\eta_x < \overline{\eta}_x$).

Proof. See appendix B.5. ■

Proposition 4 points out our last result about an economy working in a sufficiently optimistic (pessimistic) environment. That is, the standard *BS* pricing formula may overestimate (underestimate) the reaction of the value of a European call option to changes in stock volatility.

Turning now to put prices, we note that put-call parity applies in the usual way, so that put prices can be computed from call prices and taking as fixed equity and bond prices (see appendix B.1). Comparative statics can also be computed within this framework. Likeness of results, however, suggests omitting details.

5. Concluding remarks

This paper has presented a model with a parameterised version of state dependent utility where external happiness is correlated with utility. Happiness is directly introduced as a non-tradable extra variable into the preference specification. We show that the standard Black-Scholes (*BS*) formula is augmented by a term depending on the parameters associated with the marginal distribution of happiness. We name this term *optimism*, and find that, in an optimistic (pessimistic) environment, *BS* may underestimate (overestimate) the value of a European call option.

We also demonstrate that while the effect of the underlying parameters on the option price is qualitatively unchanged, the magnitude of the partial derivatives of the happiness-augmented pricing formula with respect to such parameters is affected by the degree of optimism. We show that optimism has a stabilizing effect on the value of a European call option, as it decreases the absolute value of such derivatives. A similar result is also found about the effect of the parameters associated with the marginal distribution of happiness on the option price.

These findings say something about the success or failure of hedging strategies. Once we acknowledge that a variable having a log-normal distribution is a reasonable proxy for the external conditions that impinge on utility, we can set up testable hypotheses as to when hedging strategies over or under hedge.

Some colleagues in discussion have thought that this paper is a contribution to behavioural finance. We do not agree with this view. Our analysis is conducted entirely within a state dependent utility framework. Our agents are utility maximisers. Our probabilities are either objective or subjective. If a definition of behavioural finance is that assets are priced outside a utility maximising framework, then our approach is <u>not</u> behavioural. Nevertheless, we acknowledge the importance of psychological factors, influencing risk premia.

We have discussed at some length external social happiness and how it may be measured.

However, we have not tried to empirically price derivatives in this paper. We plan to do so in a following article.

Finally, it is worth noting that, although we do not compute partial derivatives of our option pricing formula with respect to time to maturity, this could be easily done, allowing us to compare our findings with those of numerous studies that uncover some anomalies in the behaviour of option pricing with respect to time to maturity. A well-documented anomaly, pointed out by Stein (1989) and according to who close to maturity options under react to volatility news, whilst long maturity options over react to such news, requires a stochastic volatility framework to be explored. Time variation in the parameters of the happiness distribution can probably capture some of these effects. We leave this for further research.

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Appendix A: Deriving the Option Pricing Formula

To ease exposition, we begin by defining $g(x) \equiv \int_{-\infty}^{+\infty} e^x f(x \mid z,c)dx$, $g(c) \equiv \int_{-\infty}^{+\infty} g(x)e^c f(c \mid z)dc$, $g(c+x) \equiv \int_a^{+\infty} g(c) f(z)dz$ and $g(z+c+x) \equiv \int_a^{+\infty} g(c)e^z f(z)dz$, so that relation (3) can be rewritten as:

$$p_j(S_j,k) = S_j g(z+c+x) - kg(c+x). \tag{10}$$

The first expression to be computed is:

$$g(x) \equiv \int_{-\infty}^{+\infty} e^x f(x \mid z,c)dx. \tag{11}$$

By using some well-known properties of the Normal distribution, we get, after some algebra:

$$g(x) = \exp\left\{\mu_x + \frac{\sigma_x^2}{2} - c - \frac{\left(c - \mu_{c|z} - \sigma_{c|z}^2 - \left(\kappa_{cx} - \kappa_{jc}\kappa_{jx}\right)\sigma_c\sigma_x\right)^2}{2\sigma_{c|z}^2} + \frac{\left(c-\mu_{c|z}\right)^2}{2\sigma_{c|z}^2}\right\} \cdot$$
$$\cdot \exp\left\{\mu_c + \frac{\sigma_c^2}{2} - \frac{\left(\kappa_{jc}\sigma_c + \kappa_{jx}\sigma_x\right)^2}{2} + \kappa_{cx}\sigma_c\sigma_x + \left(\kappa_{jc}\sigma_c + \kappa_{jx}\sigma_x\right)\frac{z-\mu_j}{\sigma_j}\right\}.$$

The second expression to be computed is:

$$g(c) \equiv \int_{-\infty}^{+\infty} g(x)e^c f(c \mid z)dc. \tag{12}$$

that becomes, after some algebra:

$$g(c)=\exp\left\{\mu_c+\frac{\sigma_c^2}{2}+\kappa_{cx}\sigma_c\sigma_x\right\}\cdot$$
$$\cdot\exp\left\{-\frac{\left(\kappa_{jc}\sigma_c+\kappa_{jx}\sigma_x\right)^2\sigma_j^2}{2\sigma_j^2}+\frac{2\left(\kappa_{jc}\sigma_c+\kappa_{jx}\sigma_x\right)\sigma_j\left(z-\mu_j\right)}{2\sigma_j^2}\right\}.$$

For convenience, we state two versions of this equation. The first is given by:

$$g(c)=\exp\left\{\mu_c+\frac{\sigma_c^2}{2}+\kappa_{cx}\sigma_c\sigma_x+\frac{\left(z-\mu_j\right)^2}{2\sigma_j^2}-\frac{\left(z-\mu_j-\kappa_{jc}\sigma_j\sigma_c-\kappa_{jx}\sigma_j\sigma_x\right)^2}{2\sigma_j^2}\right\}. \tag{13}$$

whereas the second is as follows:

$$g(c)=\exp\left\{\mu_j+\mu_c+\frac{\sigma_j^2+2\kappa_{jx}\sigma_j\sigma_x+2\kappa_{jc}\sigma_j\sigma_c+2\kappa_{cx}\sigma_c\sigma_x+\sigma_c^2}{2}-z+\frac{\left(z-\mu_j\right)^2}{2\sigma_j^2}\right\}\cdot$$
$$\cdot\exp\left\{-\frac{\left(z-\mu_j-\kappa_{jc}\sigma_j\sigma_c-\kappa_{jx}\sigma_j\sigma_x-\sigma_j^2\right)^2}{2\sigma_j^2}\right\}. \tag{14}$$

Considering (11) and (12), equation (13) is readily replaced into the definition of $g(c+x)$ to have:

$$g(c+x)=\exp\left\{\mu_c+\frac{\sigma_c^2}{2}+\kappa_{cx}\sigma_c\sigma_x\right\}N\left(\frac{-a+\mu_j}{\sigma_j}+\kappa_{jc}\sigma_c+\kappa_{jx}\sigma_x\right). \tag{15}$$

On the other hand, considering (11) and (12), and replacing (14) into the definition of $g(z+c+x)$ gives:

$$g(z+c+x)=\exp\left\{\mu_j+\mu_c+\frac{\sigma_j^2+2\kappa_{jx}\sigma_j\sigma_x+2\kappa_{jc}\sigma_j\sigma_c+2\kappa_{cx}\sigma_c\sigma_x+\sigma_c^2}{2}\right\}\cdot$$
$$\cdot N\left(\frac{-a+\mu_j}{\sigma_j}+\kappa_{jc}\sigma_c+\kappa_{jx}\sigma_x+\sigma_j\right). \tag{16}$$

By replacing (15) and (16) into (10), we obtain:

$$p_j(S_j,k)=S_j e^{\mu_j+\mu_c+\left(\sigma_j^2+2\kappa_{jx}\sigma_j\sigma_x+2\kappa_{jc}\sigma_j\sigma_c+2\kappa_{cx}\sigma_c\sigma_x+\sigma_c^2\right)/2}\cdot$$
$$\cdot N\left(\frac{-a+\mu_j}{\sigma_j}+\kappa_{jc}\sigma_c+\kappa_{jx}\sigma_x+\sigma_j\right)-$$
$$-ke^{\mu_c+\frac{\sigma_c^2}{2}+\kappa_{cx}\sigma_c\sigma_x}N\left(\frac{-a+\mu_j}{\sigma_j}+\kappa_{jc}\sigma_c+\kappa_{jx}\sigma_x\right) \tag{17}$$

Under the assumption of joint log-normality, equation (1) can be computed by using (16) and considering $a = -\infty$, so that $N(\cdot) = 1$. Dividing both sides of (1) by S_j, we get:

$$
\begin{aligned}
1 &= E\left[\left(\frac{\rho C_1}{C_0}\right)^{-\alpha}\left(\frac{X_1}{X_0}\right)^{1-\beta}\frac{Z_j}{S_j}\right] \\
&= \exp\left\{\mu_j + \mu_c + \frac{\sigma_j^2 + 2\kappa_{jx}\sigma_j\sigma_x + 2\kappa_{jc}\sigma_j\sigma_c + 2\kappa_{cx}\sigma_c\sigma_x + \sigma_c^2}{2}\right\}.
\end{aligned} \tag{18}
$$

The general pricing equation (1) also holds for a riskless unit discount bond, defined above as a complex security that pays one unit of consumption at time 1 in all states. We have:

$$S_0 = E\left[\left(\frac{\rho C_1}{C_0}\right)^{-\alpha}\left(\frac{X_1}{X_0}\right)^{1-\beta}\right].$$

It is not possible, however, to compute this equation by using an analogous procedure to that used to derive (18), since (15) is conditional on the distribution of the risky *j*th security. Nevertheless, under the assumption of joint log-normality, the expected value of the term in brackets is given by:

$$S_0 = \int_{-\infty}^{+\infty}\exp\{c\}\, f(c)\int_{-\infty}^{+\infty}\exp\{x\}\, f(x \mid c)\, dx dc.$$

Using some standard properties of the Gaussian distribution, the last expression can be rewritten as:

$$S_0 = \exp\left\{\mu_c + \mu_x + \frac{\sigma_c^2 + 2\kappa_{cx}\sigma_c\sigma_x + \sigma_x^2}{2}\right\}. \tag{19}$$

Considering (18) and (19), and recalling that $a \equiv \ln(k / S_j)$, equation (17) can be rewritten as:

$$p_j(S_j, k) = S_j N(Z_k^+) - kS_0 \exp\left\{-\mu_x - \frac{\sigma_x^2}{2}\right\} N(Z_k^-), \tag{5}$$

where we have used the fact that $\mu_c + \sigma_c^2/2 + \kappa_{cx}\sigma_c\sigma_x = S_0 - \mu_x - \sigma_x^2/2$, $Z_k^+ = Z_k^- + \sigma_j$ and:

$$Z_k^- = Z_k^+ - \sigma_j = \frac{\ln(S_j / k) - \ln S_0}{\sigma_j} - \frac{\sigma_j}{2} + \frac{1}{\sigma_j}\left(\mu_x + \frac{\sigma_x^2}{2}\right). \tag{6}$$

Besides, we can obtain an alternative version of the pricing formula simply by disregarding the last passage to obtain:

$$Z_k^- = Z_k^+ - \sigma_j = \frac{\ln(S_j / k) + \mu_j + \kappa_{jc}\sigma_j\sigma_c + \kappa_{jx}\sigma_j\sigma_x}{\sigma_j}.$$

Appendix B: Comparative Statics

From (7) and (8), $p_j^{st}(S_j,k)$ depends on the time-0 price of its underlying asset S_j, the exercise price *k*, the riskless interest rate r_f, and the variance σ_j of $\ln(Z_j/S_j)$. In this section, we compute the comparative statics of $p_j(S_j,k)$ with respect to each of these values, to see whether their effects on the new pricing formula, defined by (5) and (6), differ from those on the standard *BS*. In addition, we study the comparative statics of $p_j(S_j,k)$ with respect to the new terms μ_x and σ_x.

B.1. Increasing Optimism

Prior to conducting this analysis, it is worth studying the effects of optimism (and pessimism), as defined by (9), on $p_j(S_j,k)$. Using this definition, we can also write the option pricing formula as:

(20) $$p_j(S_j,k) = S_j N(Z_k^+) - kS_0 \exp\{-\eta_x\} N(Z_k^-)$$

where:

(21) $$Z_k^- = Z_k^+ - \sigma_j = \frac{\ln(S_j/k) - \ln S_0}{\sigma_j} - \frac{\sigma_j}{2} + \frac{\eta_x}{\sigma_j}$$

Taking total differentiation of $p_j(S_j,k)$ with respect to η_x yields:

$$\frac{dp_j(S_j,k)}{d\eta_x} = S_j \frac{\partial N(Z_k^+)}{\partial Z_k^+}\frac{\partial Z_k^+}{\partial \eta_x} + \frac{kS_0}{\exp\{\eta_x\}} N(Z_k^-) - \frac{kS_0}{\exp\{\eta_x\}}\frac{\partial N(Z_k^-)}{\partial Z_k^-}\frac{\partial Z_k^-}{\partial \eta_x}$$

Using the Leibniz rule, we have that $\partial N(Z_k^+)/\partial Z_k^+ = n(Z_k^+)$, and likewise $\partial N(Z_k^-)/\partial Z_k^- = n(Z_k^-)$. From (6), it is easy to show that $\partial Z_k^-/\partial \eta_x = \partial Z_k^+/\partial \eta_x = \sigma_j^{-1}$. All this considered, we get:

$$\frac{dp_j(S_j,k)}{d\eta_x} = \frac{kS_0}{\exp\{\eta_x\}} N(Z_k^-) + \frac{1}{\sigma_j}\left[S_j n(Z_k^+) - \frac{kS_0}{\exp\{\eta_x\}} n(Z_k^-) \right]$$

It is easy to show that the term in square brackets equals zero. Isolating S_j from (6), we get:

$$S_j = \exp\left\{ \sigma_j Z_k^+ - \frac{\sigma_j^2}{2} \right\} kS_0 \exp\{-\eta_x\}$$

Considering that $Z_k^- = Z_k^+ - \sigma_j$, we can write:

$$n\left(Z_k^-\right)=n\left(Z_k^+\right)\exp\left\{\sigma_j Z_k^+-\frac{\sigma_j^2}{2}\right\}$$

Replacing these findings into the term in square brackets of the above expression, we obtain:

$$kS_0\exp\{-\eta_x\}n\left(Z_k^+\right)-kS_0\exp\{-\eta_x\}n\left(Z_k^+\right)=0$$

As a result, we can finally write:

$$\frac{dp_j\left(S_j,k\right)}{d\eta_x}=\frac{kS_0}{\exp\{\eta_x\}}N\left(Z_k^-\right)>0$$

Higher degrees of optimism lead to a more valuable call option.

Recall that the price of a European put can be computed from the prices of its underlying security and its European call counterpart through a relation called *put-call parity*, according to which:

$$P_j\left(S_j,k\right)=kS_0-S_j+p_j\left(S_j,k\right)$$

Taking total differentiation of last equation with respect to η_x, we have:

$$\frac{\partial P_j\left(S_j,k\right)}{\partial\eta_x}=\frac{dp_j\left(S_j,k\right)}{d\eta_x}=\frac{kS_0}{\exp\{\eta_x\}}N\left(Z_k^-\right)>0$$

Higher degrees of optimism also lead to a more valuable put option.

B.2. Increasing the Time-0 Stock Price

The total differentiation of (20) with respect to S_j gives:

$$\frac{dp_j\left(S_j,k\right)}{dS_j}=N\left(Z_k^+\right)+S_j\frac{\partial N\left(Z_k^+\right)}{\partial Z_k^+}\frac{dZ_k^+}{dS_j}-kS_0\exp\{-\eta_x\}\frac{\partial N\left(Z_k^-\right)}{\partial Z_k^-}\frac{dZ_k^-}{dS_j}$$

From (21), it is easy to show that $\partial Z_k^+/\partial S_j=\partial Z_k^-/\partial S_j=\left(\sigma_j S_j\right)^{-1}$. The last expression thus becomes:

$$\frac{dp_j\left(S_j,k\right)}{dS_j}=N\left(Z_k^+\right)+\frac{1}{\sigma_j S_j}\left[S_j n\left(Z_k^+\right)-kS_0\exp\{-\eta_x\}n\left(Z_k^-\right)\right]$$

Since the term in square brackets equals zero, we simply get:

$$\frac{dp_j\left(S_j,k\right)}{dS_j}=N\left(Z_k^+\right)>0$$

This result is analogous to the standard comparative statics giving:

$$\frac{dp_j^{st}(S_j,k)}{dS_j} = N(Z_k + \sigma_j) > 0$$

Increasing the time-0 price of a security *j*, therefore, raises the price of the European call written on the same security.

If $\eta_x = 0$, then $Z_k^- = Z_k$, and the last two equations coincide. The effect of increasing optimism on $dp_j(S_j,k)/dS_j$ is computed as follows:

$$\frac{d^2 p_j(S_j,k)}{dS_j d\eta_x} = \frac{\partial N(Z_k^+)}{\partial Z_k^+}\frac{\partial Z_k^+}{\partial \eta_x} = \frac{n(Z_k^+)}{\sigma_j} > 0$$

Higher degrees of optimism lead to increasing magnitude of the hedging coefficient of the partial derivative of the call option price with respect to the underlying security price.

B.3. Increasing the Exercise Price

The total differentiation of (5) with respect to *k* gives:

$$\frac{dp_j(S_j,k)}{dk} = S_j \frac{\partial N(Z_k^+)}{\partial Z_k^+}\frac{dZ_k^+}{dk} - S_0 \exp\{-\eta_x\} N(Z_k^-) - kS_0 \exp\{-\eta_x\}\frac{\partial N(Z_k^-)}{\partial Z_k^-}\frac{dZ_k^-}{dk}$$

From (6), it is easy to show that $\partial Z_k^+ / \partial k = \partial Z_k^- / \partial k = -(\sigma_j k)^{-1}$. The last expression becomes:

$$\frac{dp_j(S_j,k)}{dk} = -S_0 \exp\{-\eta_x\} N(Z_k^-) < 0$$

Once again, this result is analogous to the standard comparative statics giving:

$$\frac{dp_j^{st}(S_j,k)}{dk} = -S_0 N(Z_k) < 0$$

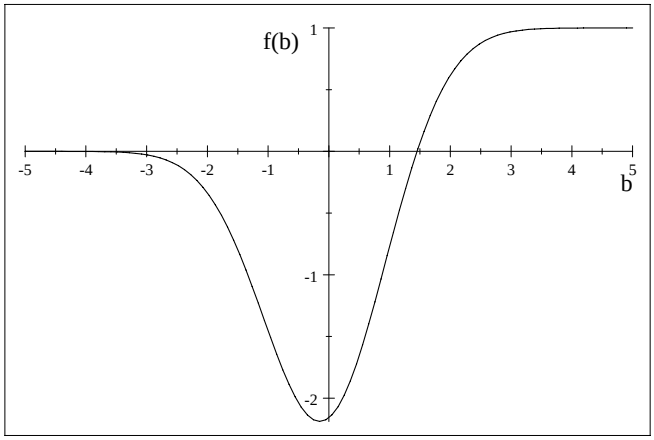
Increasing the exercise price, therefore, lowers the price of the European call, a well-known result from no-arbitrage considerations.

If $\eta_x = 0$, then $Z_k^- = Z_k$, and the last two equations coincide. The effect of increasing optimism on $dp_j(S_j,k)/dk$ is computed as follows:

$$\frac{d^2 p_j(S_j,k)}{dk d\eta_x} = S_0 \exp\{-\eta_x\}\left[N(Z_k^-) - \frac{n(Z_k^-)}{\sigma_j} \right]$$

The sign of this derivative depends on the expression in square brackets. Given the functional form of

the Normal distribution, this expression is negative up to some value $\overline{Z}_k^-$, and becomes positive thereafter. We report one example of this expression, obtained by imposing $\sigma_j = 0.15$, in the figure below:



where $b = Z_k^-$ and $f(b) = N(Z_k^-) - n(Z_k^-)/\sigma_j$. In this case, the hedging value is about $\overline{Z}_k^- = 1.452$. The curve is affected by stock volatility. Rising σ_j shifts $f(b)$ leftward. Given the values of all parameters, rising η_x implies moving rightwards along the curve in the figure. From the definitions of Z_k and Z_k^-, respectively given by (8) and (21), we can derive the hedging value of optimism:

$$\overline{\eta}_x = \sigma_j\left(\overline{Z}_k^- - Z_k\right)$$

Degrees of optimism higher than $\overline{\eta}_x$ thus lead to a decreasing magnitude (in absolute value) of the hedging coefficient of the partial derivative of the call option price with respect to the exercise price.

B.4. Increasing the Riskless Rate of Return

The total differentiation of (20) with respect to r_f gives:

$$\frac{dp_j(S_j,k)}{dr_f} = S_j\frac{\partial N(Z_k^+)}{\partial Z_k^+}\frac{dZ_k^+}{dr_f} - k\exp\{-\eta_x\}N(Z_k^-)\frac{dS_0}{dr_f} - kS_0\exp\{-\eta_x\}\frac{\partial N(Z_k^-)}{\partial Z_k^-}\frac{dZ_k^-}{dr_f}$$

From (21), it is easy to show that $\partial Z_k^+ / \partial r_f = \partial Z_k^- / \partial r_f = \sigma_j^{-1}$ and, from the definition $S_0 = \exp\{-r_f\}$, that $\partial S_0 / \partial r_f = -S_0$. The last expression becomes:

$$\frac{dp_j(S_j,k)}{dr_f} = kS_0 \exp\{-\eta_x\} N(Z_k^-) > 0$$

Once again, this result is analogous to the standard comparative statics giving:

$$\frac{dp_j^{st}(S_j,k)}{dr_f} = kS_0 N(Z_k) > 0$$

Increasing the discount rate, therefore, raises the price of the European call.

If $\eta_x = 0$, then $Z_k^- = Z_k$, and the last two equations coincide. The effect of increasing optimism on $dp_j(S_j,k)/dr_f$ is computed as follows:

$$\frac{d^2 p_j(S_j,k)}{dr_f d\eta_x} = -kS_0 \exp\{-\eta_x\}\left[N(Z_k^-) - \frac{n(Z_k^-)}{\sigma_j}\right]$$

The sign of this derivative is the opposite of $d^2 p_j(S_j,k)/(dkd\eta_x)$. Degrees of optimism higher than $\overline{\eta}_x$ thus lead to decreasing values of the hedging coefficient of the partial derivative of the call option price with respect to the riskless asset returns.

B.5. Increasing Stock Volatility

The total differentiation of (20) with respect to σ_j gives:

$$\frac{dp_j(S_j,k)}{d\sigma_j} = S_j \frac{\partial N(Z_k^+)}{\partial Z_k^+}\frac{dZ_k^+}{d\sigma_j} - kS_0 \exp\{-\eta_x\}\frac{\partial N(Z_k^-)}{\partial Z_k^-}\frac{dZ_k^-}{d\sigma_j}$$

From (21), it is easy to show that:

$$\frac{\partial Z_k^-}{\partial \sigma_j} = \frac{\partial Z_k^+}{\partial \sigma_j} - 1 = -\frac{\ln(S_j/k) + r_f}{\sigma_j^2} - \frac{1}{2} - \frac{\eta_x}{\sigma_j^2}.$$

The former expression thus becomes:

$$\frac{dp_j(S_j,k)}{d\sigma_j} = kS_0 \exp\{-\eta_x\} n(Z_k^-) > 0$$

Once again, this result is analogous to the standard comparative statics giving:

$$\frac{dp_j(S_j,k)}{d\sigma_j} = kS_0 n(Z_k) > 0$$

Increasing the variance of $\ln(Z_j / S_j)$ raises the price of the European call.

If $\eta_x = 0$, then $Z_k^- = Z_k$, and the last two equations coincide. The effect of increasing optimism on $dp_j(S_j, k)/d\sigma_j$ is computed as follows:

$$\frac{d^2 p_j(S_j,k)}{d\sigma_j d\eta_x} = -kS_0 \exp\{-\eta_x\} n(Z_k^-)\left[1 + \frac{Z_k^-}{\sigma_j}\right]$$

The sign of this derivative depends on the expression in square brackets. Given the functional form of Normal density function, this expression is negative up to $\tilde{Z}_k^- = -\sigma_j$, and becomes positive thereafter. Using (6), it is easy to show that the value $\tilde{\eta}_x = -\left[\ln(S_j / k) - \ln S_0 + \sigma_j^2 / 2\right]$ is the value of optimism corresponding to $Z_k^- = \tilde{Z}_k^-$. Degrees of optimism higher than $\tilde{\eta}_x$ thus lead to decreasing magnitude of the hedging coefficient of the partial derivative of the call option price with respect to the exercise price.

B.6. Increasing Expected Happiness

The total differentiation of (20) with respect to μ_x gives:

$$\frac{dp_j(S_j,k)}{d\mu_x} = S_j \frac{\partial N(Z_k^+)}{\partial Z_k^+}\frac{dZ_k^+}{d\mu_x} - kS_0 \frac{d\exp\{-\eta_x\}}{d\mu_x} N(Z_k^-) -$$
$$-kS_0 \exp\{-\eta_x\}\frac{\partial N(Z_k^-)}{\partial Z_k^-}\frac{dZ_k^-}{d\mu_x}.$$

From (21), it is easy to show that $\partial Z_k^+ / \partial \mu_x = \partial Z_k^- / \partial \mu_x = 1/\sigma_j$. We have:

$$\frac{dp_j(S_j,k)}{d\mu_x} = kS_0 \exp\{-\eta_x\} N(Z_k^-) > 0,$$

Increasing the expected value of happiness raises the price of the European call.

If $\eta_x = 0$, then $Z_k^- = Z_k$, and the last two equations coincide. The effect of increasing optimism on $dp_j(S_j, k)/d\mu_x$ is computed as follows:

$$\frac{d^2 p_j(S_j,k)}{d\mu_x d\eta_x} = -kS_0 \exp\{-\eta_x\}\left[N(Z_k^-) - \frac{n(Z_k^-)}{\sigma_j}\right]$$

The sign of this derivative the same as $d^2 p_j(S_j,k)/(dr_f d\eta_x)$. Degrees of optimism higher than $\overline{\eta}_x$ thus lead to decreasing values of the hedging coefficient of the partial derivative of the call option price with respect to the riskless asset returns

B.7. Increasing Happiness Volatility

The total differentiation of (20) with respect to σ_x gives:

$$\frac{dp_j(S_j,k)}{d\sigma_x} = S_j \frac{\partial N(Z_k^+)}{\partial Z_k^+}\frac{dZ_k^+}{d\sigma_x} - kS_0 \frac{d\exp\{-\eta_x\}}{d\sigma_x} N(Z_k^-) -$$
$$-kS_0 \exp\{-\eta_x\}\frac{\partial N(Z_k^-)}{\partial Z_k^-}\frac{dZ_k^-}{d\sigma_x}$$

From (21), it is easy to show that $\partial Z_k^+ / \partial \sigma_x = \partial Z_k^- / \partial \sigma_x = \sigma_x / \sigma_j$. We have:

$$\frac{dp_j(S_j,k)}{d\sigma_x} = kS_0\sigma_x \exp\{-\eta_x\} N(Z_k^-) > 0$$

Increasing the variance of consumer confidence raises the price of the European call.

If $\eta_x = 0$, then $Z_k^- = Z_k$, and the last two equations coincide. The effect of increasing optimism on $dp_j(S_j,k)/d\sigma_x$ is computed as follows:

$$\frac{d^2 p_j(S_j,k)}{d\sigma_x d\eta_x} = -kS_0\sigma_x \exp\{-\eta_x\}\left[N(Z_k^-) - \frac{n(Z_k^-)}{\sigma_j}\right]$$

The sign of this derivative the same as $d^2 p_j(S_j,k)/(d\mu_x d\eta_x)$. Degrees of optimism higher than $\overline{\eta}_x$ thus lead to decreasing values of the hedging coefficient of the partial derivative of the call option price with respect to the riskless asset returns.