Blind Stealing: Experience and Expertise in a Mixed-Strategy Poker Experiment*

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Abstract

We explore the role of experience in mixed-strategy games by comparing, for a stylized version of Texas Hold-em, the behavior of experts, who have extensive experience playing poker online, to the behavior of novices. We find significant differences. The initial frequencies with which players bet and call are closer to equilibrium for experts than novices. And, while the betting and calling frequencies of both types of subjects exhibit too much heterogeneity to be consistent with equilibrium play, the frequencies of experts exhibit less heterogeneity. We find evidence that the style of online play transfers from the field to the lab.

Keywords: expertise, mixed strategy, minimax, laboratory experiments.

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1 Introduction

Game theory has revolutionized the field of economics over the last 60 years and has had a significant impact in biology, computer science, and political science as well. Yet there is conflicting evidence on whether the theory successfully predicts human behavior. For mixed-strategy games, i.e., games requiring that a decision maker be unpredictable, these doubts have emerged as a result of laboratory experiments using student subjects. In these experiments, the behavior of student subjects is largely *inconsistent* with von Neumann's minimax hypothesis and its generalization to mixed-strategy Nash equilibrium: students do not choose actions according to the equilibrium proportions and they exhibit serial correlation in their actions, rather than the serial independence predicted by theory.[1] On the other hand, evidence from professional sports contests suggests that the on-the-field behavior of professionals in situations requiring unpredictability does conform to the theory, e.g., see Walker and Wooders (2001) who study first serves in tennis and see Chiappori, Levitt, and Groseclose (2002) and Palacios-Huerta (2003) who study penalty kicks in soccer.

This evidence suggests that behavior is consistent with game theory in settings where the financial stakes are large and, perhaps more important, where the players have devoted their lives to becoming experts, while behavior is less likely to be consistent with theory when the subjects are novices in the strategic situation at hand. The present paper explores the role of experience in mixed-strategy games by comparing the behavior of novice poker players to the behavior of expert players who have extensive experience playing online poker. We find that the behavior of experts is closer to equilibrium than the behavior of novices. Nevertheless, even our expert players exhibit significant departures from equilibrium.

Our experimental game is a stylized representation of "blind stealing," a strategic interaction that commonly arises in popular versions of poker such as Texas Hold'em. In order to maximize the saliency of the experience of the expert players, the game is endowed with a structure and context similar to an actual game of "heads up" (two player) Texas Hold'em. In the experimental game, just as in heads up Hold'em, the players alternate between one of two positions which differ in the size of the ante (known as the "blind") and who moves first. Employing the same language used in actual play, we labelled these positions as the "small blind" and the "big blind."

[1] See Figure 1 of Erev and Roth (1998) for a discussion of 12 such experiments, and see Camerer (2003) for a survey of mixed-strategy experiments.

The action labels also correspond to their real-world counterparts: The small blind position moves first, choosing whether to "bet" or "fold." Following a bet by the small blind, the big blind chooses whether to "call" or "fold."[2]

While the experimental game is a highly stylized version of Texas Hold'em, the game is sufficiently rich that the small blind has an incentive to bluff and thereby attempt to "steal" the blinds. In equilibrium, when holding a weak hand, the small blind mixes between betting or folding. He is said to have "stolen" the blinds when he bets with a weak hand and the big blind folds. Likewise, the big blind mixes between calling or folding when holding a weak hand and facing a bet.

We find that, in aggregate, both students and expert poker players bet too frequently relative to equilibrium, although poker players bet at a frequency closer to the equilibrium. Students also call too frequently, while the poker players call at the equilibrium rate. At the individual-player level, Nash (and minimax) play is rejected far too frequently to be consistent with equilibrium. However, Nash play is rejected less frequently for poker players than students, for both positions. Thus the behavior of experts is closer to equilibrium than the behavior of novices. The differences in play are statistically significant.

Novices and experts also differ in how their behavior changes over time. From the first half to the second half of the experiment, the equilibrium mixtures of novices move (in aggregate) closer to equilibrium for both the small and the big blind positions. By contrast, although the mixtures of the experts are slightly closer to the equilibrium mixtures in the second half, the change between halves is not statistically significant. Thus the closer conformity of the experts to equilibrium is a consequence of a difference in initial play. Indeed, considering only the second half of the experiment, one can not reject that novices and experts mix at the same rate. This suggests that the behavior of novices, who have limited or no experience in the field, approaches the behavior of experts, once novices obtain sufficient experience with the experimental game.

A unique feature of our study is that we obtain the "hand histories" of the online play (e.g., at Poker Stars, Full Tilt Poker, etc.) for some of our expert players. Hand histories are text files that show a complete record of the cards a player receives, the

[2]Rapoport, Erev, Abraham, and Olsen (1997) employ students, who were not selected for experience playing poker, to test the minimax hypothesis in a simplified poker game in which only the first player to move has private information. Unlike in the present paper, it is largely framed in an abstract context.

actions he takes, and the actions he observes of his opponents, once he joins a game. A player may choose to have this data automatically downloaded onto his computer as he plays. Using the hand history data, we compare the subjects' behavior in our game to their online behavior. We find that the playing style of experts is correlated between the field and the lab: players who are aggressive online (i.e., they bet with a high frequency) are also aggressive in our experimental game. Hence the style of play transfers from one setting to another, when the context is similar.

Related Literature

Several experimental studies have highlighted the importance of field experience on behavior in markets and games.[3] For mixed-strategy games, Palacios-Huerta and Volij (2008) argue that Spanish professional soccer players exactly follow minimax in O'Neill's (1987) classic mixed-strategy game when in the laboratory, and very nearly follow minimax in a 2×2 "penalty kick" game they develop. This is evidence, so they argue, that experience with mixed-strategy equilibrium play on the field (e.g., Palacios-Huerta (2003)) transfers to the play of abstract normal form mixed-strategy games in the laboratory. In other words, subjects who play mixed-strategy equilibrium in one setting will play it in another.

This finding has been challenged from two directions. Levitt, List, and Reiley (2010) are unable to replicate it, using either professional American soccer players or professional poker players, two groups of subjects that are experts in settings requiring randomization. They report that "... professional soccer players play no closer to minimax than students ... and far from minimax prediction." Indeed, their soccer players deviate *more* from minimax in the O'Neill game than do students or poker players. Thus they find no support for the hypothesis that experience in mixed-strategy play transfers from the field to the laboratory. Wooders (2010) takes another tack and reexamines the PH-V data. He finds that their data is inconsistent with minimax play in several respects, the most important being that the distribution of action frequencies across players is far from the distribution implied by the minimax hypothesis. Put simply, actual play is too close to expected play.

In light of these conflicting results, there is considerable doubt that expertise in mixed-strategy play transfers from the field to the laboratory. There is, however, intriguing evidence that providing subjects with a meaningful context facilitates such

[3] See, for example, List (2003), Levitt, List, and Sadoff (2011), and Garratt, Walker, and Wooders (2012). Fréchette (2010) provides a nice survey of experiments that compare the behavior of students and professionals.

transfers. Cooper, Kagel, Lo and Gu (1999), in a study of the ratchet effect and using Chinese managers and students as subjects, finds that context facilitates the development of strategic play among managers, but has little impact on the behavior of students. They write (p. 783) "The fact that context had a much larger effect on PRC managers than on students suggests that context must be eliciting something from managers' experience as *managers*." In other words, meaningful context is not enough alone, but experience interacts with context to promote the transfer of expertise.[4]

The experiment reported here was designed to give the transfer of expertise its best possible chance by providing subjects with a meaningful context, and it is the first to do so for mixed-strategy games. Subjects in Palacios-Huerta and Volij (2008) and Levitt, List, and Reiley's (2010) replication, in contrast, faced abstract contexts, and hence were not provided with a cognitive trigger which might facilitate the transfer of expertise from the field to the lab. Indeed, Levitt, List, and Reiley (2010) report for a post-experiment survey of their subjects that "... not one soccer player who participated in the experiment spontaneously responded that the experiment reminded him of penalty kicks."

Our finding that, when provided with a meaningful context, the play of expert poker players is closer to equilibrium than the play of students is in accordance with the findings of Cooper, Kagel, Lu, and Gu (1999). Providing a context, however, may also lead to the transfer of other behaviors from the field to the laboratory, e.g., aggressiveness of play, which are not shaped by considerations of equilibrium in the experimental game. We find evidence of this type of transfer as well.

Section 2 describes the experimental design. Results are reported in Section 3. Section 4 discusses alternative models of equilibrium, and Section 5 concludes.

2 Experimental Design

2.1 The Subjects

Our experiment utilized subjects with and without experience playing poker. We first recruited 34 subjects with experience playing online poker via an advertisement

[4] Cooper and Kagel (2009) shows that meaningful context also facilitates learning from one game to the next in the laboratory. See that paper and Cooper, Kagel, Lo and Gu (1999) for a nice discussion of the relevant psychology literature.

in the *Arizona Daily Wildcat*, the local student paper, and through an email invitation to students registered in the Economic Science Lab's subject database. The advertisement and email directed students to a web page that collected two types of data. First, the students completed an online survey aimed at determining their level of experience playing poker. Our subjects reported an average of more than 4 years experience playing poker and more than 2 years experience playing online, with 61% playing more than 5 hours online a week. With one exception, they reported Texas Hold'em as the game played most frequently.

After completing the survey, the subjects were directed to a web page that enabled them to upload their personal "hand histories" from PartyPoker and PokerStars, two popular online poker websites. A hand history is a text file which contains the record of the play you observe at a table from the time you join the table until the time you leave. A player may choose to have these hand histories automatically stored on his computer while playing on PartyPoker and PokerStars. Our web page contained a *Java* applet which located the player's hand histories, and then uploaded them to a server when he clicked on the "Start Hand History Collection" button. These hand histories enable us to compare the behavior of our subjects in our experimental game to their behavior in the "field," when playing actual online poker. We postpone a detailed discussion of the hand histories until we use them in our analysis.

As a final check that our subjects are experienced, at the end of the experiment they took a quiz in which they were asked to identify the probability ("pre-flop") that a player will win the hand in a two-player contest if the hand goes to a "showdown," for several hypothetical starting hands dealt to the two players.[5]

We recruited an additional 42 subjects who did not have experience playing poker through an email invitation to students in the Economic Science Lab's subject data base. (Any student who responded to the first invitation was excluded from the second.) While all of our subjects were students, for expositional convenience we will henceforth refer to the subjects with experience playing poker as the "poker players" and to the other subjects simply as the "students."

[5] For example, in a heads up contest, if one player has Ad-Ah (i.e., an ace of diamonds and an ace of hearts) and the other has Kc-Ks (i.e., a king of clubs and a king of spades), then the player with aces has a pre-flop winning probability of 81%. See http://twodimes.net/poker/.

2.2 The Experimental Game: Blind Stealing

In the experiment, each subject had an initial endowment of 100 chips and played the Blind Stealing game, described below, against a fixed opponent for up to 200 hands, with each playing to wins chips from his opponent. Poker players played only against other poker players, and knew that they and their opponents had been recruited based on their experience playing poker.

In the game, there are two positions – the "small" blind and the "big" bind – and subjects alternated between positions at each hand. We refer to the overall extensive form game as a "match." The match ended as soon as either (i) 200 hands were completed, or (ii) at the beginning of an odd-numbered hand a subject had fewer than 8 chips. At the end of the match, a $50 prize was allocated to one player or the other, where the probability that the player holding k chips won the $50 was $k/200$. In addition to his earnings from the experiment, each subject received a $10 payment for participating.

In the description of the rules of the Blind Stealing game that follows, we refer to the players by their position.

1. The "Small Blind" antes 1 chip and the "Big Blind" antes 2 chips. The three chips anted are the prize (*aka* the "pot") to be won in the game.

2. Each player is dealt a single card from a four card deck, consisting of one ace and three kings.

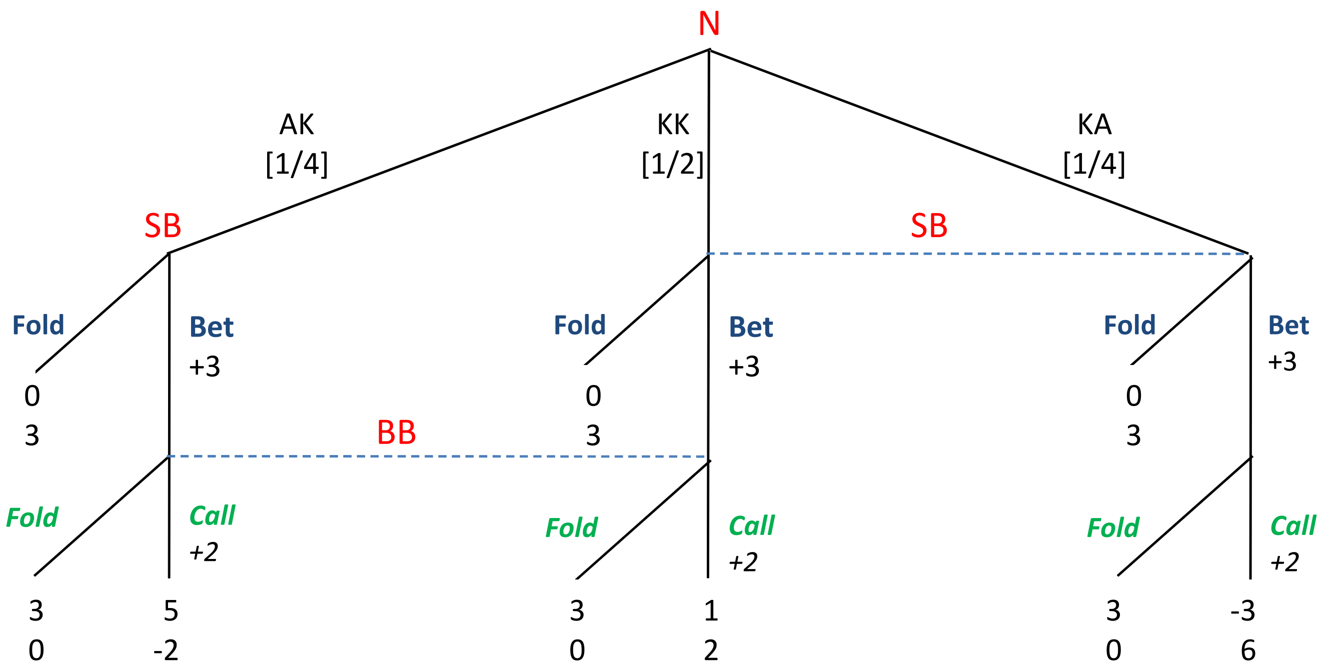
3. The Small Blind moves first, and either bets (by placing 3 additional chips into the pot) or folds. If he folds, the game ends with the Big Blind winning the pot.

4. If the Small Blind bets, then the Big Blind gets the move. He either calls (by placing 2 additional chips into the pot) or folds. If he folds, the Small Blind wins the pot.

5. If the Big Blind calls, then the players' cards are revealed and compared. If a player has the ace, then he wins the 8-chip pot. Otherwise the players split the pot, with each player winning 4 chips.

A written description of the rules of the experimental game were provided to all the subjects, which were then read out loud. To familiarize subjects with the rules of the game and the mechanics of playing, subjects played an unpaid "demo" of 16 hands against the computer (http://poker.econlab.arizona.edu/demo), prior to playing against a human opponent in the experiment. The (pure) strategy followed by the computer was provided to the subjects.

The extensive form of the Blind Stealing game is below, where "AK" denotes that the Small Blind (SB) is dealt an ace, "KA" denotes that the Big Blind (BB) is dealt an ace, and "KK" denotes that both players are dealt kings. We call one play of the Blind Stealing game a "hand."



A single hand of the Blind Stealing game is a constant 3-sum game since the players compete to win the initial ante of 3 chips. In a match a player observes his opponent's card only when the big blind calls. While this is consistent with the actual play of poker, we shall see it complicates the theoretical analysis of the match.[6]

Equilibrium Play of a Hand

The representation above of the extensive form game for a single hand implicitly assumes that it is appropriate to take the number of chips won by a player as his utility payoff. Under this assumption, the Blind Stealing game has a unique Nash equilibrium: the Small Blind bets for sure if he has an ace, and he bets with proba-

[6] If both cards were revealed at the end of each hand, then each new hand begins a proper subgame in the match.

bility 1/2 if he has a king; the Big Blind calls for sure if he has an ace, and he calls with probability 3/4 if he has a king.[7]

In equilibrium, the Small Blind position has an advantage: If the Small Blind draws an ace, then he bets and wins 3 chips if the Big Blind folds and he wins 5 chips if the Big Blind calls, with an expected number of chips won of

$$\frac{1}{4}(3) + \frac{3}{4}(5) = \frac{9}{2}.$$

If the Small Blind draws a king he has an expected payoff of zero. Since he draws an ace with probability 1/4, the Small Blind's equilibrium payoff is $\frac{1}{4}(\frac{9}{2}) = \frac{9}{8}$, and his payoff net of his 1-chip ante is 1/8.

The Small Blind guarantees himself an expected payoff of at least 1/8 of a chip by following his equilibrium strategy. Since this is the maximum payoff he can guarantee himself, 1/8 is the Small Blind's *value*.

The opportunity for the Small Blind to "bluff," i.e., to represent holding a strong card when he actually holds a weak card, allows him to win chips on average.[8] The Small Blind is said to have "stolen" the blinds when he bets with a king and the Big Blind folds. Hence we call this game the "blind stealing" game.

Equilibrium (and Minimax) Play in the Match

In the analysis above of a single hand we took each player's payoff to be the number of chips won. The players, however, are interested in winning chips only as a means of obtaining the \$50 prize for winning the match. We now turn to a characterization of equilibrium play in the match, and verify that it is an equilibrium of the match for each player to play the Nash equilibrium (described above) of each hand, regardless of the past history of play.

It is convenient and without loss of generality to assign a utility of 1 to the outcome in which a player wins the match and a utility of zero when he loses. With

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[7] In the perfect Bayesian equilibrium, when holding a king the Small Blind assigns probability 1/3 to the event that the Big Blind holds an ace. When holding a king and facing a bet, the Big Blind assigns probability 1/2 to the event that the Small Blind holds an ace since

$$\Pr(A_1|Bet, K_2) = \frac{\Pr(A_1, K_2, \mathrm{Bet})}{\Pr(A_1, K_2)\Pr(\mathrm{Bet}|A_1) + \Pr(K_1, K_2)\Pr(\mathrm{Bet}|K_2)} = \frac{\frac{1}{4}}{\frac{1}{4} \times 1 + \frac{1}{2} \times \frac{1}{2}} = \frac{1}{2},$$

where A_i and K_i denote, respectively, the event that player i holds an ace or a king.

[8] In the equilibrium of the game in which the Small Blind's card is observable (and so he can not bluff), the Big Blind folds when the Small Blind bets with an ace, but calls otherwise. Thus it is optimal for the Small Blind to bet with an ace and fold with a king. His expected payoff, net of his 1 chip ante, is only $\frac{1}{4}(3) + \frac{3}{4}(0) - 1 = -\frac{1}{4}$.

this assignment of utilities, a player's expected payoff at any point in the match can be interpreted as the probability that he ultimately wins the match. Since it is certain that one player or the other wins, the match is a 1-sum game. Henceforth we refer to the player in the Small Blind position at the first hand of the match as Player 1, and we refer to the other player as Player 2. Since the players alternate between positions from one hand to the next, Player 1 is the Small Blind on odd numbered hands and the Big Blind on even numbered hands.

Since the match is a constant sum game, von-Neumann's Minimax Theorem tells us there are probability payoffs, v_1 for Player 1 and v_2 for Player 2, with $v_1 + v_2 = 1$, such that (i) Player 1 has a mixed-strategy σ_1 for the match which *guarantees* him in expectation a payoff of at least v_1, (ii) Player 2 has a mixed-strategy σ_2 for the match which *guarantees* him at least v_2, and (iii) the mixed-strategy profile (σ_1, σ_2) is a Nash equilibrium. The payoff v_i is called player i's *value*. The Minimax Theorem, however, doesn't identify each player's value, nor the mixed strategy which assures him his value.

Proposition 1 in the Appendix proves a stronger result for the match. It shows for each $t \in \{1, \ldots, 200\}$ that at the beginning of the t-th hand, each player i has a value v_i^t (i.e., a probability that i can guarantee himself at the t-th hand that he ultimately wins the match) that depends only on the number of chips he holds and whether t is even or odd. Furthermore, it identifies a particular strategy that guarantees him his value. Specifically, if Player 1 holds k_1^t chips at the beginning of the t-th hand, then $v_1^t(k_1^t) = k_1^t/200$ if t is odd and $v_1^t(k_1^t) = (k_1^t - 1/8)/200$ if t is even; for Player 2 we have $v_2^t(k_2^t) = k_2^t/200$ if t is odd and $v_2^t(k_2^t) = (k_2^t + 1/8)/200$ if t is even. Player i obtains this value by following the strategy for the match which calls for playing, at each hand, the Nash equilibrium of the hand, ignoring the history of all prior hands – ignoring his own and his rivals prior cards, ignoring his own and his rival's prior actions, and ignoring the number of chips he holds. Furthermore, it is a Nash equilibrium of the match when each player follows this strategy. Proposition 2 in the Appendix shows that the strategy just described is the unique stationary equilibrium.

Risk Attitudes and the Possibility of Bankruptcy

The match has only two outcomes – a player either wins \$50 or nothing, with the probability of winning \$50 proportional to the number of chips he holds when the match terminates. Hence utility maximization is equivalent to maximizing the expected number of chips, and risk aversion plays no role.

A more subtle issue is the appropriate stopping rule to deal with the possibility that a player runs out of chips prior to the completion of 200 hands. Since a player can lose up to 4 chips in a hand, a natural stopping rule would be to terminate play and implement the lottery if, at the beginning of a hand, either player had fewer than 4 chips.

This "Stop-at-4" rule is inadequate since it is no longer a Nash equilibrium of the match for each player to play the Nash equilibrium of the Blind Stealing game at each hand. To see this, consider Player 1 at hand 199 (and in the small blind) when he has 4 chips prior to anteing. Table 1 describes the possible outcomes. In the table, we denote by $v_1^{200}(k)$ the probability that Player 1 wins the match when he holds k chips at the beginning of hand 200, and each player follows the Nash stationary strategy.

Own Card	Own Action	Rival's Card	Rival's Action	Δ Chips	Winning Prob.
King	Bet	Ace	Call	-4	0/200
King/Ace	Fold	n/a	n/a	-1	3/200
King	Bet	King	Call	0	$v_1^{200}(4)$
King	Bet	King	Fold	$+2$	$v_1^{200}(6)$
Ace	Bet	King	Call	$+4$	$v_1^{200}(8)$

Table 1: Possible Outcomes for Player 1 with 4 chips at Hand 199

As shown in the first row, if Player 1 bets with a king and his rival calls with an ace, then he has zero chips at the end of the hand and loses the match. If Player 1 folds, then he has 3 chips at the end of the hand, the lottery is implemented, and he wins with probability 3/200. In the remaining contingences, Player 1 holds at least 4 chips at the end of the hand and the match continues to hand 200 (the final hand), where he is in the Big Blind.

We show that it is not a Nash equilibrium for each player to follow his Nash-stationary strategy with the Stop-at-4 bankruptcy rule. In particular, Player 1 has an incentive to deviate at hand 199 if dealt a king. If Player 1 folds a king at hand 199, he obtains a payoff of 3/200. If he bets, his payoff is only

$$\frac{1}{3}(0) + \frac{2}{3}\left[\frac{3}{4}\frac{v_1^{200}(4)}{200} + \frac{1}{4}\frac{v_1^{200}(6)}{200}\right] = \frac{7}{480},$$

which is less than 3/200, where $v_1^{200}(k) = (k - 1/8)/200$.[9] Thus Player 1 obtains a higher payoff folding a king, when holding 4 chips in hand 199.

[9]If he bets, then with probabilty 1/3 Player 2 has an ace and Player 1's payoff is 0. With

Intuitively, it is advantageous for Player 1 to fold the king since this ends the match, and he thereby avoids being in the Big Blind at hand 200. (Recall that the Big Blind loses 1/8 of a chip in expectation.) Hence "Nash at every hand" is not a Nash equilibrium with the Stop-at-4 rule. With our stopping rule, by contrast, "Nash at every hand" is not only an equilibrium, it is also (by Proposition 2) the unique equilibrium in stationary strategies.[10]

In the experiment no subject went bankrupt. The stopping rule is nonetheless important since it affects equilibrium play at every hand, not just those hands in which a subject is on the verge of bankruptcy.

3 Results

3.1 Equilibrium Mixtures

Aggregate Play

No poker player ever folded an ace; four students folded a total of 9 aces in a total of 8400 hands.[11] Thus we focus on the players' decisions when holding a king. Table 2 shows the frequency that poker players and students bet with a king (when in the small blind) and call with a king (when in the big blind) over all 200 hands. Poker players, for example, bet in 1692 of the 2579 hands in which a player held a king in the small blind.

	Bet K	Call K
Poker Players	65.6% (1692/2579)	74.3% (1458/1963)
Students	69.0% (2198/3187)	78.5% (1912/2436)
Theory	50.0%	75.0%

Table 2: Aggregate Play over 200 Rounds

It's evident from the table that both students and poker players bluff too frequently. Let N_j^i denote the number of times a subject of type $i \in \{poker, student\}$ takes action $j \in \{B, F\}$ when dealt a king in the small blind, and let $N^i = N_B^i + N_F^i$. Under the

probability 2/3 Player 2 has a king. Given a king, Player 2 calls with probability 3/4 and Player 1's payoff is $v_1^{200}(4)$; Player 2 folds with probability 1/4 and Player 1's payoff is $v_1^{200}(6)$.

[10] "Nash at every hand" will be an equilibrium for any stopping rule that guarentees a player is in each position the same number of times.

[11] Of these, 6 instances were in the first 100 hands of a match.

null hypothesis of minimax play, i.e., $p_B = p_F = 1/2$, the Pearson goodness of fit test statistic

$$Q = \sum_{j \in \{B,F\}} \frac{(N_j^{poker} - N^{poker} p_j)^2}{N^{poker} p_j}$$

is distributed chi-square with 1 degree of freedom.[12] This null is decisively rejected for poker players ($Q = 251.26$, $p = 1.37 \times 10^{-56}$). The same null is also rejected for students ($Q = 458.64$, $p = 9.51 \times 10^{-102}$).

Both types of subjects, however, call with a king at rates much closer to the theoretical one. Remarkably, one can not reject the null hypothesis that poker players call according to the theoretical mixture ($Q = 0.55$, $p = 0.46$) using the anologous Pearson test for the big blind. The same null is, however, rejected for students ($Q = 15.81$, $p = 6.97 \times 10^{-5}$), who call too frequently relative to the theory.

Table 2 shows that the aggregate frequencies with which poker players bluff and call are each closer to the equilibrium frequencies than those of the students. The differences in behavior are statistically significant. In particular, let p_j^i denote the true (but unknown) probability that a subject of type i takes action j when in the small blind. Under the null hypothesis that poker players and students follow the same mixture, i.e., $p_j^{poker} = p_j^{student}$, the test statistic for the Pearson test of the equality of two multinomial distributions is

$$Q = \sum_{i \in \{poker, student\}} \sum_{j \in \{B,F\}} \frac{(N_j^i - N^i \hat{p}_j)^2}{N^i \hat{p}_j},$$

where

$$\hat{p}_j = \frac{\sum_{i \in \{poker, student\}} N_j^i}{\sum_{i \in \{poker, student\}} N^i},$$

and is distributed chi-square with one degree of freedom. This null hypothesis is decisively rejected ($Q = 7.34$, $p = 0.007$), as is the null that poker players call with the same probability as students ($Q = 10.78$, $p = 0.001$).

Aggregate Play – By Half

Poker players and students also differ in how their behavior changes between the first and the second half of the match. Table 3 shows the aggregate betting and

[12] See p. 444-449 of Mood, Graybill, and Boes (1974) for a description of the Pearson tests we employ.

calling frequencies for the first and last 100 hands.

Hands		Bet K	Call K
1-100	Poker Players	65.5% (833/1272)	73.3% (736/1004)
	Students	72.5% (1167/1609)	79.9% (990/1239)
101-200	Poker Players	65.7% (859/1307)	75.3% (722/959)
	Students	65.3% (1031/1578)	77.0% (922/1197)
Theory		50.0%	75.0%

Table 3: Aggregate Play By Half

There is no tendency for poker players to change their behavior between the first and last 100 hands. In particular, the Pearson test of the equality of two multinomial distributions does not reject the null hypothesis that they bluff at the same rate in each half ($Q = 0.016$, $p = .900$). And, while poker players call at a rate slightly closer to equilibrium in the second than in the first half, the difference between the two rates is not statistically significant ($Q = 1.01$, $p = 0.316$).

The aggregate behavior of students, in contrast, changes between the two halves with the betting and calling frequencies both moving closer to the equilibrium frequencies. The betting frequency of students is 7.2% lower in the second half. The Pearson test of the equality of two multinomial distributions rejects the null hypothesis that the aggregate betting frequencies are the same in each half ($Q = 19.26$, $p = 1.14 \times 10^{-05}$). The aggregate calling frequency declines by 2.9 percentage points. One can reject the null hypothesis that the aggregate calling frequencies are the same in each half ($Q = 2.99$, $p = 0.084$) at the 10% significance level.

As a result of the change in student behavior, the aggregate betting and calling frequencies of poker players and students are statistically indistinguishable in the second half. One can not reject the null hypothesis that the betting frequencies of poker players (65.7%) and students (65.3%) are the same ($Q = 0.047$, $p = 0.828$). Nor can one reject that the calling frequencies are the same ($Q = 0.889$, $p = 0.346$). The analogous null hypotheses are both decisively rejected for the first half.[13]

These results suggest that experience playing poker causes the initial behavior of poker players to conform more closely to equilibrium than the behavior of students who do not have this experience. As students gain experience with the experimental

[13]The analogous p-values are 4.63×10^{-05} and 2.26×10^{-04}.

game, however, their (aggregate) behavior quickly becomes indistinguishable from that of poker players.

Individual Level Play

We examine whether behavior at the individual player level is consistent with minimax. Let n_K^i denote the number of times player i received a king when in the small blind in the first 100 hands. Under the null hypothesis of minimax play, the number of times player i bets with a king is distributed $B(n_K^i, p)$, with *cdf* denoted by $F_{bin}(n_{bet}^i; n_K^i, p)$, where n_{bet}^i is the number of bets and $p = .5$. Given n_{bet}^i, we form the random test statistic t^i where $t^i \sim U[0, F_{bin}(0; n_K^i, .5)]$ if $n_{bet}^i = 0$ and $t^i \sim U[F_{bin}(n_{bet}^i - 1; n_K^i, .5), F_{bin}(n_{bet}^i; n_K^i)]$ otherwise, where U denotes the uniform distribution. Under the null hypothesis of minimax play, the statistic t^i is distributed $U[0, 1]$. For each t^i, the associated p-value is $p^i = \min\{2t^i, 2(1 - t^i)\}$, which is also distributed $U[0, 1]$.[14]

At the individual-player level, both poker players and students frequently depart from minimax play. Table 4 shows the empirical betting frequencies of poker players, for the first and last 100 hands, when holding a king.[15] The null hypothesis that in the first 100 hands a poker player bets with a king with probability .5 is rejected at the 5% level for 18 of 34 players (52%). Consistent with the excessive betting observed in aggregate, 17 of these 18 players bet too frequently. In the last 100 hands the same null is reject for 19 players (56%), with 16 of the 19 betting too frequently.

Table 5 shows the same empirical betting frequencies for students. In the first 100 hands, minimax is rejected for 30 of 42 students (71%), with 28 students betting too frequently. In the last 100 hands, it is also rejected for 30 students, but with only 24 students betting too frequently.

Despite the fact that poker players and students bet with similar frequencies in the last 100 hands (65.7% versus 65.3%), the minimax binomial model is rejected more frequently for students (71% versus 56%). In particular, students exhibit more heterogeneity in their betting frequencies than do poker players.

[14] The randomized binomial test based on the p^i's has two advantages over a deterministic decision rule. First, even with a finite sample, the randomized test is symmetric and of exactly size α. More important, each p^i is drawn from the same continuous distribution (viz. the U[0,1] distribution) and hence it is valid to apply the Kolmogorov-Smirnov (KS) goodness of fit test to the empirical *cdf* of the p^i's in order to test the joint null hypothesis that all the players bet according the minimax hypothesis.

[15] A player is in the small blind position 50 times in the first 100 hands, and the expected number of kings is 37.5.

Tables 6 and 7 show, respectively, the empirical calling frequencies of individual poker players and students in the big blind. As noted earlier, in the big blind position poker players in aggregate call according to the equilibrium frequencies. Nonetheless, the null hypothesis that in the first 100 hands a player calls with a king with probability .75 is rejected for 13 of the 34 players (38%) at the 5% level, with 6 of the 13 calling too infrequently. The analogous null hypothesis for the last 100 hands, is rejected for 15 players (44%), also with 6 players calling too infrequently. In each case only 1.7 rejections are expected. Hence, while poker players on average bet according to the equilibrium frequencies, there is far more heterogeneity in their betting frequencies than predicted by the theory.

For students the analogous null hypothesis is rejected for 19 of the 42 players (45%) in the first 100 hands. It is rejected for 24 players (57%) in the last 100 hands. Although the aggregate calling frequencies of poker player and students in the last 100 hands are close, we reject minimax play more frequently for students which suggests there is even greater heterogeneity in their mixtures than in the mixtures followed by poker players.

KS Tests for Differences Between Poker Players and Students

Figure 1 reports the empirical *cdf*s of the p-values obtained from testing, for poker players and students, the null hypothesis that in the first 100 hands a subject bets with probability .5 in the small blind. (There are 34 such p-values for poker players and 42 for students. They are reported on the left hand sides of Tables 4 and 5, respectively.) Figure 2 shows the same *cdf*s for the last 100 hands, and Figures 3 and 4 show the same *cdf*s for the big blind. The empirical distribution of p-values for the poker players and students are given, respectively, by $\hat{F}_{poker}(x) = \frac{1}{34}\sum_{i=1}^{34} I_{[0,x]}(p^i_{poker})$ and $\hat{F}_{student}(x) = \frac{1}{42}\sum_{i=1}^{42} I_{[0,x]}(p^i_{student})$.[16]

We first consider whether the behavior of poker players is "closer" to equilibrium than the behavior of students, i.e., whether the p-values for these tests are stochastically larger for poker players than students. Consistent with this hypothesis, it is visually evident in Figures 1 to 4 that the empirical *cdf*s of p-values for poker players very nearly first order stochastically dominate the same *cdf*s for students

[16] The indicator function is defined as

$$I_{[0,x]}(p^i) = \begin{cases} 1 & \text{if } p^i \leq x \\ 0 & \text{otherwise.} \end{cases}$$

(viz. $\hat{F}_{poker}(x) \leq \hat{F}_{student}(x)$ for all x), in both positions and in both halves. To determine whether the difference is statistically significant we consider the null hypothesis $H_0 : F_{poker}(x) = F_{student}(x)\ \forall x \in [0,1]$ versus the one-tailed alternative $H_1 : F_{poker}(x) < F_{student}(x)\ \forall x \in [0,1]$. Let

$$D_{\text{1-side}} = \max_{x \in [0,1]} \left[\hat{F}_{student}(x) - \hat{F}_{poker}(x) \right].$$

Under the null hypothesis, the statistic $4D^2_{\text{1-side}} \frac{mn}{m+n}$ is distributed chi-square with two degrees of freedom (see p. 148 of Siegel and Castellan), where in this application $m = 42$ and $n = 34$.

As shown in the Table 8, the null hypothesis that the p-values of poker players are drawn from the same distribution as for students is rejected in favor of the alternative for the first 100 hands in the small blind (p-value of .078) and in the last 100 hands in the big blind (p-value of .021). Hence two of the four pairwise comparisons are statistically significant. All four p-values are small, which provides strong evidence that the behavior of poker players is indeed closer to equilibrium than the behavior of students."

Hands		$D_{\text{1-side}}$	$4D^2_{\text{1-side}} \frac{mn}{m+n}$	p-value
1-100	Small Blind	0.261	5.100	0.078
	Big Blind	0.210	3.317	0.190
101-200	Small Blind	0.234	4.022	0.128
	Big Blind	0.321	7.562	0.021

Table 8: KS Test of Closeness to Equilibrium, $m = 42$ and $n = 34$

3.2 Predictability of Play

There are notable differences between poker players and students of their predictability of play that are not captured by the usual runs tests for serial independence, which we report shortly. Three students followed pure strategies when in the small blind, always betting with a king. Facing such an opponent, the big blind optimally always calls and the small blind's 1/8 chip advantage is eliminated.[17] There were also four

[17] In this case, the expected number of chips won by the small blind, net of his 1 chip ante, is $\frac{1}{4}(5) + \frac{1}{2}(1) + \frac{1}{4}(-3) - 1 = 0$.

students who always called in the big blind; an opponent in the small blind increases his expected advantage to 1/4 chips if he optimally responds by betting only when he holds an ace. There was, by contrast, only one poker player who followed a pure strategy.[18]

Students were also more likely to follow predictable rules. Consider the rule "When in the small blind always bet with a king if the last time you held a king you folded." A player who follows this rule is exploitable since if he is observed folding in the small blind, then he is sure to bet when next in the small blind (and hence his bet should be called).[19] There were four students whose choices were consistent with the rule, but only two poker players. There was one student whose choices were consistent with the opposite rule "When in the small blind, always fold with a king if the last time you held a king you bet."

A player whose choices are serially correlated is, in principle, exploitable. We now test the hypothesis that the players' actions are serially independent. Let $a^i = (a^i_1, \ldots, a^i_{n^i_B+n^i_F})$ be the list of actions – bet or fold – in the order they occurred for player i when in the small blind and when dealt a king, where n^i_B and n^i_F are the number of times player i bet and folded. Our test of serial independence is based on the number of runs in the list a^i, which we denote by r^i.[20] We reject the hypothesis of serial independence if there are "too many" runs or "too few" runs. Too many runs suggests negative correlation in betting, while too few runs suggests that the player's choices are positively correlated.

Under the null hypothesis of serial independence, the probability that there are exactly r runs in a list made up of n_B and n_F occurrences of B and F is known (see for example Gibbons and Chakraborti (2003) p. 80). Denote this probability by $f(r; n_B, n_F)$, and let $F(r; n_B, n_F)$ denote the value of the associated c.d.f., *i.e.*, $F(r; n_B, n_F) = \sum_{k=1}^{r} f(k; n_B, n_F)$, the probability of obtaining r or fewer runs. At the 5% significance level, the null hypothesis of serial independence for player i is rejected if either $F(r^i; n^i_B, n^i_F) < .025$ or $1 - F(r^i - 1; n^i_B, n^i_F) < .025$, *i.e.*, if the probability of r^i or fewer runs is less than .025 or the probability of r^i or more runs is less than .025.

[18] This player faced an opponent whose empirical betting frequency was above the equilibrium frequency, and thus always calling was an optimal response.

[19] Since players virtually always bet with an ace, if a player following this rule folds, then he must have a king. When next in the small blind he bets both an ace or a king, i.e., he bets for sure.

[20] A *run* is a maximal string of consecutive identical symbols, either all B's or all F's, *i.e.*, a string which is not part of any longer string of identical symbols.

Tables 9a and 9b shows the data and results for our tests for serial independence. Since players virtually always bet or call with an ace, we focus on their behavior when dealt a king. The left hand side of these tables shows the number of times a player bet and folded when holding a king in the small blind. The "Runs" column indicates the number of runs.[21] The right hand side shows the analogous data for the big blind. At the 5% significance level, serial independence is rejected for poker players in 4 instances (11.7%) in the small blind and an additional 4 instances in the big blind.[22] In both cases, 3 of the rejections are a result of a player's choices exhibiting too few runs. At this significance level, only 1.7 rejections are expected for each position. For students, there are, respectively, 4 (9.5%) and 3 (7.1%) rejections for the small and big blind. Hence, at the level of an individual player, the runs test reveals little difference between poker players and students.

Next consider the joint null hypothesis that each player in a group chooses his actions serially independently. If r^i is the realized number of runs for player i, we form the random test statistic t^i as a draw from the $U[F(r^i - 1; n_B^i, n_F^i), F(r^i; n_B^i, n_F^i)]$ distribution. Under the null hypothesis of serial independence, t^i (the "t-value") is distributed $U[0, 1]$. On the other hand, if players tend to switch too often, there will tend to be too many runs and more than the expected number of large values of t. In this case the empirical c.d.f. $\hat{F}(x)$ of t values will be far from the theoretical c.d.f., *viz.*, $F(x) = x$ for $x \in [0, 1]$.

The realized values of these t^i's are shown in the columns labeled $U[F(r-1), F(r)]$ in Tables 8a and 8b. Figures 5 and 6 show, respectively, the empirical c.d.f.'s of the t values for poker players and students in the small blind and the big blind. (The empirical *cdf*s tend to be above the 45 degree line, which shows there are more than the expected number of subjects whose choices exhibit few runs.) Under the null hypothesis of serial independence, the test statistic $K = \sqrt{n}|\hat{F}(x) - x|$ has a known distribution (see p. 509 of Mood, Boes, and Graybill (1974)), where n is the number of players in the group. The first and third row of Table 10 reports the results of these KS tests. Serial independence is rejected at the 5% level for Poker players in the small blind and for students in the big blind, when in each case we condition on

[21]The amount in the "Tot." column is the number of times a player had to make a decision when holding a king. In the small bind it is the number of kings he received; in the big blind it is the number of times he faced a bet while holding a king.

[22]Serial independence is rejected for both roles for two players.

the player holding a king.[23]

	Poker Players			Students		
	n	K	p-value	n	K	p-value
Small Blind (holding King)	34	1.4496	0.0299	39	1.1880	0.1189
Small Blind (Unconditional)	34	1.2886	0.0722	39	0.8280	0.4993
Big Blind (holding King)	33	0.5398	0.9327	38	1.4518	0.0295
Big Blind (Unconditional)	33	0.9185	0.3677	38	0.6597	0.7769

Table 10: KS Test of Joint Hypothesis of Serial Independence

These results suggest that neither poker players nor students completely successfully choose their actions in a serial independent fashion.

This analysis focuses on the players' decisions to bet/fold (or call/fold) conditional on holding a king. In the play of the match, however, a player doesn't observe his rival's card. Hence it is natural to look for serial correlation in the players' *unconditional* action choices, e.g., his bet/fold decision without conditioning on holding a king. The second and fourth rows of Table 10 shows that the joint null hypothesis that all players choose their (unconditional) actions serially independently can not be rejected for either poker player or students, in either the small or the big blind.[24] Hence, from the perspective of an observer who does not know the players' cards, serial independence is not rejected.

Both students and poker players exhibit far less serial correlation than did the subjects in O'Neill's (1987) experiment. In his data, serial independence is rejected at the 5% significance level for 15 of 50 players (30%). The KS test just described yields a value of $K = 2.503$, with a p-value of 0.000007. We conjecture that the fact that subjects alternated between positions in our experiment accounts for the difference.

[23] Since the runs test is not meaningful when a player always choose the same action, Table 9 excludes the poker player who always called, the four students who always called, and three students who always bet. For these tests we have $n = 33$, 38, and 39, respectively.

[24] The test statistic K in Table 9 tends to be smaller for the players' unconditional actions than for their actions conditional on holding a king. This is intuitive since the random arrival of aces leads to random bets (or calls), as players virtually always bet (or call) with an ace. This tends to reduce the degree of serial correlation in action choices.

3.3 Hand Histories

As noted earlier, a hand history is a text file which contains the record of the play you observe at a table from the time you join until the time you leave the table, and it typically has the results of the play of many hands. We obtained hand histories for 16 of the poker players, which included hand histories for both "ring games" and "tournaments." A ring game is a cash game where the chips and bets correspond to actual money amounts. In tournaments, a player pays a fixed amount of money (a "buy in") to play and then receives a number of chips. A player is eliminated from the tournament when he runs out of chips. In a tournament that pays the top three, the player who remains after all the others are eliminated receives the first-place prize, the last player to be eliminated obtains the second-place prize, and the second-to-last player to be eliminated obtains the third-place prize. In a tournament a player is interested in winning chips only insofar as it prevents (or delays) his elimination.

We used commercial software provided by *PokerTracker* to generate several summary statistics from the hand histories. The first statistic is the number of hands played (HANDS). It is a rough proxy for experience. The second is the percentage of times a player voluntarily put money into the pot before the flop (VOL$).[25] It measures how tightly or loosely a player plays. The third statistic is the percentage of times that a player, in a non-blind position, makes a bet larger than the amount of the big blind (STEAL) when no other player has bet before him. Such bets may be attempts to "steal" the blind, and hence this statistic provides a measure of the aggressiveness of play. These statistics are well-defined for hand histories from both ring games and tournaments, and we pooled both types of histories when generating them. The data is provided in Table 11.

Table 11 goes here.

We are interested in whether behavior in the field is related to behavior in the laboratory. A linear regression in which the dependant variable is the frequency a player bets when holding a king in the small blind (BLUFF) and the independent

[25] Putting money in the blinds is not considered voluntary unless you call from the small blind or call a raise from the big blind.

variables are HANDS, VOL$, and STEAL yields the following result.

$BLUFF$	Coefficient	Std. Error	t	$p > \|t\|$
$HANDS$	8.65E-06	4.77E-06	1.81	0.095
$VOL\$$	0.0104626	.0030866	3.39	0.005
$STEAL$	-0.0024973	.0032357	-0.77	0.455
Constant	0.3726509	.0975951	3.82	0.002
Prob>F	0.0393			
R^2	0.4884			
Observ.	16			

Table 12: Regression Results

The variable VOL$ is highly statistically significant, with a p-value of 0.005. Poker players who bet frequently in the field, playing with their own money, also tend to bet (and bluff) more frequently in our experimental setting. In particular, the regression model suggests that for every 1% increase in pre-flop voluntary betting online, players exhibit 0.01% more bluffing in the lab. These results suggest that behaviors in the field transfer to the laboratory, at least when the contexts are similar.

Since the number of observations is relatively small, we verify the robustness of the regression results by computing the Spearman rank correlation coefficient R between the variables *BLUFF* and *VOL$*. Under the null hypothesis that the two variables are uncorrelated, the distribution of the correlation coefficient is known and therefore the correlation coefficient yields a non-parametric test of the null. For our data, $R = 0.4912$. The associated two-tailed p-value is 0.0534, and hence the null is rejected at the 6% significance level even using this conservative test.

4 Discussion

Our results suggest that the behavior of both poker players and students approaches an "equilibrium," or stable point, of some kind: By the second half of the experiment both groups exhibit the same betting and calling frequencies. Poker players start at a 65% betting frequency and 75% calling frequency and remain there. Students initially bet and call at higher frequencies, but converge to the same 65% and 75% frequencies by the second half of the experiment.

In this section we consider several models that generate an equilibrium betting probability above the Nash equilibrium level of .5. Agent quantal response equilibrium (AQRE, McKelvey and Palfrey (1998)) replaces sequential equilibrium as a solution concept in extensive form games by incorporating decision errors by players via random payoff disturbances. In our application of AQRE to the Blind Stealing game, we assume that players only make decision errors when holding a king since it is transparently dominant to bet and call with an ace, and thus the random utility assumption does not seem to be appropriate. In the logistic AQRE model the payoff disturbance ε_i of player i to each action is assumed to have an extreme distribution $F(\varepsilon_i) = e^{-e^{-\lambda\varepsilon_i}}$ with variance $\frac{\pi^2}{6\lambda^2}$.

Consider first the Small Blind. Denote by $\sigma_B(C)$ the probability that the Big Blind calls with a king. The payoff to the Small Blind to betting with a king, denoted by $u_S(B|K)$, is

$$u_S(B|K) = \frac{1}{3}(-3) + \frac{2}{3}\left[\sigma_B(C) + 3(1 - \sigma_B(C))\right],$$

where the 2/3 is the probability the Small Blind assigns to the Big Blind holding a king, conditional on he himself holding a king. The payoff to folding is zero, i.e., $u_S(F|K) = 0$. The perturbed payoffs to betting and folding are $\hat{u}_S(B|K) = u_S(B|K) + \varepsilon'_S$ and $\hat{u}_S(F|K) = u_S(F|K) + \varepsilon''_S$, where $\varepsilon'_S, \varepsilon''_S \sim F$.

If the Small Blind chooses the action with the highest perturbed payoff, then he chooses $a_S \in \{B, F\}$ with probability

$$\sigma_S(a_S) = \frac{e^{\lambda u_S(a_S|K)}}{e^{\lambda u_S(B|K)} + e^{\lambda u_S(F|K)}}.$$

(See McKelvey and Palfrey (1998)). Rewriting, he bets with probability

$$\sigma_S(B) = \frac{1}{1 + e^{-\lambda(u_S(B|K) - u_S(F|K))}} = \frac{1}{1 + e^{-\lambda(1 - \frac{4}{3}\sigma_B(C))}}. \quad (1)$$

Consider now the Big Blind when facing a bet and holding a king. He believes that the Small Blind holds an ace with probability[26]

$$\frac{\frac{1}{4}}{\frac{1}{2}\sigma_S(B) + \frac{1}{4}} = \frac{1}{2\sigma_S(B) + 1}.$$

His payoff to calling is therefore

$$u_B(C|K) = \frac{1}{2\sigma_S(B) + 1}(-2) + \frac{2\sigma_S(B)}{2\sigma_S(B) + 1}(2).$$

[26] The numerator in this expression is the probability the Small Blind is dealt an ace and bets. The denominator is the probability that the Small Blind bets and the Big Blind holds a king.

He obtains zero by folding, i.e., $u_B(C|K) = 0$. Choosing the action with the highest perturbed payoff, the Big Blind calls with probability

$$\sigma_B(C) = \frac{1}{1 + e^{-\lambda(u_B(C|K) - u_B(F|K))}} = \frac{1}{1 + e^{-\lambda\left(\frac{4\sigma_S(B)-2}{2\sigma_S(B)+1}\right)}}. \tag{2}$$

For each $\lambda > 0$, the **agent quantal response equilibrium** (AQRE) is the pair $(\sigma_S^\lambda(B), \sigma_B^\lambda(C))$ that solves (1) and (2).[27] Larger values of λ correspond to smaller decision errors. As λ approaches ∞ the AQRE approaches the Nash (and perfect Bayesian) equilibrium of the Blind Stealing game; as λ approaches 0 the solution approaches purely random choice.

Figure 7 below shows the reaction function of the Small Blind without decision errors (solid-bold) and with $\lambda = 10$ (solid). Dashed lines show the analogous reaction functions of the Big Blind. The solid double line shows the locus of AQRE obtained by varying λ. From the figure it is clear that AQRE explains betting rates above .5, but at the same time predicts calling rates below .75. The later feature of AQRE is inconsistent with the data for most pairs.[28] Moreover, the maximum betting frequency that AQRE generates is approximately 60%, which is below the aggregate betting frequency observed in that data.

We have estimated λ for every pair of subjects to obtain two sample distributions of λ's, one for poker players and one for students. The average λ for the poker players is 23.22 and for students is 9.51, which suggests that decision errors are smaller for poker players. Using the standard two sample t test with unequal variances, we reject the null hypothesis that the mean for poker players is less than or equal to the mean for students at the 10 percent significance level (p-value of 0.0655).

The high betting frequency can be rationalized if the Big Blind suffers a disutility to calling (in addition to his chip loss) when the Small Blind holds an ace. Denote this disutility by d. The indifference condition that determines the equilibrium rate at which the Small Blind bets with a king is then

$$u_B(C|K) = \left[1 - \frac{2\sigma_S(B)}{2\sigma_S(B)+1}\right](-2-d) + \frac{2\sigma_S(B)}{2\sigma_S(B)+1}(2) = 0 = u_B(F|K).$$

It's easy to verify that $d > 0$ implies $\sigma_S(B) > .5$. An equilibrium betting frequency

[27] The AQRE is unique in our application

[28] The figure shows the AQRE equilibria when the same error parameter λ applies to both positions. If one allows the error parameter to vary across positions, the model still predicts that the calling rate is at most 75%.

of 65% implies a value of d equal to .6 chips, which seems too large to be plausible.[29]

An equilibrium betting frequency above .5 can not be rationalized by a "joy of betting." If the Small Blind obtains a utility bonus for betting, this has no effect on the equilibrium betting probability but it raises the equilibrium calling probability.

5 Conclusions

Our results show that experience in the field matters in mixed strategy games – the behavior of subjects with experience playing online poker accords more closely to mixed-strategy Nash equilibrium than the behavior of inexperienced subjects. The difference in behavior is largely manifested as a difference in early play. In the last 100 hands the aggregate betting and call frequencies of poker players and students are indistinguishable, although the behavior of students is more heterogeneous.

Experience in the field contributes to equilibrium behavior in the lab. To be successful playing poker player in the field, one must quickly identify and exploit deviations from optimal play by one's opponents. The potential of players to exploit any deviation from equilibrium is the force that drives play towards equilibrium. We conjecture that greater skill at exploitation is what drives the behavior of poker players to conform more closely to equilibrium.

An unexpected result is that poker players transfer a style of play from the field to the lab. Players who are involved in many hands when they play online are more likely to be involved in a hand, choosing to bet rather than fold, in our experimental game. Since equilibrium play in the experimental game is the same for all the players, the transfer of a style of play from the field to the lab is inappropriate and would appear to make it less likely that poker players would behave in accordance with equilibrium.

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6 Appendix

Here we formally state and prove Propositions 1 and 2. We begin by defining histories and strategies. A hand history is a record of the cards and actions observed by a single player. Player i's history for a single hand is denoted by (c_i) if player i is dealt the card $c_i \in \{A, K\}$ and the hand ends immediately with the Small Blind folding; it is $(c_i, *)$ if his card is c_i and the Big Blind folds to a bet; it is (c_1, c_2) if the Small Blind bets and the Big Blind calls, in which case both players observe both cards.[30] Thus a *hand history* h for a single hand is an element of $H = \{(c_i)\} \cup \{(c_i, *)\} \cup \{(c_1, c_2)\}$, where $(c_1, c_2) \in \{(A, K), (K, K), (K, A)\}$. There are 7 possible hand histories resulting from the play of a single hand: (A), (K), $(A, *)$, $(K, *)$, (A, K), (K, K), and (K, A). A hand history at the start of the t-th hand, after $t-1$ hands have been completed, is an element of $H^{t-1} = H \times \ldots \times H$ (repeated $t-1$ times), with generic element h^{t-1}, where $H^0 = \{h^0\}$ and h^0 denotes the null history. Denote by $\mathcal{H}$ the set of all possible hand histories, i.e., $\mathcal{H} = \cup_{t=0}^{200} H^t$.

A strategy for a player maps his hand history and current card into an available action. Formally, a *strategy for Player 1* is a function σ_1 which, for every

[30] For example, for a player in the Small Blind the history $(K, *)$ means his card was a King, he bet, and the Big Blind folded. For a player in the Big Blind the same history means his card was a King and he folded to a bet.

$t \in \{1, \ldots, 200\}$, every history $h^{t-1} \in H^{t-1}$ and card $c^t \in \{A, K\}$ prescribes a probability distribution over the actions "Bet" and "Fold" when t is odd and a probability distribution over "Call" and "Fold" when t is even.[31] In particular, for each $t \in \{1, \ldots, 200\}$, $h^{t-1} \in H^{t-1}$ and $c^t \in \{A, K\}$ we have

$$\sigma_1(h^{t-1}, c^t) \in \begin{cases} \Delta\{Bet, Fold\} & \text{if } t \text{ is odd} \\ \\ \Delta\{Call, Fold\} & \text{if } t \text{ is even,} \end{cases}$$

where $\Delta\{Bet, Fold\}$ is the set of all probability distributions on the actions Bet and Fold. A strategy for Player 2, who is in the Small Blind in even hands, is defined analogously.

A match history is a complete record of the cards received and the actions taken by both players in the course of a match. The set of possible action profiles in a hand is given by $\{F, BF, BC\}$, where F denotes the Small Blind folded, BF denotes the Small Blind bet and the Big Blind folded, while BC denotes the Small Blind bet and the Big Blind called. Formally, a *match history* at the start of the t-th hand, after $t-1$ hands have been completed, is the complete history of play of the preceding $t-1$ hands and is an element of $G^{t-1} = G \times \ldots \times G$ (repeated $t-1$ times) where $G = \{(A, K), (K, K), (K, A)\} \times \{F, BF, BC\}$. Let g^0 denote the null history.

Given a pair of strategies (σ_1, σ_2) and a match history g^{t-1}, let $v_i^t(\sigma_1, \sigma_2, g^{t-1})$ denote the probability at the start of the t-th hand that player i ultimately wins the match. Since either one player or the other wins the match, then for each t, each $g^{t-1} \in G^{t-1}$, and each (σ_1, σ_2) we have that $v_1^t(\sigma_1, \sigma_2, g^{t-1}) + v_2^t(\sigma_1, \sigma_2, g^{t-1}) = 1$.

We shall be particularly interested in strategies in which the behavior of a player in a hand depends only on his current position – the Small Blind or the Big Blind – and current card, but which is otherwise independent of the history of play (e.g., the number of chips he holds, or his own or his rival's cards or actions in prior hands). We say that Player 1's strategy is *Nash-stationary* if for each t, each history $h^{t-1} \in H^{t-1}$, and each card c^t that

$$\sigma_1(h^{t-1}, c^t) = \begin{cases} \sigma_S^*(\circ|c^t) & \text{if } t \text{ is odd} \\ \\ \sigma_B^*(\circ|c^t) & \text{if } t \text{ is even,} \end{cases}$$

[31]When Player 1 is in the Big Blind (i.e., t is even) it is understood that his strategy describes the mixture he follows when facing a bet as he takes no action when the Small Blind folds.

where (σ_S^*, σ_B^*) is the Nash equilibrium of a single hand of the blind stealing game, i.e., $\sigma_S^*(Bet|A) = 1$, $\sigma_S^*(Bet|K) = 1/2$, $\sigma_B^*(Call|A) = 1$, and $\sigma_B^*(Call|K) = 3/4$.

We first show that if Player 1 follows his Nash-stationary strategy σ_1^* and he holds k_1^t chips at hand t (prior to anteing) then he *guarantees* himself an (expected) payoff at hand t of at least $k_1^t/200$ if t is odd (i.e., he is in the small blind) and at least $(k_1^t - 1/8)/200$ if t is even (i.e., he is in the big blind), regardless of Player 2's strategy. An analogous result holds for Player 2.

Proposition 1: Minimax Theorem. (i) *Let σ_1^* be the Nash-stationary strategy for Player 1 and let σ_2 be an arbitrary strategy for Player 2. Then for each t and each match history $g^{t-1} \in G^{t-1}$ we have that*

$$v_1^t(\sigma_1^*, \sigma_2, g^{t-1}) \geq \begin{cases} k_1^t/200 & \text{if } t \text{ is odd} \\ \\ (k_1^t - 1/8)/200 & \text{if } t \text{ is even,} \end{cases}$$

where k_1^t is the number of chips held by Player 1 at hand t given g^{t-1}.
(ii) Let σ_2^* be the Nash-stationary strategy for Player 2 and let σ_1 be an arbitrary strategy for Player 1. Then for each t and each match history $g^{t-1} \in G^{t-1}$ we have that

$$v_2^t(\sigma_1, \sigma_2^*, g^{t-1}) \geq \begin{cases} k_2^t/200 & \text{if } t \text{ is odd} \\ \\ (k_2^t + 1/8)/200 & \text{if } t \text{ is even,} \end{cases}$$

where k_2^t is the number of chips held by Player 2 at hand t given g^{t-1}.
(iii) The inequalities in (i) and (ii) hold as equalities for $(\sigma_1, \sigma_2) = (\sigma_1^*, \sigma_2^*)$.

Since the match is a 1-sum game, when $t = 1$ we have $v_1^1(\sigma_1, \sigma_2, g^0) = 1 - v_2^1(\sigma_1, \sigma_2, g^0)$ for each σ_1 and σ_2, where g^0 is the null history. If Player 2 follows, in particular, his Nash-stationary strategy σ_2^*, then for any σ_1 we have

$$v_1^1(\sigma_1, \sigma_2^*, g^0) = 1 - v_2^1(\sigma_1, \sigma_2^*, g^0) \leq \frac{1}{2} = v_1^1(\sigma_1^*, \sigma_2^*, g^0),$$

where the inequality holds by part (ii) of Proposition 1, and the final equality holds by Part (iii) of Proposition 1 and since $k_2^1 = 100$. Therefore $v_1^1(\sigma_1^*, \sigma_2^*, g^0) \geq v_1^1(\sigma_1, \sigma_2^*, g^0)$ for any σ_1, i.e., σ_1^* is a best response to σ_2^*. The analogous argument establishes that σ_2^* is a best response to σ_1^*. Thus we have the following corollary.

Corollary 1: *The profile (σ_1^*, σ_2^*) of Nash-stationary strategies is a Nash equilibrium of the match. In every Nash equilibrium each player wins the match with probability 1/2.*

Proposition 2 establishes that the Nash-stationary strategy profile (σ_1^*, σ_2^*) is the unique Nash equilibrium in stationary strategies.

Proposition 2: *The profile (σ_1^*, σ_2^*) of Nash-stationary strategies is the unique Nash equilibrium in stationary strategies.*

PROOFS OF PROPOSITIONS 1 AND 2

Proof of Proposition 1: Denote by σ_i^* and σ_i, respectively, the Nash stationary strategy and an arbitrary strategy for player i.

We first show the result is true at the last hand, i.e., for $t = 200$. Let $g^{199} \in G^{199}$ be the match history after 199 hands have been completed and let k_i^{200} denote the number of chips held by player i at the start of the last hand.

Player 1's Nash stationary strategy σ_1^*, which calls for $\sigma_B^*(call|A) = 1$ and $\sigma_B^*(call|K) = 3/4$ at $t = 200$, guarantees that he loses in expectation at most $1/8^{th}$ of a chip. Thus when the game terminates he holds (in expectation) at least $k_1^{200} - 1/8$ chips, and hence he wins with probability at least $(k_1^{200} - 1/8)/200$, i.e., $v_1^{200}(\sigma_1^*, \sigma_2, g^{199}) \geq (k_1^{200} - 1/8)/200 \ \forall \ \sigma_2$. Player 2's Nash stationary strategy σ_2^*, which calls for $\sigma_S^*(bet|A) = 1$ and $\sigma_S^*(bet|K) = 1/2$, guarantees he wins in expectation at least $1/8^{th}$ of a chip. Thus when the game terminates he holds (in expectation) at least $k_2^{200} + 1/8$ chips, and hence he wins with probability at least $(k_2 + 1/8)/200$, i.e., $v_2^{200}(\sigma_1, \sigma_2^*, g^{199}) \geq (k_2^{200} + 1/8)/200 \ \forall \ \sigma_1$.

Since the match is a 1-sum game, we have $v_1^{200}(\sigma_1^*, \sigma_2^*, g^{200}) + v_2^{200}(\sigma_1^*, \sigma_2^*, g^{200}) = 1$ for each (σ_1, σ_2). Hence $v_1^{200}(\sigma_1^*, \sigma_2^*, g^{200}) \geq (k_1^{200} - 1/8)/200$, $v_2^{200}(\sigma_1^*, \sigma_2^*, g^{200}) \geq (k_2^{200} + 1/8)/200$, and $k_1^{200} + k_2^{200} = 200$ implies $v_1^{200}(\sigma_1^*, \sigma_2^*, g^{200}) = (k_1^{200} - 1/8)/200$ and $v_2^{200}(\sigma_1^*, \sigma_2^*, g^{200}) = (k_2^{200} + 1/8)/200$. Thus Proposition 1 holds for $t = 200$.

Assume that the result is true for $t + 1$, where $t + 1 \leq 200$. We show that it is true for t. Let $g^{t-1} \in G^t$ and let k_i^t denote the number of chips held by player i at the start of the t-th hand. We consider two cases: t is odd and t is even.

Suppose that t is odd. If $k_i^t < 8$ for some player i, then the result is trivially true since in this case the game ends immediately, Player 1 wins with probability $k_1^t/200$, and Player 2 wins with probability $k_2^t/200$. Suppose $k_i^t \geq 8$ for both players. Player 1's Nash-stationary strategy σ_1^* guarantees he wins in expectation at least $1/8^{th}$ of a chip when in the small blind. Hence

$$v_1^t(\sigma_1^*, \sigma_2, g^{t-1}) \geq E\left[\frac{k_1^{t+1} - \frac{1}{8}}{200}\right] = \frac{E[k_1^{t+1}] - \frac{1}{8}}{200} \geq \frac{k_1^t}{200},$$

where the first inequality holds by the induction hypothesis and since Player 1 is the big blind at $t+1$, and the second inequality holds since $E[k_1^{t+1}] \geq k_1^t + 1/8$. The analogous argument establishes for Player 2 (the big blind) that $v_2^t(\sigma_1, \sigma_2^*, g^{t-1}) \geq k_2^t/200$.

Suppose that t is even. Player 1's the Nash-stationary strategy σ_1^* guarantees that he loses in expectation at most $1/8^{th}$ of a chip when in the big blind. Hence

$$v_1^t(\sigma_1^*, \sigma_2, g^{t-1}) \geq E\left[\frac{k_1^{t+1}}{200}\right] = \frac{E[k_1^{t+1}]}{200} \geq \frac{k_1^t - \frac{1}{8}}{200},$$

where the first inequality holds by the induction hypothesis and since Player 1 is the small blind at $t+1$, and the second inequality holds since $E[k_1^{t+1}] \geq k_1^t - 1/8$. The analogous argument establishes for Player 2 (the small blind) that $v_2^t(\sigma_1, \sigma_2^*, g^{t-1}) \geq (k_2^t + 1/8)/200$.

Whether t is even or odd, since $v_1^t(\sigma_1^*, \sigma_2^*, g^{t-1}) + v_2^t(\sigma_1^*, \sigma_2^*, g^{t-1}) = 1$ we have $v_i^t(\sigma_1^*, \sigma_2^*, g^{t-1}) = k_i^t/200$ if t is odd, and we have $v_1^t(\sigma_1^*, \sigma_2^*, g^{t-1}) \geq (k_1^t - 1/8)/200$ and $v_2^t(\sigma_1^*, \sigma_2^*, g^{t-1}) = (k_2^t + 1/8)/200$ if t is even. $\square$

Proof of Proposition 2: A strategy σ_1' for Player 1 is stationary if it depends on Player 1's position and card, but is otherwise independent of the history of play. In other words, if σ_1' is stationary, then for each t, each $h^t \in H^t$, and each $c^t \in \{A, K\}$ we can write

$$\sigma_1'(h^t, c^t) = \begin{cases} \sigma_S'(\circ|c^t) & \text{if } t \text{ is odd} \\ \\ \sigma_B'(\circ|c^t) & \text{if } t \text{ is even,} \end{cases}$$

for some σ_S' and σ_B', where σ_S' and σ_B' are strategies for the small and big blind of a single hand of the Blind Stealing game. A stationary strategy for Player 2 is defined analogously.

Suppose that (σ_1', σ_2') is a Nash equilibrium in stationary strategies, in which at least one player's strategy is not Nash stationary. Assume Player 1 does not follow the Nash-stationary strategy. Consider, for example, $\sigma_S'(Bet|A) = 1$ and $\sigma_S'(Bet|K) = \gamma > 1/2$, i.e., Player 1 always bets with an ace and bets with a king with probability γ. We show that $v_2^1(\sigma_1', \sigma_2', g^0) > 1/2$, which contradicts Corollary 1.

Consider the strategy $\tilde{\sigma}_2$ for Player 2 in which at the first hand he calls for sure, and thereafter he follows his Nash-stationary strategy. At the first hand, there are four possible outcomes for Player 2:

- If $(c_1, c_2) = (A, K)$, then Player 1 bets, and Player 2 calls and loses 2 chips. Since he anted two chips at the first hand, he begins the next hand with $96 = 98 - 2$ chips and, by Proposition 1(ii) he wins the match with probability of at least $(96 + 1/8)/200$. This occurs with probability $1/4$.

- If $(c_1, c_2) = (K, K)$ and Player 1 bets, then Player 2 wins 2 chips and he begins the next hand with $98 + 2$ chips. This occurs with probability $\gamma/2$. By Proposition 1(ii) he wins with probability of at least $(100 + 1/8)/200$.

- If $(c_1, c_2) = (K, A)$ and Player 1 bets, then Player 2 wins 6 chips and he begins the next hand with $104 = 98 + 6$ chips. He wins the match with probability at least $(104 + 1/8)/200$. This occurs with probability $\gamma/4$.

- If $(c_1, c_2) = (K, K)$ or $(c_1, c_2) = (K, A)$ and Player 1 folds, the Player 2 wins 3 chips and starts the next hand with $101 = 98 + 3$ chips. He wins the match with probability at least $(104 + 1/8)/200$. This occurs with probability $(1/2 + 1/4)(1 - \gamma)$.

Thus

$$\begin{aligned} v_2(\sigma_1', \tilde{\sigma}_2, g^0) &\geq \frac{1}{4}\frac{96 + \frac{1}{8}}{200} + \frac{\gamma}{2}\frac{100 + \frac{1}{8}}{200} + \frac{\gamma}{4}\frac{104 + \frac{1}{8}}{200} + \frac{3(1-\gamma)}{4}\frac{101 + \frac{1}{8}}{200} \\ &= \frac{2}{1600}\gamma + \frac{799}{1600} \\ &> \frac{1}{2}, \end{aligned}$$

since $\gamma > 1/2$. Since σ_2' is a best response to σ_1', then $v_2(\sigma_1', \sigma_2', g^0) \geq v_2(\sigma_1', \tilde{\sigma}_2, g^0)$ and thus $v_2(\sigma_1', \sigma_2', g^0) > 1/2$. This contradicts Corollary 1 which shows that Player 1 wins with probability 1/2 in a Nash equilibrium.

If $\gamma < 1/2$ then the analogous argument shows that Player 2 has a strategy (*viz.*, fold to any bet in the first hand and play the Nash-stationary strategy thereafter) that yields a payoff strictly greater than 1/2.

If Player 1 follows a stationary strategy in which $\sigma_S'(Bet|A) < 1$, then Player 2's Nash stationary strategy gives him a payoff strictly greater than 1/2, which again yields a contradiction. □

Table 4: Poker Players - Small Blind

First versus Second Half Mixtures (with a King)

		Hands 1-100							Hands 101-200						
					Mixture		Rand					Mixture		Rand	
Pair	Player	F	B	Tot.	F	B	t	p-value	F	B	Tot.	F	B	t	p-value
1	A	21	22	43	0.488	0.512	0.513	0.974	19	19	38	0.500	0.500	0.442	0.883
	B	21	12	33	0.636	0.364	0.080	0.161	23	20	43	0.535	0.465	0.275	0.551
2	C	16	25	41	0.390	0.610	0.912	0.176	21	17	38	0.553	0.447	0.225	0.450
	D	11	22	33	0.333	0.667	0.965	0.070 *	15	24	39	0.385	0.615	0.921	0.157
3	E	13	17	30	0.433	0.567	0.810	0.380	8	29	37	0.216	0.784	1.000	0.000 **
	F	11	26	37	0.297	0.703	0.992	0.016 **	15	26	41	0.366	0.634	0.952	0.096 *
4	G	6	29	35	0.171	0.829	1.000	0.000 **	10	28	38	0.263	0.737	0.998	0.003 **
	H	8	26	34	0.235	0.765	0.999	0.002 **	19	15	34	0.559	0.441	0.283	0.566
5	I	23	12	35	0.657	0.343	0.031	0.062 *	23	16	39	0.590	0.410	0.147	0.295
	J	16	24	40	0.400	0.600	0.903	0.193	16	20	36	0.444	0.556	0.723	0.553
6	K	4	37	41	0.098	0.902	1.000	0.000 **	3	32	35	0.086	0.914	1.000	0.000 **
	L	13	19	32	0.406	0.594	0.881	0.238	12	27	39	0.308	0.692	0.994	0.012 **
7	M	8	32	40	0.200	0.800	1.000	0.000 **	17	24	41	0.415	0.585	0.888	0.225
	N	29	10	39	0.744	0.256	0.001	0.002 **	23	9	32	0.719	0.281	0.005	0.010 **
8	O	12	22	34	0.353	0.647	0.946	0.109	22	15	37	0.595	0.405	0.140	0.280
	P	7	27	34	0.206	0.794	1.000	0.000 **	4	37	41	0.098	0.902	1.000	0.000 **
9	Q	18	20	38	0.474	0.526	0.647	0.706	17	20	37	0.459	0.541	0.704	0.593
	R	12	31	43	0.279	0.721	0.998	0.004 **	31	6	37	0.838	0.162	0.000	0.000 **
10	S	11	27	38	0.289	0.711	0.993	0.013 **	11	27	38	0.289	0.711	0.994	0.012 **
	T	17	19	36	0.472	0.528	0.668	0.664	21	14	35	0.600	0.400	0.141	0.282
11	U	3	34	37	0.081	0.919	1.000	0.000 **	8	35	43	0.186	0.814	1.000	0.000 **
	V	9	21	30	0.300	0.700	0.991	0.018 **	9	30	39	0.231	0.769	1.000	0.001 **
12	W	1	39	40	0.025	0.975	1.000	0.000 **	7	35	42	0.167	0.833	1.000	0.000 **
	X	11	30	41	0.268	0.732	0.999	0.002 **	2	35	37	0.054	0.946	1.000	0.000 **
13	Y	3	36	39	0.077	0.923	1.000	0.000 **	0	38	38	0.000	1.000	1.000	0.000 **
	Z	25	16	41	0.610	0.390	0.090	0.179	24	12	36	0.667	0.333	0.024	0.048 **
14	AA	20	18	38	0.526	0.474	0.347	0.695	21	19	40	0.525	0.475	0.398	0.795
	BB	10	22	32	0.313	0.688	0.989	0.023 **	4	38	42	0.095	0.905	1.000	0.000 **
15	CC	1	40	41	0.024	0.976	1.000	0.000 **	7	35	42	0.167	0.833	1.000	0.000 **
	DD	9	30	39	0.231	0.769	1.000	0.000 **	14	26	40	0.350	0.650	0.960	0.080 *
16	EE	26	13	39	0.667	0.333	0.025	0.051 *	16	22	38	0.421	0.579	0.833	0.333
	FF	13	26	39	0.333	0.667	0.986	0.029 **	4	33	37	0.108	0.892	1.000	0.000 **
17	GG	14	26	40	0.350	0.650	0.963	0.074 *	2	37	39	0.051	0.949	1.000	0.000 **
	HH	17	23	40	0.425	0.575	0.855	0.289	0	39	39	0.000	1.000	1.000	0.000 **
Totals		439	833	1272	0.345	0.655	1.000	0.000 **	448	859	1307	0.343	0.657	1.000	0.000 **

** Indicates rejection at the 5% level.

* Indicates rejection at the 10% level.

Table 5: Students - Small Blind

First versus Second Half Mixtures (with a King)

		Hands 1-100								Hands 101-200							
					Mixture		Rand						Mixture		Rand		
Pair	Player	F	B	Tot.	F	B	t	p-value		F	B	Tot.	F	B	t	p-value	
1	A	34	4	38	0.895	0.105	0.000	0.000	**	36	3	39	0.923	0.077	0.000	0.000	**
	B	13	29	42	0.310	0.690	0.992	0.015	**	10	19	29	0.345	0.655	0.938	0.124	
2	C	6	31	37	0.162	0.838	1.000	0.000	**	3	33	36	0.083	0.917	1.000	0.000	**
	D	19	18	37	0.514	0.486	0.411	0.822		27	8	35	0.771	0.229	0.001	0.001	**
3	E	12	26	38	0.316	0.684	0.988	0.024	**	20	13	33	0.606	0.394	0.082	0.164	
	F	5	38	43	0.116	0.884	1.000	0.000	**	12	29	41	0.293	0.707	0.996	0.008	**
4	G	5	34	39	0.128	0.872	1.000	0.000	**	4	36	40	0.100	0.900	1.000	0.000	**
	H	4	35	39	0.103	0.897	1.000	0.000	**	5	28	33	0.152	0.848	1.000	0.000	**
5	I	2	39	41	0.049	0.951	1.000	0.000	**	5	38	43	0.116	0.884	1.000	0.000	**
	J	14	23	37	0.378	0.622	0.911	0.178		25	15	40	0.625	0.375	0.052	0.104	
6	K	15	24	39	0.385	0.615	0.929	0.142		10	33	43	0.233	0.767	1.000	0.000	**
	L	31	5	36	0.861	0.139	0.000	0.000	**	31	4	35	0.886	0.114	0.000	0.000	**
7	M	10	25	35	0.286	0.714	0.995	0.011	**	10	24	34	0.294	0.706	0.994	0.012	**
	N	8	30	38	0.211	0.789	1.000	0.000	**	10	28	38	0.263	0.737	0.999	0.003	**
8	O	0	37	37	0.000	1.000	1.000	0.000	**	0	42	42	0.000	1.000	1.000	0.000	**
	P	4	37	41	0.098	0.902	1.000	0.000	**	9	30	39	0.231	0.769	1.000	0.001	**
9	Q	22	20	42	0.524	0.476	0.373	0.746		26	10	36	0.722	0.278	0.003	0.005	**
	R	1	38	39	0.026	0.974	1.000	0.000	**	2	37	39	0.051	0.949	1.000	0.000	**
10	S	14	25	39	0.359	0.641	0.968	0.064	*	14	20	34	0.412	0.588	0.844	0.311	
	T	7	33	40	0.175	0.825	1.000	0.000	**	1	36	37	0.027	0.973	1.000	0.000	**
11	U	15	24	39	0.385	0.615	0.910	0.180		23	15	38	0.605	0.395	0.084	0.168	
	V	17	20	37	0.459	0.541	0.687	0.626		20	20	40	0.500	0.500	0.455	0.910	
12	W	7	31	38	0.184	0.816	1.000	0.000	**	11	28	39	0.282	0.718	0.998	0.003	**
	X	7	28	35	0.200	0.800	1.000	0.000	**	9	25	34	0.265	0.735	0.998	0.004	**
13	Y	24	15	39	0.615	0.385	0.069	0.139		28	14	42	0.667	0.333	0.019	0.038	**
	Z	9	28	37	0.243	0.757	0.999	0.002	**	22	8	30	0.733	0.267	0.003	0.006	**
14	AA	9	29	38	0.237	0.763	1.000	0.001	**	4	37	41	0.098	0.902	1.000	0.000	**
	BB	11	29	40	0.275	0.725	0.998	0.004	**	8	30	38	0.211	0.789	1.000	0.000	**
15	CC	0	37	37	0.000	1.000	1.000	0.000	**	11	34	45	0.244	0.756	1.000	0.000	**
	DD	11	27	38	0.289	0.711	0.995	0.010	**	17	19	36	0.472	0.528	0.583	0.833	
16	EE	8	26	34	0.235	0.765	0.999	0.002	**	19	24	43	0.442	0.558	0.736	0.528	
	FF	4	34	38	0.105	0.895	1.000	0.000	**	25	7	32	0.781	0.219	0.000	0.001	**
17	GG	16	21	37	0.432	0.568	0.781	0.439		12	23	35	0.343	0.657	0.965	0.071	*
	HH	10	32	42	0.238	0.762	1.000	0.001	**	11	28	39	0.282	0.718	0.996	0.009	**
18	II	2	36	38	0.053	0.947	1.000	0.000	**	0	35	35	0.000	1.000	1.000	0.000	**
	JJ	15	26	41	0.366	0.634	0.968	0.064	*	8	32	40	0.200	0.800	1.000	0.000	**
19	KK	16	23	39	0.410	0.590	0.877	0.246		19	17	36	0.528	0.472	0.380	0.759	
	LL	16	20	36	0.444	0.556	0.693	0.613		24	13	37	0.649	0.351	0.031	0.062	*
20	MM	0	34	34	0.000	1.000	1.000	0.000	**	0	35	35	0.000	1.000	1.000	0.000	**
	NN	0	42	42	0.000	1.000	1.000	0.000	**	0	36	36	0.000	1.000	1.000	0.000	**
21	OO	10	33	43	0.233	0.767	1.000	0.000	**	2	40	42	0.048	0.952	1.000	0.000	**
	PP	9	21	30	0.300	0.700	0.979	0.042	**	14	25	39	0.359	0.641	0.960	0.080	*
Totals		442	1167	1609	0.275	0.725	1.000	0.000	**	547	1031	1578	0.347	0.653	1.000	0.000	**

** Indicates rejection at the 5% level.

* Indicates rejection at the 10% level.

Table 6: Poker Players - Big Blind
First versus Second Half Mixtures (with a King)

		Hands 1-100							Hands 101-200						
					Mixture		Rand					Mixture		Rand	
Pair	Player	F	C	Tot.	F	C	t	p-value	F	C	Tot.	F	C	t	p-value
1	A	4	19	23	0.174	0.826	0.804	0.393	10	11	21	0.476	0.524	0.018	0.035 **
	B	6	15	21	0.286	0.714	0.325	0.650	5	23	28	0.179	0.821	0.810	0.380
2	C	11	20	31	0.355	0.645	0.077	0.153	7	15	22	0.318	0.682	0.241	0.482
	D	7	18	25	0.280	0.720	0.394	0.788	8	16	24	0.333	0.667	0.228	0.456
3	E	21	13	34	0.618	0.382	0.000	0.000 **	10	16	26	0.385	0.615	0.085	0.170
	F	9	23	32	0.281	0.719	0.402	0.804	10	21	31	0.323	0.677	0.181	0.362
4	G	7	29	36	0.194	0.806	0.774	0.451	2	24	26	0.077	0.923	0.977	0.047 **
	H	6	30	36	0.167	0.833	0.891	0.217	3	27	30	0.100	0.900	0.978	0.044 **
5	I	6	18	24	0.250	0.750	0.442	0.884	6	24	30	0.200	0.800	0.764	0.472
	J	8	16	24	0.333	0.667	0.145	0.289	7	12	19	0.368	0.632	0.146	0.292
6	K	9	21	30	0.300	0.700	0.256	0.512	10	19	29	0.345	0.655	0.100	0.200
	L	3	31	34	0.088	0.912	0.991	0.018 **	3	34	37	0.081	0.919	0.993	0.014 **
7	M	1	16	17	0.059	0.941	0.980	0.041 **	4	20	24	0.167	0.833	0.822	0.357
	N	13	20	33	0.394	0.606	0.026	0.051 *	1	25	26	0.038	0.962	0.997	0.006 **
8	O	11	28	39	0.282	0.718	0.275	0.549	3	28	31	0.097	0.903	0.990	0.019 **
	P	13	18	31	0.419	0.581	0.015	0.031 **	11	12	23	0.478	0.522	0.008	0.017 **
9	Q	4	29	33	0.121	0.879	0.946	0.108	4	15	19	0.211	0.789	0.537	0.926
	R	8	20	28	0.286	0.714	0.346	0.693	15	9	24	0.625	0.375	0.000	0.000 **
10	S	3	20	23	0.130	0.870	0.906	0.187	4	22	26	0.154	0.846	0.834	0.332
	T	0	30	30	0.000	1.000	1.000	0.000 **	2	29	31	0.065	0.935	0.994	0.013 **
11	U	11	23	34	0.324	0.676	0.160	0.320	7	23	30	0.233	0.767	0.649	0.703
	V	23	18	41	0.561	0.439	0.000	0.000 **	13	16	29	0.448	0.552	0.007	0.014 **
12	W	0	25	25	0.000	1.000	1.000	0.001 **	9	26	35	0.257	0.743	0.387	0.773
	X	10	26	36	0.278	0.722	0.343	0.685	12	27	39	0.308	0.692	0.208	0.416
13	Y	2	15	17	0.118	0.882	0.873	0.254	8	15	23	0.348	0.652	0.125	0.250
	Z	0	34	34	0.000	1.000	1.000	0.000 **	0	40	40	0.000	1.000	1.000	0.000 **
14	AA	15	20	35	0.429	0.571	0.011	0.022 **	12	26	38	0.316	0.684	0.186	0.371
	BB	5	18	23	0.217	0.783	0.534	0.932	2	17	19	0.105	0.895	0.912	0.176
15	CC	1	31	32	0.031	0.969	0.999	0.001 **	1	24	25	0.040	0.960	0.996	0.008 **
	DD	4	35	39	0.103	0.897	0.992	0.016 **	4	23	27	0.148	0.852	0.883	0.233
16	EE	18	7	25	0.720	0.280	0.000	0.000 **	19	18	37	0.514	0.486	0.000	0.001 **
	FF	12	9	21	0.571	0.429	0.001	0.002 **	16	11	27	0.593	0.407	0.000	0.000 **
17	GG	9	21	30	0.300	0.700	0.284	0.568	2	29	31	0.065	0.935	0.992	0.017 **
	HH	8	20	28	0.286	0.714	0.380	0.760	7	25	32	0.219	0.781	0.573	0.855
Totals		268	736	1004	0.267	0.733	0.104	0.208	237	722	959	0.247	0.753	0.570	0.861

** Indicates rejection at the 5% level.

* Indicates rejection at the 10% level.

Table 7: Students - Big Blind
First versus Second Half Mixtures (with a King)

		Hands 1-100							Hands 101-200						
					1st Mixture		Rand					2nd Mixture		Rand	
Pair	Player	F	C	Tot.	F	C	t	p-value	F	C	Tot.	F	C	t	p-value
1	A	12	9	21	0.571	0.429	0.001	0.002 **	26	8	34	0.765	0.235	0.000	0.000 **
	B	8	5	13	0.615	0.385	0.002	0.004 **	6	5	11	0.545	0.455	0.030	0.061 *
2	C	10	13	23	0.435	0.565	0.039	0.077 *	4	18	22	0.182	0.818	0.834	0.331
	D	2	32	34	0.059	0.941	0.998	0.004 **	3	33	36	0.083	0.917	0.991	0.019 **
3	E	1	30	31	0.032	0.968	0.999	0.003 **	0	30	30	0.000	1.000	1.000	0.000 **
	F	5	22	27	0.185	0.815	0.703	0.594	3	24	27	0.111	0.889	0.964	0.072 *
4	G	0	35	35	0.000	1.000	1.000	0.000 **	3	34	37	0.081	0.919	0.995	0.010 **
	H	8	27	35	0.229	0.771	0.527	0.946	13	24	37	0.351	0.649	0.062	0.124
5	I	4	24	28	0.143	0.857	0.882	0.235	3	17	20	0.150	0.850	0.904	0.193
	J	6	31	37	0.162	0.838	0.903	0.195	4	25	29	0.138	0.862	0.954	0.092 *
6	K	3	14	17	0.176	0.824	0.778	0.445	5	12	17	0.294	0.706	0.271	0.542
	L	9	13	22	0.409	0.591	0.072	0.144	17	11	28	0.607	0.393	0.000	0.000 **
7	M	8	27	35	0.229	0.771	0.543	0.914	9	20	29	0.310	0.690	0.175	0.349
	N	11	18	29	0.379	0.621	0.064	0.129	12	21	33	0.364	0.636	0.064	0.129
8	O	0	36	36	0.000	1.000	1.000	0.000 **	0	31	31	0.000	1.000	1.000	0.000 **
	P	0	36	36	0.000	1.000	1.000	0.000 **	0	36	36	0.000	1.000	1.000	0.000 **
9	Q	5	28	33	0.152	0.848	0.928	0.145	14	21	35	0.400	0.600	0.033	0.065 *
	R	3	18	21	0.143	0.857	0.887	0.226	7	16	23	0.304	0.696	0.321	0.643
10	S	4	31	35	0.114	0.886	0.983	0.035 **	4	31	35	0.114	0.886	0.960	0.081 *
	T	8	17	25	0.320	0.680	0.214	0.429	3	31	34	0.088	0.912	0.987	0.025 **
11	U	12	15	27	0.444	0.556	0.014	0.028 **	12	11	23	0.522	0.478	0.002	0.004 **
	V	4	23	27	0.148	0.852	0.894	0.211	2	22	24	0.083	0.917	0.991	0.019 **
12	W	7	28	35	0.200	0.800	0.704	0.592	6	25	31	0.194	0.806	0.788	0.425
	X	16	16	32	0.500	0.500	0.001	0.003 **	10	17	27	0.370	0.630	0.055	0.110
13	Y	2	28	30	0.067	0.933	0.991	0.017 **	0	24	24	0.000	1.000	0.999	0.001 **
	Z	0	19	19	0.000	1.000	0.999	0.001 **	11	8	19	0.579	0.421	0.002	0.003 **
14	AA	5	25	30	0.167	0.833	0.849	0.303	0	28	28	0.000	1.000	1.000	0.000 **
	BB	6	22	28	0.214	0.786	0.710	0.580	6	28	34	0.176	0.824	0.840	0.321
15	CC	2	31	33	0.061	0.939	0.999	0.003 **	1	28	29	0.034	0.966	0.998	0.004 **
	DD	6	29	35	0.171	0.829	0.865	0.270	1	25	26	0.038	0.962	0.995	0.010 **
16	EE	2	25	27	0.074	0.926	0.990	0.020 **	5	17	22	0.227	0.773	0.543	0.914
	FF	5	31	36	0.139	0.861	0.953	0.093 *	1	22	23	0.043	0.957	0.991	0.018 **
17	GG	13	17	30	0.433	0.567	0.014	0.027 **	10	19	29	0.345	0.655	0.118	0.236
	HH	14	13	27	0.519	0.481	0.001	0.002 **	12	17	29	0.414	0.586	0.027	0.055 *
18	II	4	24	28	0.143	0.857	0.897	0.206	0	28	28	0.000	1.000	1.000	0.000 **
	JJ	4	34	38	0.105	0.895	0.988	0.023 **	5	30	35	0.143	0.857	0.950	0.101
19	KK	11	15	26	0.423	0.577	0.030	0.061 *	15	7	22	0.682	0.318	0.000	0.000 **
	LL	9	17	26	0.346	0.654	0.166	0.332	16	13	29	0.552	0.448	0.000	0.001 **
20	MM	0	35	35	0.000	1.000	1.000	0.000 **	0	39	39	0.000	1.000	1.000	0.000 **
	NN	0	36	36	0.000	1.000	1.000	0.000 **	0	32	32	0.000	1.000	1.000	0.000 **
21	OO	11	20	31	0.355	0.645	0.122	0.244	15	12	27	0.556	0.444	0.000	0.001 **
	PP	9	21	30	0.300	0.700	0.252	0.504	11	22	33	0.333	0.667	0.123	0.245
Totals		249	990	1239	0.201	0.799	1.000	0.000 **	275	922	1197	0.230	0.770	0.963	0.075 *

** Indicates rejection at the 5% level.

* Indicates rejection at the 10% level.

Table 9a: Poker Players - Runs (with a King)

		Small Blind							Big Blind						
Pair	Player	F	B	Tot.	Runs	F(r-1)	F(r.)	U[F(r-1),F(r.)]	F	C	Tot.	Runs	F(r-1)	F(r.)	U[F(r-1),F(r
1	A	40	41	81	48	0.910	0.942	0.926	14	30	44	20	0.419	0.543	0.432
	B	44	32	76	44	0.902	0.937	0.922	11	38	49	18	0.413	0.532	0.432
2	C	37	42	79	47	0.920	0.948	0.947	18	35	53	24	0.348	0.457	0.407
	D	26	46	72	40	0.916	0.947	0.925	15	34	49	29	0.991 **	0.998	0.996
3	E	21	46	67	25	0.063	0.111	0.109	31	29	60	33	0.656	0.745	0.727
	F	26	52	78	30	0.059	0.092	0.076	19	44	63	29	0.599	0.727	0.612
4	G	16	57	73	26	0.439	0.535	0.502	9	53	62	14	0.087	0.143	0.130
	H	27	41	68	37	0.772	0.844	0.825	9	57	66	16	0.305	0.405	0.401
5	I	46	28	74	32	0.143	0.203	0.163	12	42	54	23	0.854	0.959	0.935
	J	32	44	76	42	0.793	0.854	0.810	15	28	43	20	0.363	0.487	0.433
6	K	7	69	76	13	0.151	0.456	0.405	19	40	59	30	0.799	0.865	0.847
	L	25	46	71	38	0.862	0.909	0.896	6	65	71	13	0.477	1.000	0.880
7	M	25	56	81	33	0.206	0.296	0.251	5	36	41	9	0.139	0.427	0.324
	N	52	19	71	36	0.987 **	0.993	0.990	14	45	59	16	0.009	0.019 **	0.013
8	O	34	37	71	39	0.690	0.768	0.751	14	56	70	24	0.522	0.618	0.535
	P	11	64	75	23	0.874	1.000	0.898	24	30	54	23	0.075	0.104	0.101
9	Q	35	40	75	45	0.926	0.953	0.935	8	44	52	14	0.300	0.414	0.356
	R	43	37	80	17	0.000	0.000 ***	0.000	23	29	52	23	0.119	0.185	0.175
10	S	22	54	76	34	0.639	0.722	0.718	7	42	49	13	0.311	0.634	0.540
	T	38	33	71	34	0.248	0.331	0.255	2	59	61	5	0.097	1.000	0.254
11	U	11	69	80	17	0.051	0.140	0.054	18	46	64	15	0.000	0.000 ***	0.000
	V	18	51	69	24	0.104	0.158	0.112	36	34	70	41	0.863	0.909	0.872
12	W	8	74	82	11	0.003	0.019 **	0.010	9	51	60	14	0.095	0.155	0.147
	X	13	65	78	20	0.112	0.167	0.135	22	53	75	35	0.739	0.837	0.739
13	Y	3	74	77	6	0.078	0.150	0.127	10	30	40	15	0.244	0.419	0.415
	Z	49	28	77	43	0.927	0.958	0.958	0	74	74	1	na	na	na
14	AA	41	37	78	27	0.001	0.002 ***	0.001	27	46	73	21	0.000	0.000 ***	0.000
	BB	14	60	74	19	0.026	0.065	0.030	7	35	42	15	0.801	1.000	0.923
15	CC	8	75	83	13	0.035	0.137	0.106	2	55	57	5	0.103	1.000	0.654
	DD	23	56	79	34	0.490	0.582	0.534	8	58	66	17	0.712	1.000	0.821
16	EE	42	35	77	37	0.268	0.348	0.325	37	25	62	25	0.046	0.078	0.046
	FF	17	59	76	29	0.612	0.770	0.733	28	20	48	31	0.969 *	0.986	0.975
17	GG	16	63	79	22	0.048	0.078	0.065	11	50	61	20	0.591	0.690	0.649
	HH	17	62	79	22	0.024	0.043 *	0.037	15	45	60	27	0.838	0.933	0.932

*** Indicates rejection at the 1% level.
** Indicates rejection at the 5% level.
* Indicates rejection at the 10% level.

Table 9b: Students - Runs (with a King)

		Small Blind							Big Blind						
Pair	Player	F	B	Tot.	Runs	F(r-1)	F(r.)	U[F(r-1),F(r.)]	F	C	Tot.	Runs	F(r-1)	F(r.)	U[F(r-1),F(r
1	A	70	7	77	11	0.020	0.092	0.087	38	17	55	26	0.629	0.727	0.724
	B	23	48	71	33	0.533	0.651	0.552	14	10	24	14	0.637	0.784	0.711
2	C	9	64	73	17	0.352	0.676	0.399	14	31	45	13	0.003	0.009 **	0.009
	D	46	26	72	27	0.024	0.043 *	0.042	5	65	70	11	0.370	1.000	0.998
3	E	32	39	71	35	0.345	0.437	0.387	1	60	61	3	0.033	1.000	0.194
	F	17	67	84	27	0.266	0.423	0.286	8	46	54	14	0.283	0.392	0.348
4	G	9	70	79	19	0.725	1.000	0.836	3	69	72	6	0.083	0.160	0.132
	H	9	63	72	16	0.266	0.359	0.326	21	51	72	34	0.791	0.853	0.817
5	I	7	77	84	15	0.517	1.000	0.959	7	41	48	13	0.321	0.643	0.363
	J	39	38	77	27	0.001	0.003 ***	0.002	10	56	66	19	0.524	0.801	0.619
6	K	25	57	82	33	0.192	0.280	0.279	8	26	34	12	0.208	0.331	0.214
	L	62	9	71	19	0.767	1.000	0.827	26	24	50	22	0.100	0.162	0.129
7	M	20	49	69	30	0.513	0.610	0.564	17	47	64	31	0.926	0.974	0.938
	N	18	58	76	27	0.248	0.382	0.279	23	39	62	25	0.068	0.113	0.086
8	O	0	79	79	1	na	na	na	0	67	67	1	na	na	na
	P	13	67	80	25	0.705	0.896	0.895	0	72	72	1	na	na	na
9	Q	48	30	78	32	0.062	0.096	0.086	19	49	68	24	0.074	0.117	0.080
	R	3	75	78	7	0.148	1.000	0.639	10	34	44	17	0.468	0.685	0.685
10	S	28	45	73	43	0.960 *	0.978	0.960	8	62	70	15	0.266	0.596	0.286
	T	8	69	77	11	0.004	0.023 **	0.014	11	48	59	20	0.614	0.711	0.654
11	U	38	39	77	44	0.821	0.875	0.860	24	26	50	25	0.339	0.447	0.370
	V	37	40	77	42	0.681	0.759	0.745	6	45	51	11	0.178	0.487	0.335
12	W	12	73	85	19	0.077	0.188	0.140	26	32	58	30	0.479	0.586	0.553
	X	23	46	69	31	0.368	0.483	0.391	20	43	63	35	0.967 *	0.987	0.967
13	Y	0	69	69	1	na	na	na	0	74	74	1	na	na	na
	Z	0	78	78	1	na	na	na	0	68	68	1	na	na	na
14	AA	35	40	75	40	0.607	0.694	0.655	26	22	48	30	0.915	0.953	0.917
	BB	40	33	73	42	0.849	0.898	0.887	25	30	55	35	0.957 *	0.977	0.974
15	CC	2	71	73	3	0.001	0.028 *	0.028	4	52	56	4	0.000	0.001 ***	0.000
	DD	23	58	81	29	0.067	0.116	0.092	9	64	73	19	0.756	1.000	0.853
16	EE	28	44	72	32	0.177	0.247	0.234	23	36	59	28	0.333	0.435	0.351
	FF	21	60	81	33	0.523	0.662	0.549	26	30	56	27	0.262	0.356	0.320
17	GG	27	50	77	42	0.918	0.948	0.938	7	42	49	11	0.068	0.690	0.128
	HH	29	41	70	16	0.000	0.000 ***	0.000	6	53	59	10	0.078	0.138	0.107
18	II	11	71	82	22	0.789	0.845	0.829	3	59	62	7	0.184	1.000	0.224
	JJ	28	46	74	37	0.564	0.663	0.652	7	54	61	15	0.647	1.000	0.670
19	KK	13	66	79	21	0.161	0.318	0.273	5	53	58	9	0.073	0.315	0.251
	LL	19	59	78	22	0.007	0.014 **	0.010	12	50	62	19	0.202	0.370	0.232
20	MM	52	29	81	24	0.000	0.000 ***	0.000	2	52	54	5	0.109	1.000	0.129
	NN	31	36	67	26	0.014	0.026 *	0.018	11	27	38	10	0.003	0.008 **	0.008
21	OO	18	59	77	29	0.461	0.622	0.468	13	53	66	16	0.010	0.020 **	0.013
	PP	16	53	69	29	0.824	0.926	0.901	26	33	59	37	0.957 *	0.977	0.960

*** Indicates rejection at the 1% level.
** Indicates rejection at the 5% level.
* Indicates rejection at the 10% level.

Table 11: Hand History Statistics

				Hands 1-200	Poker Tracker		
Player	F	B	Tot.	Bet with King	HANDS	VOL $	STEAL
B	44	32	76	0.421	9123	8.89%	12.23%
C	37	42	79	0.532	3951	17.28%	14.34%
E	21	46	67	0.687	1657	29.23%	22.60%
G	16	57	73	0.781	81	26.84%	0.00%
H	27	41	68	0.603	10764	22.76%	27.43%
K	7	69	76	0.908	247	57.52%	30.15%
M	25	56	81	0.691	332	22.89%	8.07%
O	34	37	71	0.521	30	18.42%	12.50%
P	11	64	75	0.853	281	35.94%	44.16%
Q	35	40	75	0.533	726	37.32%	21.87%
U	11	69	80	0.863	32683	21.66%	24.10%
Z	49	28	77	0.364	24	22.92%	25.00%
AA	41	37	78	0.474	372	29.17%	4.17%
BB	14	60	74	0.811	1054	50.21%	11.14%
CC	8	75	83	0.904	427	42.60%	4.61%
DD	23	56	79	0.709	16	15.63%	0.00%
				Minimum	16	8.89%	0.00%
				Maximum	32683	57.52%	44.16%
				Average	3861	28.70%	16.40%

Figure 1: 1st Half p-values, Small Blind

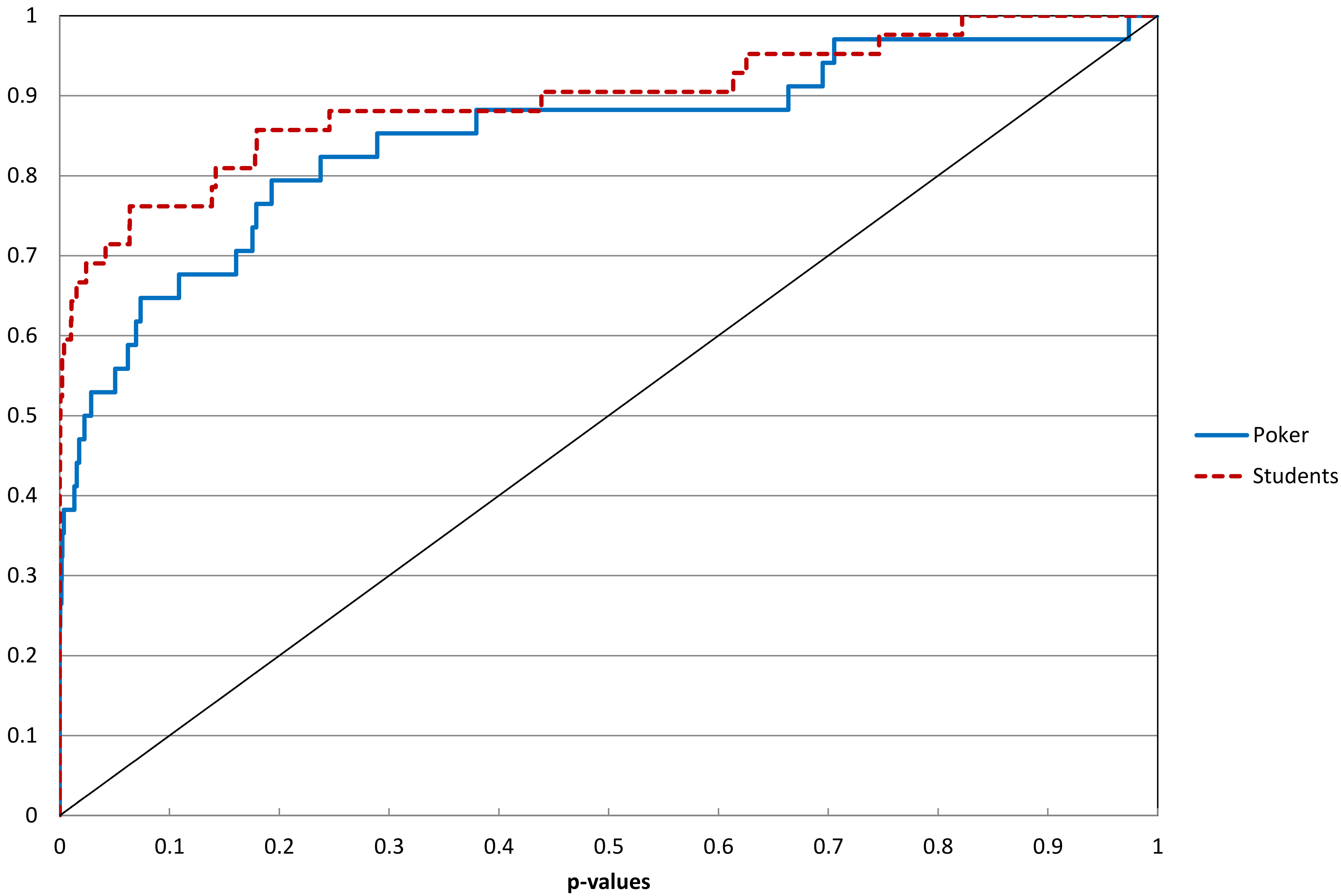

Figure 2: 1st Half p-values, Big Blind

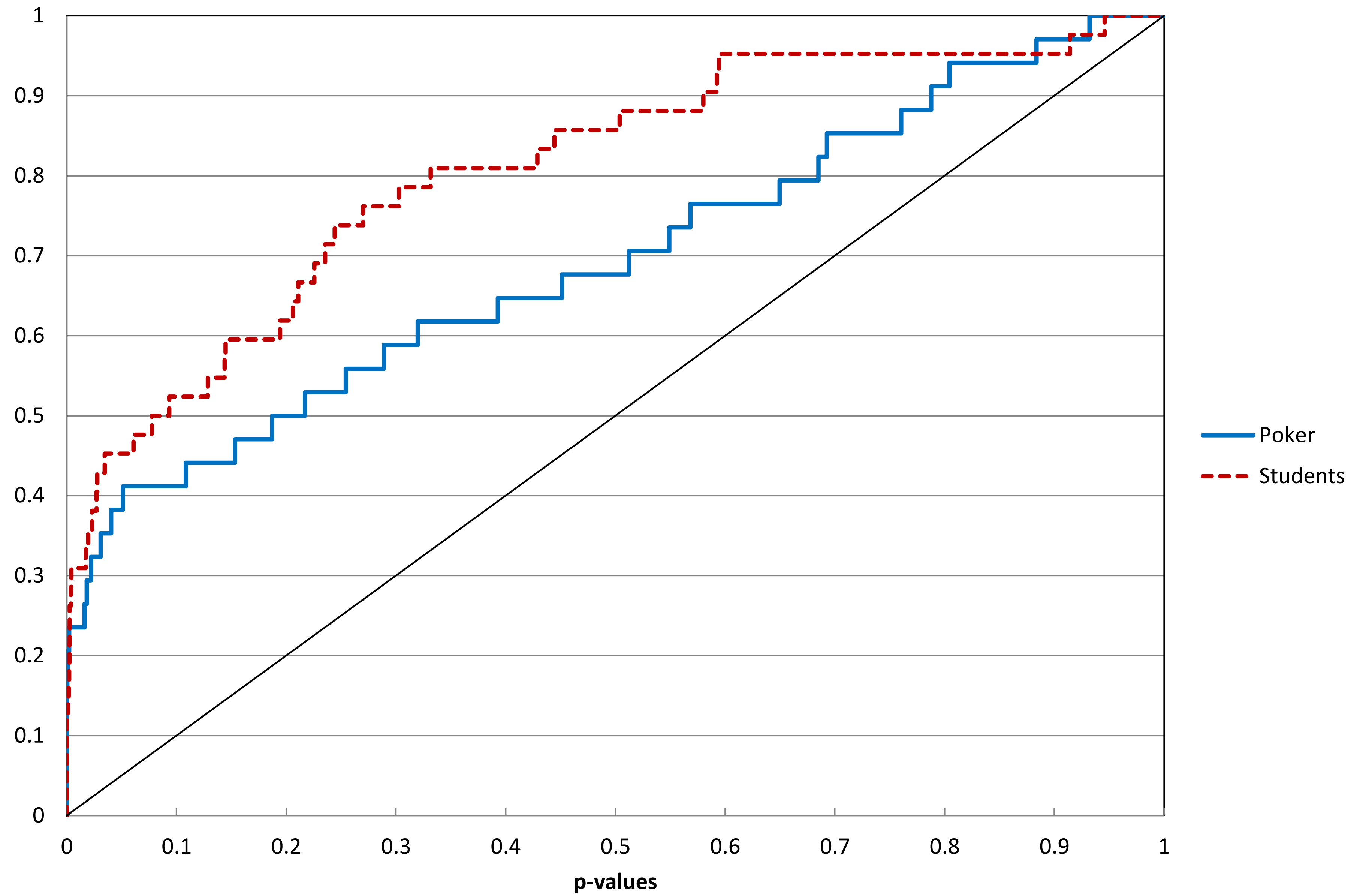

Figure 3: 2nd Half p-values, Small Blind

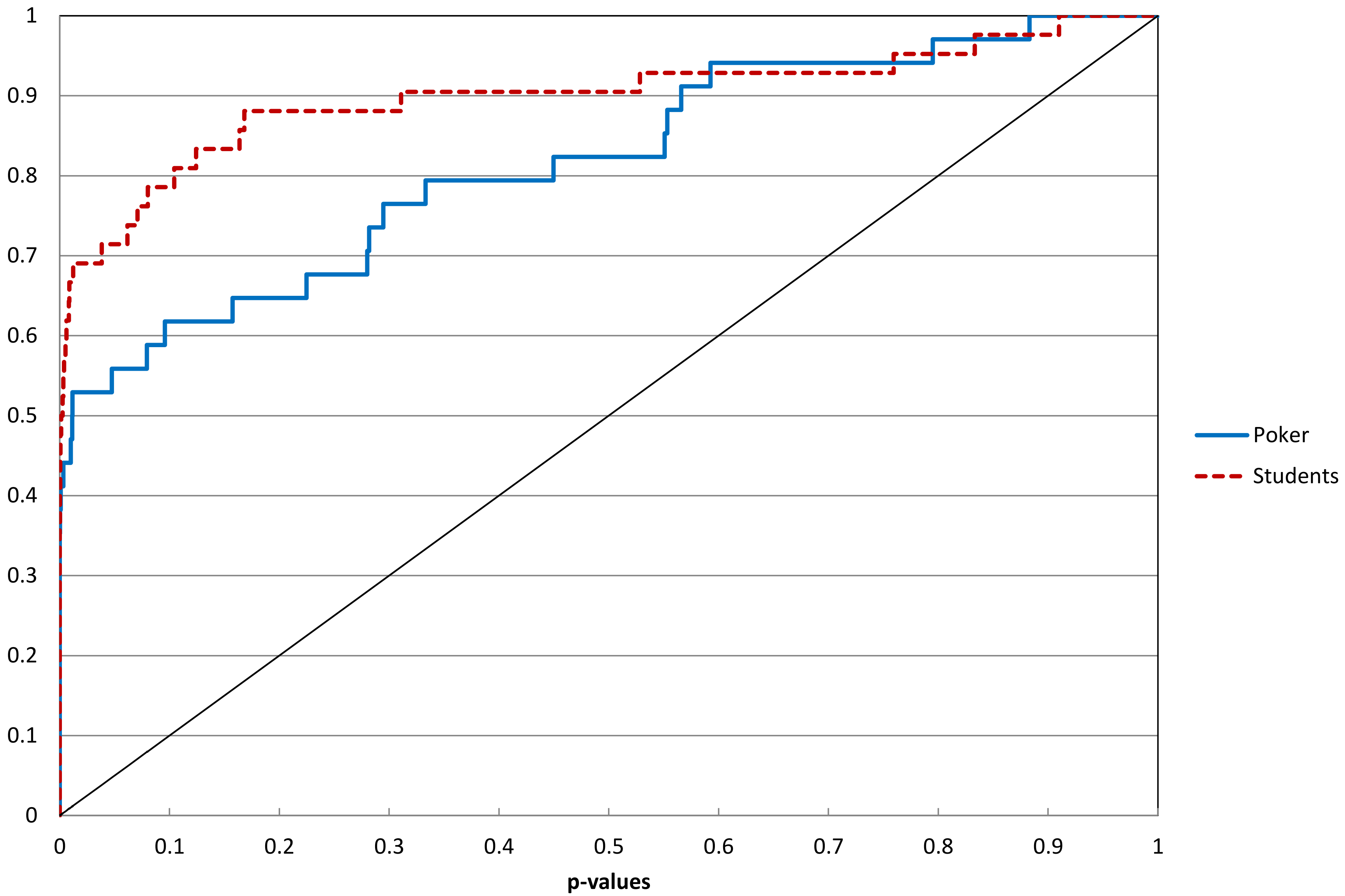

Figure 4: 2nd Half p-values, Big Blind

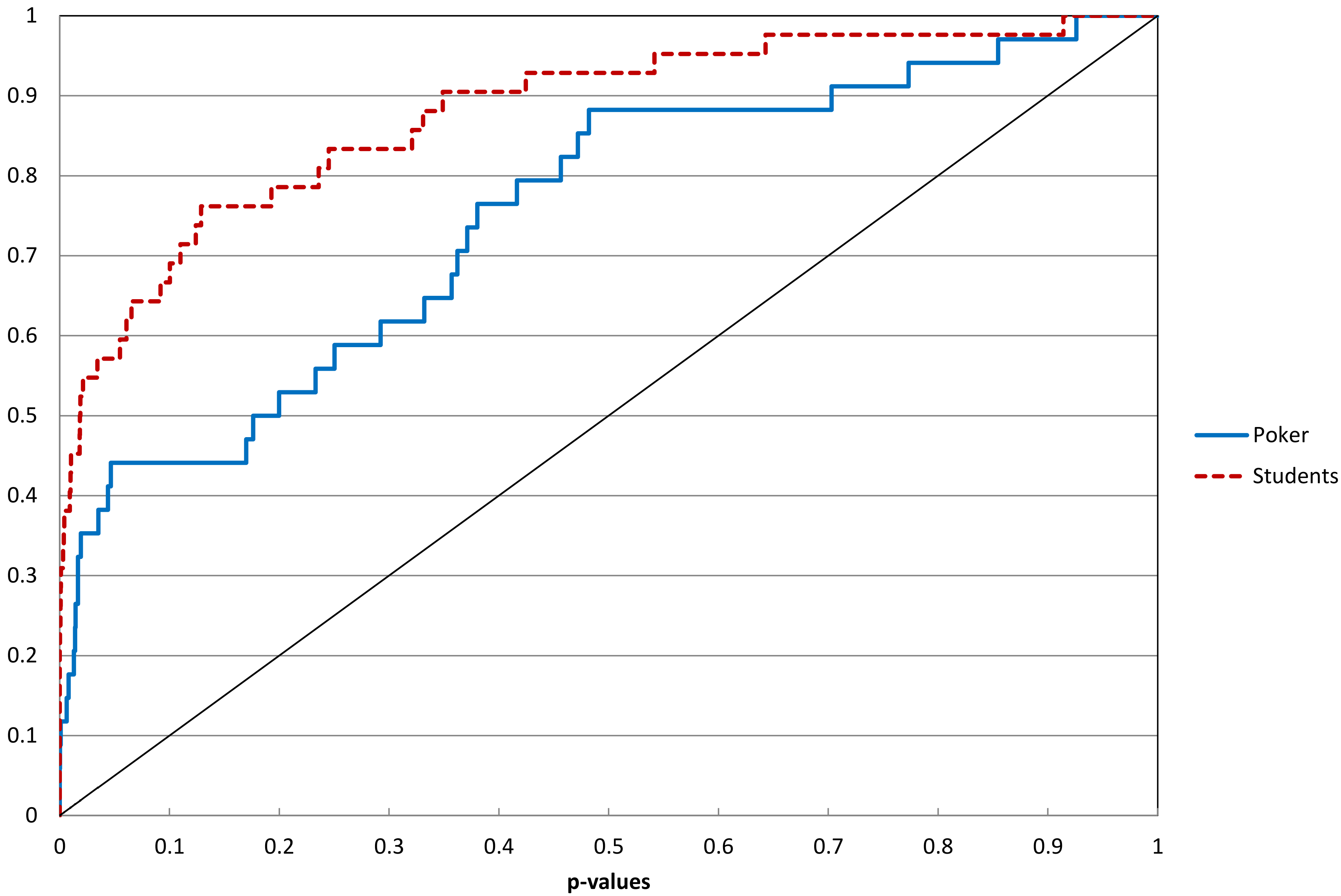

Figure 5: Empirical cdf of *t*-values for Runs Test
Small Blind

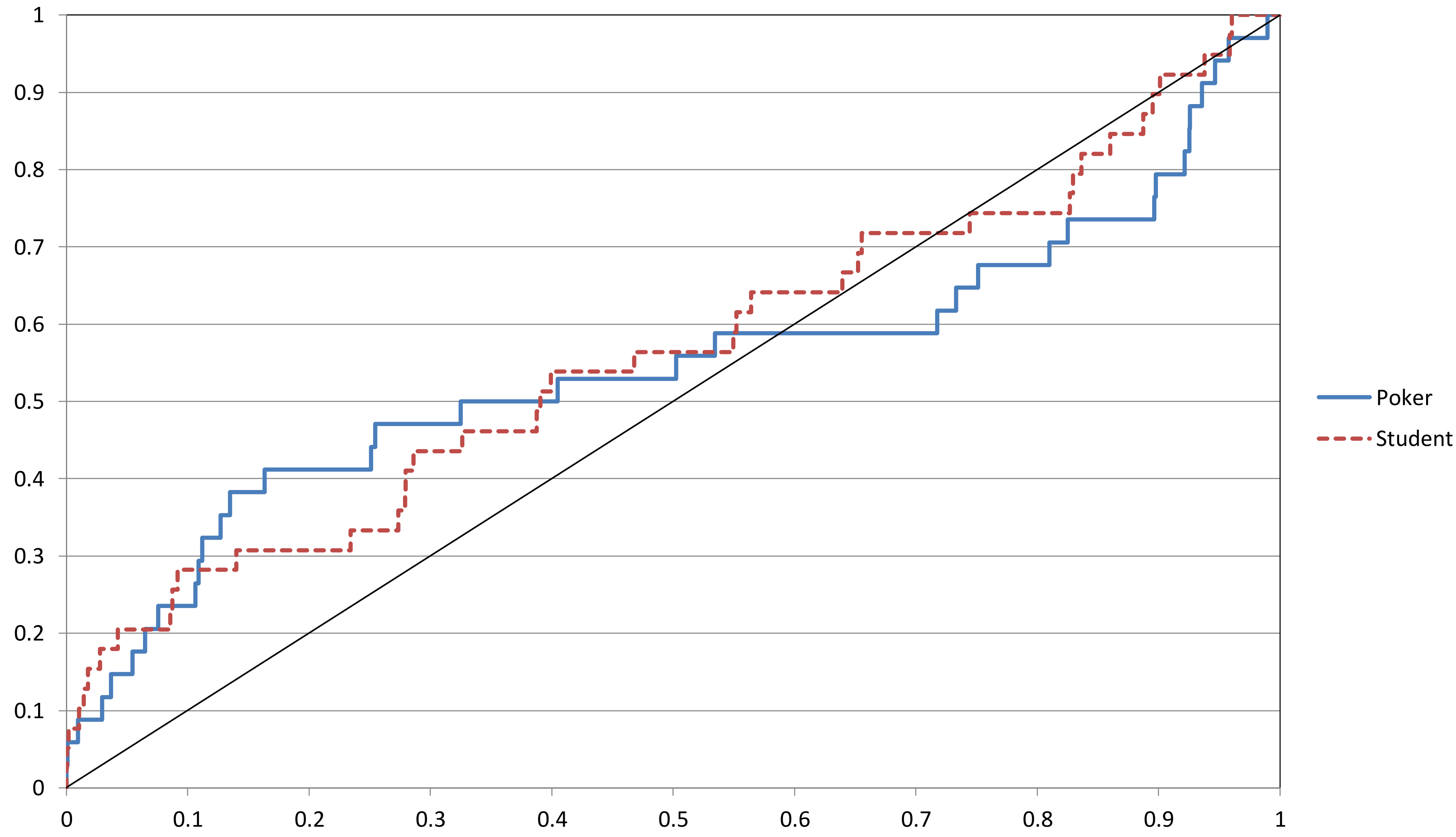

Figure 6: Empirical cdf of *t*-values for Runs Test
Big Blind

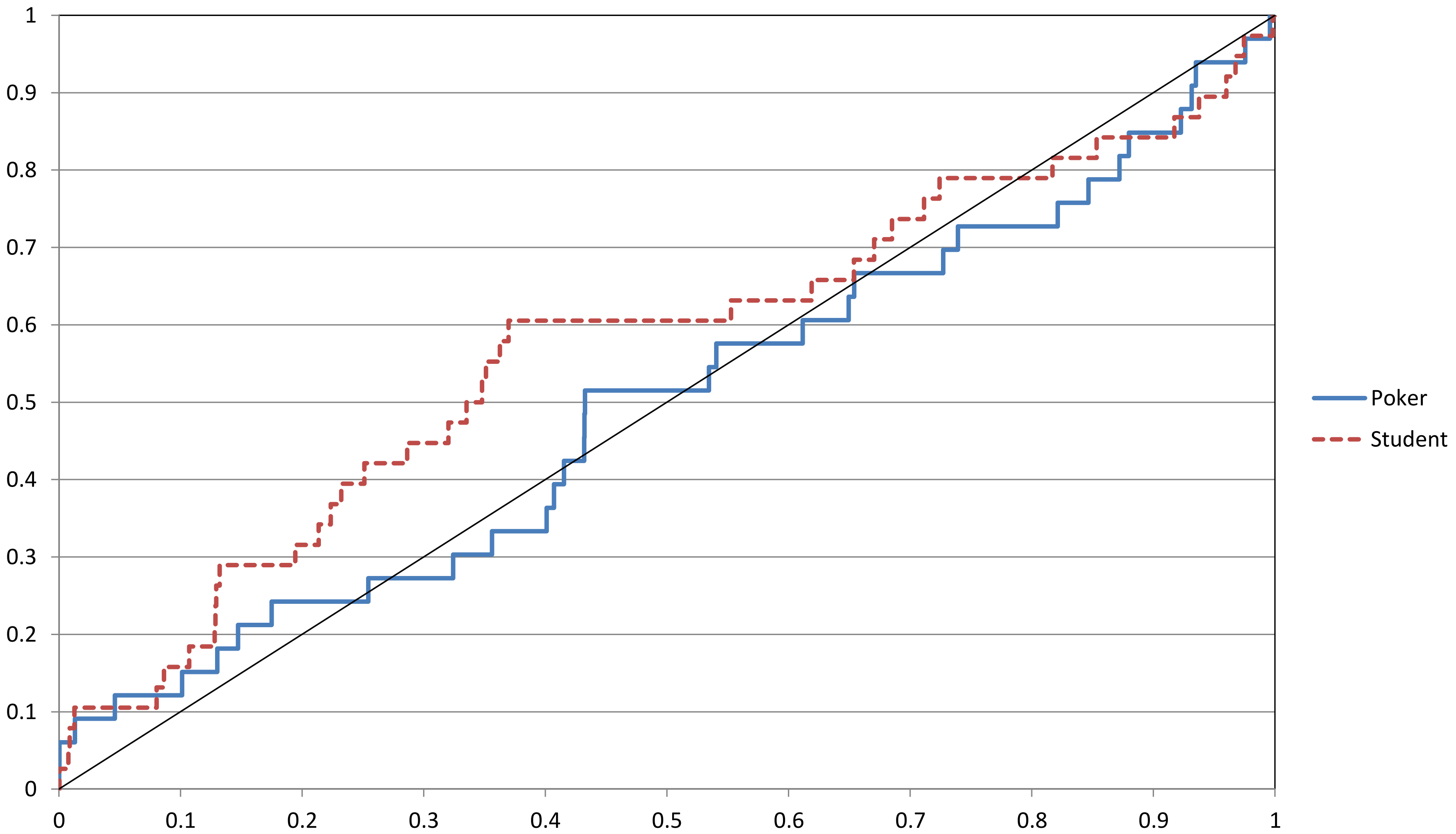

Figure 7: Agent Quantal Response Equilibria