Accounting Standards, Earnings Management, and Earnings Quality

Ralf Ewert

and

Alfred Wagenhofer

University of Graz

We thank Wolfgang Ballwieser (discussant), Anna Boisits, Judson Caskey (discussant), Frank Gigler, Mirko Heinle, Rick Lambert, Stefan Schantl, Marco Trombetta (discussant), and participants at the Ausschuss Unternehmensrechnung of the Verein für Socialpolitik, EAA Congress 2013, Accounting Research Workshop at the University of Basel, Tel Aviv International Conference in Accounting, and workshop participants at the University of Minnesota and at the University of Pennsylvania for helpful comments.

Ralf Ewert
University of Graz, Universitaetsstrasse 15, A-8010 Graz, Austria
Tel.: +43 (316) 380 7168, Email: ralf.ewert@uni-graz.at

Alfred Wagenhofer
University of Graz, Universitaetsstrasse 15, A-8010 Graz, Austria
Tel.: +43 (316) 380 3500, Email: alfred.wagenhofer@uni-graz.at

November 2013

Accounting Standards, Earnings Management, and Earnings Quality

This paper examines how the characteristics of accounting systems and management incentives interact and collectively determine financial reporting quality. We develop a rational expectations equilibrium model that features a steady-state firm with investments, financial and non-financial information, and earnings management. Our measure of earnings quality is the equilibrium information content of reported earnings. We study effects on earnings quality by varying the accruals, the precision of base earnings and private non-financial information, the cost of earnings management, and operating risk. The analysis confirms some intuitive results, such as that earnings quality increases in the precision of earnings and non-financial information. However, we also find counter-intuitive results, such as that an accounting standard that unambiguously provides more information early can reduce earnings quality. Since the equilibrium market reaction is akin to value relevance measures, we study how these measures trace the changes in earnings quality. We show that the earnings response coefficient is more closely related to earnings quality than the correlation of a price-earnings regression.

Keywords: Earnings quality; earnings management; accounting standards; value relevance; earnings response coefficient.

JEL classification: D80; G12; G14; M41; M43.

1. Introduction

Earnings quality is an important characteristic of financial reporting systems. Most recent regulatory changes in accounting standards, auditing, and corporate governance have been motivated by attempts to increase transparency of financial reporting. Earnings quality is also used in many empirical studies to examine changes in earnings characteristics over time, to assess the effects of changes in accounting standards and the institutional environment, to compare financial reporting across countries, and to measure market price and return effects of firms exhibiting different earnings quality. Dechow, Ge, and Schrand (2010) provide a broad survey of this literature.

Despite this great interest, there is little theory examining the consequences of a change in accounting standards and the institutional setting on earnings quality or, more generally, financial reporting quality. One reason is that earnings quality is an elusive notion that is being used to capture a variety of attributes. Another is that many regulatory initiatives and empirical studies are based on intuitive reasoning such as, for example, that an earlier recognition of components of future cash flows or a reduction of the discretion for earnings management increase earnings quality. This paper provides a theoretical analysis of the effects of a variation of accounting system characteristics on earnings quality, where earnings quality is the information content carried in reported earnings for pricing the firm. We show that several, a priori intuitively plausible, relationships are not warranted and provide reasons why and when this is the case.

We develop a simple stochastic steady-state model of a firm that invests in two-period projects in every period. An accounting system recognizes a portion of the future expected cash flows early, but with noise. The model is rich enough to give rise to investment, depreciation, working capital accruals, and earnings. The manager privately observes the accounting signal and obtains additional non-financial information that is not recognized in the financial reporting system. We allow for earnings management through which the manager can issue an earnings report to the capital market that includes the accounting signal with bias. Since the manager conditions the bias on all available information, including the non-financial information, reported earnings include two pieces of information, information from the accounting system and the non-financial information. The total information content of reported earnings depends on the characteristics of the information and on the incentives of the manager. We examine price, earnings, and smoothing incentives and show that particularly smoothing leads to two distinct effects which affect earnings quality in opposite directions: On one hand, to smooth earnings the manager considers current as well as future earnings, so that the resulting bias incorporates the manager's private information and this generally improves earnings quality. On the other hand, the manager's ability to smooth is affected by the smoothing inherent in the accounting system, so that in equilibrium a more informative accounting system can actually reduce earnings quality.

We establish a unique linear rational expectations equilibrium and describe the equilibrium earnings management and market price reaction. Earnings management exhibits a period-specific bias and forward and backward smoothing. In equilibrium, investors are able to make inferences about this information and use it in pricing the firm. To do so, they use information in reported earnings as well as balance sheet and cash flow information. Earnings quality is defined in the model in terms of the information content of reported earnings in equilibrium.

This economic setting is useful to study the effects of variations in accounting standards and the institutional environment on earnings quality. The effects of such variations are not straightforward because the equilibrium implies a subtle interrelationship between the effect on the accounting system and the earnings management bias. The characteristics endogenously influence the weights with which the manager's information is integrated in reported earnings, which again affects the amount of information investors can infer from the earnings report about the manager's information.

The characteristics of the accounting system we consider are the degree to which it reports components of the projects' expected cash flows that realize before the cash flows obtain, and how precise this early recognition is. We find that a greater portion of anticipated future cash flows embedded in earnings does not always increase earnings quality. The reason is that it dampens the inclusion of non-financial information in the bias, which can dominate the increase in the information content of the accounting signal. Next, we establish that an increase in the precision of the accounting system and the private information always increases earnings quality. However, an increase in the cost of earnings management (e.g., less discretion, higher audit and enforcement quality) reduces earnings quality. This result obtains because a higher cost reduces earnings management in equilibrium, which does not improve earnings quality as investors correctly undo it on average, but is harmful as it lowers the weight with which the manager's non-financial information enters reported earnings.

We also provide insights into the effect of a change in the amount of operative (cash flow) risk that is captured by the accounting system or by the manager's non-financial information relative to the total risk. A higher operative risk of cash flow components for which information becomes available generally increases earnings quality; however, we identify circumstances in which the converse happens. Again, the reason is the interaction of the equilibrium weights of accounting and non-financial information in the earnings report.

Finally, we examine how value relevance measures that are used as empirical proxies for earnings quality react to changes in our earnings quality benchmark. Value relevance is closely related to our rational expectations capital market setting, as it measures the price changes based on reported earnings. We study the earnings response coefficient and the correlation of a price-earnings regression and show how they are related to earnings quality. The main finding is that the earnings response coefficient is closely related to earnings quality, whereas the correlation measure does not capture earnings quality well. Still, even the earnings response coefficient is not a perfect proxy, mainly because the market is able to rely on more information than earnings in forming the price. We also show that the behavior of the proxies differs whether they are used in a study that compares two equilibria with different accounting characteristics or that assesses the effects of a change in the characteristics in the same setting over time.

There are several caveats that we should note at the outset. For simplicity, we do not model a full agency model with optimal compensation, but exogenously assume management incentives to show that particularly earnings smoothing can have detrimental effects on earnings quality. The model does not capture productive decisions by the manager, apart from earnings management (which can be interpreted as real activities to shift earnings across periods) and shows that *even* in a pure exchange economy a negative value of more informative standards can occur. The technical requirements of the equilibrium does not allow us to model many aspects of accounting standards, such as asymmetric information introduced by conservatism. Presumably, extending the model structure would result in even more frictions than those we identify.

There is little theoretical literature that directly addresses earnings quality and its measures, although a notion of earnings quality is embedded in many analytical studies. Closely related is Fischer and Verrecchia (2000) who study the value relevance of an earnings report in equilibrium and find that informativeness increases in the cost of bias. Unlike the present paper they do not specifically model an accounting structure with accruals and consider a one-period setting. Sankar and Subrahmanyam (2001) study a two-period model with a risk averse manager with a time-additive utility function where smoothing results from the manager's desire to smooth consumption. They find that allowing for earnings management improves the information in the capital market because the bias allows the manager to incorporate private value relevant information early. Dye and Sridhar (2004) study relevance and reliability of accounting information and model the accountant as the gatekeeper to trade off these two characteristics in a capital market equilibrium. In their model, the accountant aggregates the two pieces of information, whereas in the present model the aggregation arises in the market under rational expectations.

Closest to the present paper is Ewert and Wagenhofer (2013). They study a two-period rational expectations equilibrium model and examine a variation of accounting, incentive, and risk parameters on earnings quality. Further, they study value relevance, persistence, predictability, smoothness, and discretionary accruals in this setting. Different from their setting, we study a steady state model and exploit the natural accounting structure with depreciation and working capital accruals. Some of the results of the present paper mirror those in Ewert and Wagenhofer (2013), but the main focus is on the interaction of accounting standards and incentives.[1] Marinovic (2013) examines earnings management and capital market reactions when there is uncertainty whether the manager can bias the earnings report. He establishes a mixed-strategy equilibrium, in which price volatility around earnings announcements and persistence are useful measures, whereas predictability and smoothness behave non-monotonically in the information content of reported earnings. Fischer and Stocken (2004) examine how varying a speculator's information affects earnings management incentives. They define earnings quality as squared difference between reported and fundamental earnings and find that the effect depends on whether the speculator's information complements or substitutes for earnings.

These models focus on the decision usefulness of accounting earnings in a capital market equilibrium. Other papers study aspects of earnings quality in agency models, e.g., Christensen, Feltham, and Şabac (2005) and Christensen, Frimor, and Şabac (2013), and Drymiotes and Hemmer (2013). These papers generally find that value relevance metrics often do not capture underlying economic effects, which is in spirit similar to our results, although the economic forces are very different.

The rest of the paper is organized as follows: In the next section we set up the model with the investment projects, the accounting system, earnings management incentives, and the structure and information content of the financial reports. Section 3 establishes the rational expectations equilibrium and describes the properties of the equilibrium earnings report and market pricing mechanism. The main analysis of earnings quality and its determining factors is included in section 4. Section 5 considers value relevance and how the two value relevance proxies trace earnings quality. Finally, section 6 concludes and discusses potential extensions.

---

[1] Christensen and Frimor (2007) study the interaction between accounting information and other information in a capital market setting. Similar in spirit to our paper, they find that more information may be undesirable because it affects the weights with which they are enter the pricing mechanism.

2. The Model

Firms are regularly operative over an open time-horizon; they generate cash flows, realize new projects and terminate existing projects in each period; they operate an accounting system that allocates past and anticipated cash flows over time by means of accruals and the capitalization of assets and liabilities; and the firm's management changes after some periods. Earnings quality can only be analyzed in the context of these factors, and we capture these essential aspects in a parsimonious way.

Investment projects

We model a simple ongoing firm, which obtains an investment opportunity in each period. All projects are structurally equal.[2] Each project requires a certain investment cost $I > 0$ at the beginning of a period and yields a risky operating cash flow $\tilde{x}$ at the end of the second period. This situation is descriptive of construction contracts or other contracts in which cash flows and the underlying operations occur at different times. The main results would continue to hold for projects with more cash flows at different times. The cash flow $\tilde{x}$ is normally distributed with a mean μ and the expected net present value of the project is strictly positive,

$$NPV = \mu\rho^2 - I > 0,$$

where $\rho \in (0, 1]$ denotes the discount factor. Hence, each investment project that arises will be implemented; for simplicity, we abstract from non-trivial investment decisions and ignore potential abandonment decisions. To eliminate effects of financing decisions we assume that the firm distributes excess cash flows and raises equity if cash flows fall short of investment requirements.

The cash flow $\tilde{x}$ consists of three operating risk factors,

$$\tilde{x} = \mu + \tilde{\varepsilon} + \tilde{\delta} + \tilde{\omega} \quad (1)$$

[2] It would be possible to assume different projects, e.g., projects with a growth rate that is less than the discount rate. However, the additional insights gained would be low.

which are normally and independently distributed with zero mean and variances $\sigma_{\varepsilon}^2, \sigma_{\delta}^2$ and σ_{ω}^2. For example, it describes a situation in which the risk factors realize at different times. The cash flow structure of the projects is common knowledge. The investment *I* and the cash flow $\tilde{x}$ are publicly observable, whereas the individual stochastic components of $\tilde{x}$ are not.

Accounting system

The firm operates an accounting system that provides signals about the future cash flows of each active project to the manager. We distinguish between information that is recognized in the financial statements and other, information that we label non-financial. We assume that the recognized accounting earnings $\tilde{y}_1$ are informative about one component of the uncertain future cash flow, $\tilde{\varepsilon}$, in the following way:

$$\tilde{y}_1 = k\tilde{\varepsilon} + \tilde{n}. \tag{2}$$

This formulation captures two important characteristics of an accounting system.[3] First, the parameter $k \in (0, 1]$ determines the portion of the future cash flow caused by the factor $\tilde{\varepsilon}$. For example, $k = 1$ implies that $\tilde{y}_1$ fully contains $\tilde{\varepsilon}$; $k = 0$ prohibits early recognition of any future cash flows and would lead to an uninformative accounting system, which is the reason that we exclude it.[4] Second, the accounting system imperfectly traces $\tilde{\varepsilon}$ in that the signal is noisy, where the noise $\tilde{n}$ is independent of the accrual $k\tilde{\varepsilon}$ and normally distributed with zero mean and variance σ_n^2. Lower σ_n^2 indicates that the accounting system is more precise.[5]

---

[3] In line with most accounting standards, we assume that the expected profit μ is recognized in the second period. Since μ carries no information to investors, this assumption is not restrictive. Moreover, in our steady state structure this assumption is without loss of generality even for reported earnings, as there are always two active projects in each period, so that the sum of the individual accounting signals contains μ of one project.

[4] It should be noted that varying the degree of inclusion of $\tilde{\varepsilon}$ also affects the overall precision of the manager's private information because it is "real" variance in contrast to the constant accounting noise.

[5] An alternative assumption is $\tilde{y}_1 = k(\tilde{\varepsilon} + \tilde{n})$, so higher *k* increases both the information about $\tilde{\varepsilon}$ and the noise in $\tilde{y}_1$. A standard setter would then trade off these two effects. We do not use this assumption in order to separate information content and precision.

For example, suppose the project is a long-term contract, then $k\tilde{\varepsilon}$ is the portion of the contracted revenue or gain from work completed in period *t*. Alternatively, $k\tilde{\varepsilon}$ may be the costs incurred or the sold products or services from the project which is paid later. The accounting signal $\tilde{y}_1$ is a typical accrual in the accounting system, as it recognizes a portion of future expected cash flows early. We assume the accounting system is exogenously given, e.g., by an accounting standard, and we vary the information content by changing *k* and σ_n^2 to examine their impact on earnings quality. Note that both *k* and σ_n^2 affect the variance of $\tilde{y}_1$, albeit in an economically different way, as we show later.

In the second period the cash flow $\tilde{x}$ realizes. The accounting system generates the signal

$$\tilde{y}_2 = \tilde{x} - \tilde{y}_1 = \mu + (1-k)\tilde{\varepsilon} - \tilde{n} + \tilde{\delta} + \tilde{\omega}.$$

The accounting system obeys the clean surplus condition, which requires that the total accounting earnings are equal to total cash flows, i.e., for each project $y_1 + y_2 = x$.

The investment cost *I* is depreciated at commonly known (or observable) rates; periodic depreciation is d_1 and d_2, where $I = d_1 + d_2$.[6] Thus, we distinguish working capital accruals $\tilde{y}_t$ and other accruals (depreciation d_t).

The manager obtains additional private information, referred to as non-financial information. The source of the information may be the internal management accounting system or other external information. This information is not recognized in the financial statements and cannot credibly be communicated by the manager. We model the non-financial information by a signal

$$\tilde{z} = \tilde{\delta} + \tilde{u}$$

about the cash flow component $\tilde{\delta}$ with noise $\tilde{u}$, which is normally distributed with zero mean and variance σ_u^2. All random variables are mutually independent. Assuming that the

---

[6] Since d_1 and d_2 are common knowledge, they carry no incremental information, but are needed to capture the accounting depiction of the investment outlay.

internal accounting system also traces non-financial information, we treat σ_u^2 as another characteristic of the accounting system.

Neither the accounting system nor other sources of the manager provide information about the third cash flow component $\tilde{\omega}$, so σ_ω^2 is the residual risk of the project's cash flow. Thus, the accounting system is characterized by $(k, \sigma_n^2, \sigma_u^2)$ which is common knowledge.

We allow the manager to influence the earnings from the accounting system by introducing a bias so that reported earnings can deviate from the the original accounting earnings. In the first period of a project, the manager chooses a bias *b* (where *b* can be positive or negative), which is reversed in the second period of a project. The bias can be a result of applying a particular revenue recognition or cost allocation procedure or of exercising discretion in the estimation of future revenues and costs or may consist of real activities that shift income from one period to the other. While the literature often views the introduction of a bias as undesirable earnings management that destroys information, our formulation also comprises the possibility that a bias carries more information (in our model about the manager's private non-financial information).

In sum, the reported earnings for each project in period 1 and 2 are

$$\begin{aligned} \tilde{m}_1 &= \tilde{y}_1 - d_1 + b = k\tilde{\varepsilon} + \tilde{n} - d_1 + b \\ \tilde{m}_2 &= \tilde{y}_2 - d_2 - b = \mu + (1-k)\tilde{\varepsilon} - \tilde{n} + \tilde{\delta} + \tilde{\omega} - d_2 - b, \end{aligned} \tag{3}$$

where $\tilde{m}_1 + \tilde{m}_2 = \tilde{x} - I$.

Manager's incentives

The choice of the bias *b* depends on the manager's private information and incentives for bias. At the time the manager decides on *b*, she has available the accounting earnings *y* from the projects in the respective period and the non-financial information *z* of the new project. Therefore, *b* = *b*(*y*, *z*). We assume a manager is hired and works for two periods, then leaves the firm and is replaced by a new manager with the same characteristics.[7] Figure 1 summarizes the sequence of events.

[Insert Figure 1 about here]

We do not explicitly model reasons for specific management incentives, but assume incentives of managers that are most common in practice. In particular, survey results by Graham, Harvey and Rajgopal (2005) indicate that the overwhelming majority of surveyed managers have objectives that are tied to market prices and to earnings[8] and include a desire to smooth earnings over time.[9] Since we are interested in the interaction between accounting standards and management incentives, we include all of these possible incentives. In particular, we assume risk neutral managers and consider short-term market price, reported earnings, smoothing reported earnings over the two periods, and that earnings management is privately costly to the manager. In the first period, the manager has the following utility function:

$$U_1 = p_1 P_1 + g_1 M_t - s\mathrm{E}[(\tilde{M}_{t+1} - M_t)^2] - r\frac{b_1^2}{2}. \tag{4}$$

In the second period, the utility function is

$$U_2 = p_2 P_2 + g_2 M_{t+1} - r\frac{b_2^2}{2}. \tag{5}$$

[7] Longer tenure would significantly increase the complexity of the analysis because each bias choice must take into account subsequent biases. Since period 2 is the ending period, we view period 1 as a representative period for our analysis.

[8] Possible reasons for the optimality of incentive systems combining market price and accounting numbers can be found in Dutta and Reichelstein (2005) and Dutta (2007).

[9] Dichev et al. (2013) report that hitting earnings benchmarks, influencing executive compensation, and smoothing earnings are among the most frequent motivations for earnings management.

The number indexes $j = 1, 2$ indicate the tenure of the manager. For simplicity, we assume a zero discount rate for the manager's utility.

First, the manager' utility can be based on the respective contemporaneous market prices of the firm P_t (determined in the rational expectations equilibrium), which enter with weights p_1 and p_2. For example, the manager is evaluated on market price, holds stock (ignoring bankruptcy), or plans to raise external capital and wants to boost the market price. For most of the analysis, we assume $p_j > 0$ in explaining the results, but the analysis is not restricted to a positive p_j.

Second, the manager's utility can be based on reported period earnings, where M_t is the total of the projects' individual earnings reports (which we describe later). The weights are g_1 and g_2. This interest may arise from a compensation scheme that depends on reported earnings, earnings targets the manager wants to reach, from political cost considerations or covenants that depend on earnings.

Third, the manager may be interested in smoothing reported earnings over the two periods of tenure. There are several reasons that smoothing abounds. One reason is risk aversion of a manager, but it can exist under risk neutrality to reduce earnings volatility (Trueman and Titman 1988) or to improve the market's inference of the report precision (Kirschenheiter and Melumad 2002); it can be the result of an optimal contract (Dye 1988, Demski 1998) or it can emerge from the existence of earnings targets, career concerns, and the like.[10] We formalize smoothing in (4) by the expectation of the squared differences between first-period earnings and second-period earnings, weighted with $s \geq 0$. While smoothing is forward-looking and seeks to smooth current and expected future reported earnings, our subsequent analysis shows that backward smoothing occurs as well, because the manager "inherits" earnings from a project invested by the predecessor. At the end of the second period, the manager leaves the firm and does not care about smoothing second-period

---

[10] De Jong et al. (2012) report that even analysts recognize positive consequences of smooth earnings, although they dislike earnings smoothing if it reduces transparency.

earnings.[11] Thus, there is a "horizon effect" in that the manager is interested in a smooth earnings stream only as long as she operates the current firm, but not so when leaving the firm.

Finally, the manager bears a cost from undertaking earnings management. In line with many papers on earnings management,[12] we assume that this cost is convex in the bias b_j (the manager can bias earnings in each period j) and scaled by a weight $r \geq 0$. The cost captures personal discomfort and other costs of earnings management, and it is affected by accounting standards and institutional factors, such as auditing and enforcement effectiveness, liability risk, and corporate governance provisions. We assume that the cost occurs in (or can be attributed to) the period in which the manager biases the earnings report, but not in subsequent periods in which the bias reverses. Subsequently, we use bias and earnings management interchangeably because the manager has an incentive to manage earnings rather than directly providing private information to the market. In equilibrium the bias is in fact informative, so allowing for "earnings management" turns out to be desirable.

We assume the weights are exogenous constants and common knowledge.[13] The use of four weights (rather than three, which would be sufficient to capture the substitution rates between the components) allows us to identify the effects of each individual component separately in equilibrium.

Financial reports

Next we describe the financial reports in the steady state setting. The financial report consists of aggregate reported earnings, the two projects' cash flows and the net assets. Since we do not explicitly track cash flows from financing, the net assets are not necessarily equal

[11] Huson et al. (2012) provide empirical evidence that compensation committees adjust the weights in the variable compensation in the final year of a CEO to counter the CEO's greater incentives for earnings-increasing manipulation.

[12] See, e.g., Fischer and Verrecchia (2000), Stocken and Verrecchia (2004), Ewert and Wagenhofer (2005).

[13] Thus, we exclude private information about the weights. For an analysis of an uncertain market price weight p see Fischer and Verrecchia (2000).

to the accumulated differences between reported earnings and cash flows. To distinguish between projects, we index projects that start at the beginning of period t with t.[14] In each period there are two active projects, one that is started and the other that ends. Since the situation is similar for all managers, we choose a representative manager and examine the two periods of her tenure (j = 1, 2) in detail, which are the periods t and t+1.

[Insert Figure 2 about here]

As depicted in Figure 2, reported earnings in each period are the aggregate individual earnings of the two active projects. These are for the current manager's tenure:

$$\begin{aligned} M_1 &= m_2(t-1) + m_1(t) \\ &= y_2(t-1) - d_2 - b_2 + y_1(t) - d_1 + b_1 \\ &= y_2(t-1) - b_2 + y_1(t) + b_1 - I. \end{aligned} \tag{6}$$

$$\begin{aligned} M_2 &= m_2(t) + m_1(t+1) \\ &= y_2(t) - d_2 - b_1 + y_1(t+1) - d_1 + b_2 \\ &= y_2(t) - b_1 + y_1(t+1) + b_2 - I. \end{aligned} \tag{7}$$

The manager „inherits" earnings $m_2(t–1)$ from the previous manager, including the bias b_2 which the outgoing manager had chosen. The current manager selects the bias b_1 for the recent project, which reverses in the second period. Moreover, the current manager chooses a bias b_2 in the second period. While this b_2 in expression (7) is formally different from that in expression (6), it is of the same amount because the managers are similar.

Although the earnings components from two projects are reported in aggregate, the information in the financial statements allows sophisticated investors to disentangle them and, therefore, investors use the information about the two active projects separately when determining the value of the firm. Since I is common knowledge, $m_1(t)$ can be calculated from the cash flows $x(t–1) – I$ and the end-of-period net assets,

[14] To facilitate readability, we do not index the random cash flow components if it is not confusing.

$$x(t-1)+I+m_1(t)=\underbrace{x(t-1)}_{\substack{\text{Cash from}\\\text{ending project}}}+\underbrace{I-d_1}_{\substack{\text{Carrying amont}\\\text{of current project}}}+\underbrace{y_1(t)+b_1}_{\substack{\text{Working capital}\\\text{accruals of}\\\text{current project}}}.$$

Define $W_1(t) = y_1(t) + b_1$ as working capital accruals of the project starting in period t in the first period of the manager's tenure. Since project t–1 ends in period t, the market observes $x(t–1)$, so $W_1(t)$ is the component that carries all available information for valuing the firm at the end of period t.

The cum dividend market price of the firm at t in the first period of the manager's tenure is

$$P_t = x(t-1)+\mathrm{E}_t\left[\tilde{x}(t)\middle|W_1(t)\right]\rho+\frac{1}{1-\rho}NPV \tag{8}$$

where ρ is the market discount factor and the last term is the net present value of an infinite period of future projects. Since projects are equal, the net present value does not depend on current projects and information. The market price at the end of t+1 is

$$P_{t+1} = x(t)+\mathrm{E}_{t+1}\left[\tilde{x}(t+1)\middle|W_2(t+1)\right]\rho+\frac{1}{1-\rho}NPV\,. \tag{9}$$

where $W_2(t+1)=y_1(t+1)+b_2$ and where the manager selects b_2 in the last period of her tenure. Thus, the market uses the reported earnings (and, equivalently, accruals) to update its beliefs about the future cash flows of the project started in the respective period. As we show in the subsequent analysis, accounting information from the current and the prior year is a sufficient statistic for investors to infer pricing information.

3. Equilibrium

A rational expectations equilibrium consists of a reporting strategy by the manager (the bias) and a capital market pricing mechanism that correctly infers the information contained in the earnings report on average. Each of these strategies is an optimal response based on the conjectures of the other player's strategy. In equilibrium these conjectures are fulfilled. In line with much of the rational expectations equilibrium literature we restrict attention to linear equilibria.[15]

The manager maximizes her expected utility by choosing b contingent on her information set and her conjecture about the market price contingent on the earnings report, which are[16]

$$\hat{P}_t = \hat{\alpha}_t + \hat{\beta}_t W_t \rho$$

where hats on variables indicate conjectures and W_t stands for the information variable in period t.

Starting backwards with the second period, the manager's utility is

$$U_2 = p_2\left(\alpha_2 + \beta_2 W_2(t+1)\right) + g_2 M_2 - \frac{r b_2^2}{2}. \tag{10}$$

Since $W_2(t+1) = y_1(t+1) + b_2$, the first-order condition determines the optimal accounting bias:

$$b_2^* = \frac{p_2 \hat{\beta}_2 + g_2}{r}. \tag{11}$$

While b_2 can be a function of $y_1(t+1)$ and $z(t+1)$, the optimal bias is a constant that is independent of that information. This is a consequence of the fact that the manager's utility function is common knowledge, that the formal structure is linear, and that the manager leaves the firm at the end of this period.

In the first period of tenure, the manager selects the bias b_1 by maximizing the first-period utility U_1 and the expected second-period utility U_2, given the sequentially rational bias b_2^*. The first-period utility is

---

[15] See, e.g., Fischer and Verrecchia (2000). Guttman, Kadan, and Kandel (2006) prove the existence of equilibria with partial pooling. Einhorn and Ziv (2012) show that such equilibria do not survive the D1 criterion of Cho and Kreps (1987).

[16] In this equation, we show the market discount factor ρ separately, so that β is only driven by the market's revision of expectations.

$$U_1 = p_1 P_1 + g_1 M_t - s\mathrm{E}\left[\left(\tilde{M}_{t+1} - M_t\right)^2 \middle| y_1(t), z(t)\right] - \frac{r}{2} b_1^2$$
$$= p_1\left(\hat{\alpha}_1 + \hat{\beta}_1 W_1(t)\right) + g_1 M_t - s\,\mathrm{Var}\left(\left(\tilde{y}_2(t) + \tilde{y}_1(t+1)\right) \middle| y_1(t), z(t)\right) \tag{12}$$
$$- s\left(\mathrm{E}\left[\tilde{y}_2(t) \middle| y_1(t), z(t)\right] + 2\frac{p_2\hat{\beta}_2 + g_2}{r} - y_2(t-1) - y_1(t) - 2b_1\right)^2 - \frac{r}{2} b_1^2$$

where the second expression results from substituting for M_t and $\tilde{M}_{t+1}$ from (6) and (7), that is

$$\tilde{M}_{t+1} - M_t = \tilde{y}_2(t) + \tilde{y}_1(t+1) - y_2(t-1) - y_1(t) + 2\frac{p_2\hat{\beta}_2 + g_2}{r} - 2b_1$$

and noting that $\mathrm{E}\left[\tilde{y}_1(t+1) \middle| y_1(t), z(t)\right] = 0$. Expression (12) shows that the previous manager's earnings management decision enters the utility, which is equal to the current manager's expected second-period bias b_2^*, as does the reversal of the first-period bias b_1. Therefore, past and anticipated future earnings management affect the manager's decision in equilibrium.

The expected second-period earnings from the current project is

$$\mathrm{E}\left[\tilde{y}_2(t) \middle| y_1(t), z(t)\right] = \mu + \frac{\mathrm{Cov}\left(\tilde{y}_2(t), \tilde{y}_1(t)\right)}{\mathrm{Var}\left(\tilde{y}_1(t)\right)}\left(y_1(t) - \mathrm{E}\left[\tilde{y}_1(t)\right]\right) + \frac{\mathrm{Cov}\left(\tilde{y}_2(t), z(t)\right)}{\mathrm{Var}\left(z(t)\right)}\left(z(t) - \mathrm{E}\left[\tilde{z}(t)\right]\right)$$
$$= \mu + \frac{\mathrm{Cov}\left(\tilde{\varepsilon}(1-k) + \tilde{\delta} + \tilde{\omega} - \tilde{n}, k\tilde{\varepsilon} + \tilde{n}\right)}{\mathrm{Var}\left(k\tilde{\varepsilon} + \tilde{n}\right)} y_1(t) + \frac{\mathrm{Cov}\left((1-k)\tilde{\varepsilon} + \tilde{\delta} + \tilde{\omega} - \tilde{n}, \tilde{\delta} + \tilde{u}\right)}{\mathrm{Var}\left(\tilde{\delta} + \tilde{u}\right)} z(t)$$
$$= \mu + \frac{k(1-k)\sigma_\varepsilon^2 - \sigma_n^2}{k^2\sigma_\varepsilon^2 + \sigma_n^2} y_1(t) + \frac{\sigma_\delta^2}{\sigma_\delta^2 + \sigma_u^2} z(t).$$

The optimal bias in the first period results from the partial derivatives

$$\frac{\partial U_1}{\partial b_1} = p_1\hat{\beta}_1 + g_1 + 4s\left(\mathrm{E}\left[\tilde{y}_2(t) \middle| y_1(t), z(t)\right] + 2\frac{p_2\hat{\beta}_2 + g_2}{r} - y_2(t-1) - y_1(t) - 2b_1\right) - rb_1$$

and $$\frac{\partial U_2}{\partial b_1} = -g_2.$$

Adding both partial derivatives and setting the sum to zero results in

$$b_1^* = R(p_1\hat{\beta}_1 + g_1 - g_2)$$
$$+ Z\left(\mu + \left(\frac{k(1-k)\sigma_\varepsilon^2 - \sigma_n^2}{k^2\sigma_\varepsilon^2 + \sigma_n^2} - 1\right) y_1(t) + Hz(t) + 2\frac{p_2\hat{\beta}_2 + g_2}{r} - y_2(t-1)\right) \tag{13}$$

where $R \equiv \dfrac{1}{8s+r} > 0$, $Z \equiv \dfrac{4s}{8s+r} > 0$, and $H \equiv \dfrac{\sigma_\delta^2}{\sigma_\delta^2 + \sigma_u^2} > 0$. Further, let $Q \equiv Z\left(\dfrac{k\sigma_\varepsilon^2}{k^2\sigma_\varepsilon^2 + \sigma_n^2}\right) + rR > 0$. Investors can only observe the total working accruals, which are

$$W_1 = y_1(t) + b_1^* = A + Qy_1(t) + ZHz(t) \tag{14}$$

with $A \equiv \underbrace{R(p_1\hat{\beta}_1 + g_1 - g_2) + 2Z\dfrac{p_2\hat{\beta}_2 + g_2}{r} + Z\mu - Zy_2(t-1)}_{\text{Known or inferred by market}}.$

Investors conjecture that the manager's earnings report is linear in her private information $y_1(t)$ and $z(t)$. Inspecting expression (14) confirms this conjecture. It also reveals that W_1 negatively (and linearly) depends on $y_2(t–1)$, the second-period unmanaged earnings of the project, which enters the equilibrium bias through the smoothing incentive. Thus, the bias exhibits not only forward smoothing, but also backward smoothing. $y_2(t–1)$ is not directly observable but can be inferred from other information available at t: The bias $\hat{b}_2^*$ is a constant (and included in the second term in (14)), and since investors can infer the previous reported earnings $m_1(t–1)$ and, hence, $y_1(t–1)$ from the last period, the clean surplus property implies $y_2(t–1) = x(t–1) – y_1(t–1)$ where $x(t–1)$ are the cash assets in the recent period's balance sheet. Thus, even though accounting information about the project t–1 that ends in t is not informative for valuing the firm, it is nevertheless important for interpreting the current reported earnings M_t because it is necessary to enable inference of the bias b_1^* the manager adds to the earnings number of the new project t.

The market infers the working capital accruals $W_1(t) = y_1(t) + \hat{b}_1^*\left(y_1(t), z(t)\right)$ and uses them to elicit information about two pieces of information $y_1(t)$ and $z(t)$ that the manager privately knows. Investors revise their expectation of the future cash flow $\tilde{x}(t)$ of the recent project as follows:

$$\begin{aligned}\mathrm{E}_t\left[\tilde{x}(t) \middle| W_1(t)\right] &= \mu + \frac{\mathrm{Cov}\left(\tilde{x}(t), W_1(t)\right)}{\mathrm{Var}\left(W_1(t)\right)}\left(W_1(t) - \mathrm{E}[W_1(t)]\right) \\ &= \mu + \frac{Qk\sigma_\varepsilon^2 + ZH\sigma_\delta^2}{Q^2(k^2\sigma_\varepsilon^2 + \sigma_n^2) + Z^2H\sigma_\delta^2}\left(W_1(t) - \mathrm{E}\left[W_1(t)\right]\right).\end{aligned}$$

Examination of this expression confirms that the market price effect is a linear function of $W_1(t)$ with

$$\beta_1 = \frac{Qk\sigma_\varepsilon^2 + ZH\sigma_\delta^2}{Q^2\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + Z^2H\sigma_\delta^2}. \tag{15}$$

The other terms from the above revised expectation are constant and known or inferred by investors. In particular, investors are able to directly back out the bias from the price and earnings incentives.

In the second period, investors use $W_2(t+1)$ to revise their beliefs in a similar way. Since the manager's bias is a constant, $b_2^* = \frac{p_2\beta_2 + g_2}{r}$, the revised expectation is:

$$\begin{aligned} \mathrm{E}_{t+1}\left[\tilde{x}(t+1)\middle|W_2(t+1)\right] &= \mu + \frac{\mathrm{Cov}\left(\tilde{x}(t+1), W_2(t+1)\right)}{\mathrm{Var}\left(W_2(t+1)\right)}\left(W_2(t+1) - \mathrm{E}\left[W_2(t+1)\right]\right) \\ &= \mu + \frac{k\sigma_\varepsilon^2}{k^2\sigma_\varepsilon^2 + \sigma_n^2}\, y_1(t+1). \end{aligned} \tag{16}$$

Let $$\beta_2 \equiv \frac{k\sigma_\varepsilon^2}{k^2\sigma_\varepsilon^2 + \sigma_n^2}. \tag{17}$$

Proposition 1 summarizes the analysis and formally states the equilibrium.

Proposition 1: There exists a unique linear equilibrium in which $b_j^* = \hat{b}_j^*$ and $\beta_j = \hat{\beta}_j$, j = 1, 2. The equilibrium strategies are stated in equations (13), (11), (15), and (17).

Since our main interest is on earnings quality, we only briefly highlight some characteristics of the earnings management biases b_j^* and the market reaction β_j. Earnings management in the first period is different to that in the second period. In the second period, which is the last period in which the manager is active, she is only interested in maximizing her short-term utility regardless of what happened earlier. This horizon effect makes it easy for investors to look through the earnings management.

The first-period earnings management strategy is more complex. It includes incentives that are also effective in the second period, but they also exhibit a desire of the manager to smooth earnings over periods. Indeed, smoothing is beneficial in this setting because it induces the manager to use the private information. This smoothing desire leads to forward smoothing, in which the manager considers the effect of a bias in the first period on second-period earnings. It also leads to backward smoothing because the manager considers the "inherited" earnings of the project that had been invested by her predecessor. Even though these earnings cannot be manipulated any more, they affect the level of first-period earnings, but not of second-period earnings. Therefore, the manager wants to smooth them over the two periods. In a (unmodeled) longer-term tenure of a manager these interdependencies of past reported and future anticipated earnings management continue to exist and make the bias even more complex due to forward smoothing.

The second-period bias b_2^* exhibits intuitive comparative statics: It increases in the weight p_2 the manager assigns to the market price reaction and the weight g_2 on reported earnings, and decreases in the cost r of earnings management. A higher market reaction β_2 has more effect on the bias, and since β_2 depends on k, σ_ε^2, and σ_n^2, these parameters indirectly affect the bias.

The first-period bias b_1^* comprises the effects of more variables. A higher weight p_1 increases the bias, as does a higher weight g_1; since the bias reverses in the subsequent period, the weight g_2 enters negatively, however, the anticipated bias b_2^* affects the first-period bias as well, hence, it increases in g_2. The net effect depends on the weight s. Then, as mentioned above, high earnings from the previous year's project enter negatively to smooth them over two periods, and both $y_1(t)$ and $z(t)$ enter positively. As apparent from (14), the earnings report is adjusted such that a portion $Z = 4s/(8s + r)$ of the "shocks" $y_1(t)$ and $z(t)$ (appropriately adjusted for their information content) is allocated to the first period, whereas the portion $(1 – Z)$ is shifted to the second period. If biasing were costless ($r = 0$), then $Z = 1/2$ and the shocks were equally distributed across the two periods and, thus, perfectly smoothed. Z decreases for higher r.

The coefficients β_1 and β_2 reflect the sensitivity of investors' beliefs with respect to the earnings component that they use to update their expectations. They are given by

$$\beta_1 = \frac{Qk\sigma_\varepsilon^2 + ZH\sigma_\delta^2}{Q^2\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + Z^2H\sigma_\delta^2} \text{ and } \beta_2 = \frac{k\sigma_\varepsilon^2}{k^2\sigma_\varepsilon^2 + \sigma_n^2}.$$

These coefficients are designed to elicit the actual information content of earnings on the cash flow components $\tilde{\varepsilon}$ and $\tilde{\delta}$, that are embedded in the accounting signal and the bias. Notice that β_1 and β_2 are strictly greater than zero (for parameters bounded away from extreme values), but can be greater than 1, for example, if k is small. The reason is that k scales the information contained in $y_1(t)$ and, in equilibrium, the market reaction reverses the scaling effect. More properties of the coefficients β_1 and β_2 are discussed below.

4. Earnings quality

4.1. Definition of earnings quality

In this section, we study how the design of the accounting system affects earnings quality. As already mentioned in the Introduction, earnings quality is a notion that captures a diverse spectrum of attributes of financial reporting and needs specification. We adopt an information content perspective of financial reports: Reported earnings are of higher quality the more information they contain with respect to future cash flows.[17] This definition is based on the value of information rather than statistical properties and other concepts.

For normal distributions, the information content of a signal is independent of the realization of the signals and can be measured by comparing the ex post variance with the prior variance of the future cash flows.

Definition: Earnings quality EQ is the reduction of the market's uncertainty about the future cash flows due to the reported earnings reported in a period t, formally:

$$EQ \equiv \mathrm{Var}\left(\tilde{x}\right) - \mathrm{Var}\left(\tilde{x} \middle| M\right) \tag{18}$$

or equivalently, since the W carries the contemporaneous information in our setting,

$$EQ = \frac{\mathrm{Cov}\left(\tilde{x}, \tilde{W}\right)^2}{\mathrm{Var}\left(\tilde{W}\right)}. \tag{19}$$

[17] This definition is similar to that, e.g., in Francis, Olsson, and Schipper (2006).

A greater *EQ* implies higher earnings quality. Note that in our steady state setting each new project adds uncertainty, but still, reported earnings reduce the conditional variance of the cash flow $\tilde{x}(t)$ of the most recent project and provide more information about future cash flows. Furthermore, *EQ* is based on the entire set of information that investors receive and interpret in the rational expectations equilibrium. Therefore, we use the accruals $\tilde{W}$ as the main argument in the *EQ* function.

4.2. Effects of accounting system characteristics

We study how a variation in the characteristics of the accounting system and the cost of earnings management affect earnings quality. These are the factors that accounting standards and regulation directly affect. Due to its ceteris paribus character, this analysis does not necessarily provide the total effects on earnings quality for a simultaneous change of more than one characteristic or an endogenous interaction with other characteristics, but it shows the isolated effect for a variation of each of the characteristics individually.

The accounting system is characterized by the three parameters *k*, σ_n^2, and σ_u^2. Accounting standards determine *k* and affect σ_n^2 (e.g., if measurement is strongly based on estimates, such as fair value estimates). σ_n^2 and σ_u^2 are also characteristics of the underlying quality of the internal accounting and reporting system. As we discuss in a later section, accounting standards also shape the information about the risk components, which can change the impact of σ_n^2 and σ_u^2. Finally, the cost scaling parameter *r* depends on the tightness of accounting standards, but additionally on internal control systems, corporate governance measures, auditing, and enforcement.

In the second period, earnings quality EQ_2 is given by

$$\begin{aligned} EQ_2 &= \frac{\operatorname{Cov}\left(\tilde{x}(t+1), \tilde{W}_2(t+1)\right)^2}{\operatorname{Var}\left(\tilde{W}_2(t+1)\right)} = \frac{\operatorname{Cov}\left(\tilde{x}(t+1), \tilde{y}_1(t+1)\right)^2}{\operatorname{Var}\left(\tilde{y}_1(t+1)\right)} \\ &= \beta_2 k \sigma_\varepsilon^2 = \frac{k^2 \sigma_\varepsilon^4}{k^2 \sigma_\varepsilon^2 + \sigma_n^2}. \end{aligned} \tag{20}$$

Proposition 2: Earnings quality EQ_2

(i) strictly increases in the accrual parameter *k*;

(ii) strictly increases in accounting precision ($1/\sigma_n^2$);

(iii) is independent of the precision of non-financial information ($1/\sigma_u^2$);

(iv) is independent of the cost r of earnings management.

The proof follows immediately from the inspection of (20). In particular, EQ_2 increases in the accrual parameter k because a higher k makes contemporaneous earnings unambiguously more informative about future cash flows. Perhaps less intuitive is the fact that earnings quality is independent of the precision of non-financial information; the reason is that in the second period, the manager does not care about injecting her private non-financial information $z(t+1)$ as she is not interested in future reported earnings and their interdependence with contemporaneous earnings. EQ_2 is unaffected by a variation of the cost of earnings management. While the amount of bias b_2^* clearly depends on the cost of earnings management r, the market is able to back it out and adjusts the price reaction, so earnings quality does not change for a change in r.

Earnings quality in the second period is a limiting result as the manager is short-term interested and does not care about future earnings any more. Earnings quality in the first period is more descriptive of a typical situation of an ongoing firm. EQ_1 is

$$EQ_1 = \frac{\operatorname{Cov}\left(\tilde{x}(t), \tilde{W}_1(t)\right)^2}{\operatorname{Var}\left(\tilde{W}_1(t)\right)} = \frac{\left(Qk\sigma_\varepsilon^2 + ZH\sigma_\delta^2\right)^2}{Q^2\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + Z^2H\sigma_\delta^2} = \beta_1\left(Qk\sigma_\varepsilon^2 + ZH\sigma_\delta^2\right). \tag{21}$$

The effects of a variation of the accounting system on EQ_1 are more complex because most of the parameters have an effect on the covariance and the variance terms in (21). To see what determines the difference between EQ_1 and EQ_2 it is instructive to briefly consider two special cases. The first case is when $s = 0$, i.e., the manager has no incentives to smooth reported earnings. Then, $Z = 0$ and $Q = 1$, and

$$EQ_1(s=0) = \frac{k^2\sigma_\varepsilon^4}{k^2\sigma_\varepsilon^2 + \sigma_n^2} = EQ_2.$$

Without smoothing incentives, the manager does not care about the stochastic time structure of earnings. Thus, the optimal bias is constant and completely determined by the incentive weights on price and earnings, which is similar to the situation in the second period. Hence, the market reaction is similar, too.

The second special case is when the manager has no non-financial information, i.e., $z(t)$ is uninformative, which is equivalent to setting $\sigma_u^2 \to +\infty$. Then $H = 0$ and, again,

$$EQ_1(\sigma_u^2 \to \infty) = \frac{k^2\sigma_\varepsilon^4}{k^2\sigma_\varepsilon^2 + \sigma_n^2} = EQ_2.$$

The reason is that the equilibrium bias b_1^* does not contain information on $\tilde{\delta}$ and reduces to a linear function of $y_1(t)$. If $s > 0$, the manager smoothes earnings which leads to $Q \neq 1$, but this does not cause any loss of information for investors since they know Q. Thus, using (14) they can invert each reported value of the working capital accruals $W_1(t)$ to arrive at the actual value of $y_1(t)$. It follows that the information content solely depends on factors driving $y_1(t)$, which mirrors the result for the second period.

Lemma 1: A smoothing incentive ($s > 0$) and private non-financial information of the manager outside the accounting signal ($\sigma_u^2 < +\infty$) are necessary and sufficient conditions for reported earnings to carry incremental information over the accounting signal.

The sufficiency result can be directly seen from the expression of the equilibrium bias b_1^*, which comprises the term $ZHz(t)$. Of course, smoothing requires that the manager's decision horizon extends over more than one period. Therefore, in the last period there is no effect of $z(t+1)$ on reported earnings.

Lemma 1 highlights the importance of a smoothing incentive and earnings management for conveying additional information through the earnings report to the market. The reason is that smoothing provides a link between the two periods that the manager is motivated to embed in the bias part of her private information.

Proposition 3 reports the results for the general case, in which the parameters are not in the extremes.

Proposition 3: Earnings quality EQ_1

(i) strictly increases in the accrual parameter k for low k; for high k it can decrease (a necessary condition is $k^2 > \frac{4s}{r}\frac{\sigma_n}{\sigma_\varepsilon} - \frac{\sigma_n^2}{\sigma_\varepsilon^2}$);

(ii) strictly increases in accounting precision $(1/\sigma_n^2)$;

(iii) strictly increases in the precision of non-financial information $(1/\sigma_u^2)$;

(iv) strictly decreases in the cost r of earnings management.

The proof is in the appendix. It consists of signing the partial derivatives of EQ_1 with respect to each of the parameters. The proof is straightforward, although some of the steps are lengthy. In the following, we explain the results and provide the intuition for them.

Accrual parameter

The effect of a variation of k is complicated because k affects both components of $\tilde{W}_1(t)$, $y_1(t)$ and b_1^*. An increase in k places a greater portion of $\tilde{\varepsilon}$ into contemporaneous earnings $y_1(t)$, hence, earnings become more informative about future cash flows, which would increase earnings quality if everything else was held constant. In particular, $k = 1$ should maximize earnings quality. However, as stated in the proposition, the intuition holds for low k, whereas there are settings with high k in which an increase in k actually reduces earnings quality. The reason for this effect is that the parameter k not only changes the information content of $y_1(t)$, but also affects the weights with which financial and non-financial information are processed in the market. A more informative $y_1(t)$ can reduce the equilibrium weight of $z(t)$, thus driving out the non-financial information for pricing the firm. Note that k is a smoothing mechanism embedded in the accounting system, in addition to that in the bias b_1^*. Embedded smoothing is low for extreme values of k; and $\tilde{\varepsilon}$ would be fully smoothed across the two periods if $k = 1/2$.

To gain further insight into the result, consider a special case with a fully precise accounting system ($\sigma_n^2 = 0$) and no non-financial information by the manager ($\sigma_u^2 \to +\infty$). Then, as noted above, $EQ_1 = EQ_2$, and due to $\sigma_n^2 = 0$, $EQ_1 = \sigma_\varepsilon^2$, which is constant and independent of k. Formally, the squared covariance in the numerator of EQ_1 and the earnings variance in the denominator change exactly by the same factor, $2k$, for a change of k. Intuitively, k serves only as a scaling factor of the risk component $\tilde{\varepsilon}$ without affecting its information content. Indeed, β_2 =1/*k* in this special case, so the market reaction simply adjusts for the level of *k*. b_1^* still smoothes earnings across the two periods, but its size depends on 1/*k* as well.

Reintroducing non-financial information $(\sigma_u^2 < \infty)$ destroys this one-to-one relationship because an increase in *k* increases the (squared) covariance between earnings and the future cash flow and the variance of earnings at different rates. This is because *k* now affects the weights with which the two informative signals $y_1(t)$ and $z(t)$ enter the reported earnings. It can be shown that the variance increase dominates the covariance increase, so actually an increase in *k* *reduces* earnings quality EQ_1 as long as the accounting system is precise. Intuitively, an increase in *k* scales $\tilde{\varepsilon}$ without harming the information content of the signal $y_1(t)$, but at the same time the larger variance of earnings dilutes the information that earnings carry about $z(t)$.

Coming back to the general case, if the accounting system is noisy (σ_n^2 > 0) then an increase in *k* increases the weight of $\tilde{\varepsilon}$ in $y_1(t)$, which individually increases the information content of earnings and, hence, positively affects earnings quality. Starting at $k = 0$, an increase in *k* always increases EQ_1, but for high *k* the variance-increasing effect can dominate.

This result exhibits a subtle interaction between the accounting system (as captured by *k*), earnings management, and earnings quality. Bringing into the accounting system more information about future cash flows can reduce earnings quality, particularly if the accounting system is relatively imprecise. The proposition states a necessary condition for EQ_1 to decrease in *k*, which is $k^2 > \frac{4s}{r}\frac{\sigma_n}{\sigma_\varepsilon} - \frac{\sigma_n^2}{\sigma_\varepsilon^2}$, hence, the lower the right-hand side of this expression is the more likely is a decrease. Notice that the right-hand side is zero for $\sigma_n^2 = 0$ and achieves a unique maximum for

$$\hat{\sigma}_n^2 = 4\left(\frac{s}{r}\right)^2 \sigma_\varepsilon^2 .$$

Differentiating the right-hand side of this bound with respect to σ_ε yields

$$\frac{\partial\left(\frac{4s}{r}\frac{\sigma_n}{\sigma_\varepsilon}-\frac{\sigma_n^2}{\sigma_\varepsilon^2}\right)}{\partial\sigma_\varepsilon}=\frac{\sigma_n}{\sigma_\varepsilon^2}\left(-\frac{4s}{r}+2\frac{\sigma_n}{\sigma_\varepsilon}\right).$$

The derivative is positive if $\sigma_n > \frac{2s}{r}\sigma_\varepsilon$, which is equivalent to $\sigma_n^2 > \hat{\sigma}_n^2$, and vice versa.

Taken together, this leads to the following prediction:

Corollary 1: A decrease in earnings quality EQ_1 in *k* is more likely

(i) the lower is the accounting precision $(1/\sigma_n^2)$ if $\sigma_n^2 > \hat{\sigma}_n^2$;

(ii) the lower is the operating risk σ_ε^2 if $\sigma_n^2 > \hat{\sigma}_n^2$;

(iii) the lower is the smoothing incentive *s*;

(iv) the higher is the cost *r* of earnings management.

Accounting precision

A decrease of the accounting precision (a higher noise variance σ_n^2) lowers the covariance of earnings with future cash flows and affects the variance of earnings in two ways: First, the variance increases due to the direct impact of a larger accounting noise. Second, the variance decreases due to the negative impact of a greater σ_n^2 on the weight *Q* with which the signal $y_1(t)$ enters reported earnings. While the first variance effect works in the same direction as the covariance effect, the second is in the opposite direction. The proposition states that the latter effect is dominated, so that higher accounting noise and lower accounting precision, respectively, unambiguously reduces earnings quality. This effect is similar to the effect on EQ_2, although it is not as straightforward.

Notice that an individual variation of either *k* or σ_n^2 affects the precision of the accounting signal similarly: Higher *k* is similar to an increase in the precision (given some σ_n^2), and lower σ_n^2 is similar to an increase in the precision (given some *k*). This result reinforces the earlier discussion that the surprising effect on earnings quality for a variation of *k* is attributable to the built-in smoothing effect of *k*, which affects the weights of the two informative signals.

Precision of non-financial information

The precision of the manager's non-financial information is determined by the variance of the noise σ_u^2 in the signal $z(t)$ the manager obtains early about the cash flow component $\tilde{\delta}$. Given a certain level of operating risk σ_δ^2, higher noise σ_u^2 unambiguously reduces the information contained in $z(t)$, which is brought into reported earnings through the manager's bias. Therefore, earnings quality strictly decreases in σ_u^2 and increases in the precision of the manager's information, respectively. Note that this effect does not occur for second-period earnings quality (in Proposition 2) because the manager does not care about that information in the last period of tenure.

One might argue that more non-financial information increases the information asymmetry between the manager and investors and motivates the manager to exploit this comparative advantage by more offensive earnings management, which should decrease earnings quality. Indeed, the bias b_1^* includes a greater portion of the signal $z(t)$ and reacts more strongly to it, but investors rationally anticipate this effect and adjust the market price reaction accordingly. In total, earnings become more informative and earnings quality increases the more precise is the non-financial information of the manager.

Cost of earnings management

Without smoothing, the cost of earnings management, captured by the scaling parameter r, has no effect on earnings quality. This can be seen from EQ_2. Therefore, if a change of r has an effect on earnings quality it must be through the smoothing incentive. As stated in the proposition, an increase in r reduces earnings quality EQ_1 because it reduces the level of the bias and, hence, the information content of the bias reduces relative to that in $y_1(t)$.[18] Contemporaneous earnings reveal less information about $z(t)$, and this reduces earnings quality. As stated in Proposition 2, once $z(t)$ becomes uninformative, r dampens the level of earnings management, but this has no effect on earnings quality any more.

[18] This effect is economically similar, but converse, to an increase in the smoothing parameter s.

4.3. Effects of operating risk

To examine whether and how characteristics of the firm's operations affect earnings quality, we consider variations of the variances σ_ε^2, σ_δ^2, and σ_ω^2 of the three cash flow components $\tilde{\varepsilon}$, $\tilde{\delta}$, and $\tilde{\omega}$ on EQ_1 and EQ_2. The variances depend on the business model, the short-term or long-term volatility of the production process, and the firm's environment. Since we orthogonalize these components, the variation of one variance additively changes total operating risk.

This analysis provides insights how – these operating characteristics influence earnings quality, keeping the accounting system constant. The results are useful in at least two respects. First, they help understand the effect of information while keeping the total cash flow variance $\sigma_\varepsilon^2 + \sigma_\delta^2 + \sigma_\omega^2$, constant. For example, the accounting system may be designed to provide information about a greater portion of the future cash flows (which can be modeled by $\sigma_\varepsilon^2 + \Delta$ and $\sigma_\omega^2 - \Delta$) or the manager may be able to receive more information about the cash flow, which is not recognized by the accounting system ($\sigma_\delta^2 + \Delta$ and $\sigma_\omega^2 - \Delta$). Second, empirical studies often control for innate factors that capture the business model and the characteristics of the firms' operations (see, e.g., Francis, Olsson, and Schipper 2006) when they attempt to isolate the effects of a change in the accounting characteristics. Therefore, it is useful to know how these effects influence earnings quality measures.

Proposition 4: Operating risk affects earnings quality as follows:

(i) EQ_1 strictly increases in σ_ε^2 for high σ_ε^2 (a sufficient condition for all σ_ε^2 is $\left(\sigma_\delta^2\right)^2 - \frac{r}{2s}\sigma_n^2\sigma_\delta^2 - \frac{r}{2s}\sigma_n^2\sigma_u^2 < 0$); it can strictly decrease for low σ_ε^2;

EQ_2 strictly increases in σ_ε^2;

(ii) EQ_1 strictly increases in σ_δ^2; EQ_2 is independent of σ_δ^2;

(iii) EQ_1 and EQ_2 are independent of σ_ω^2.

The proof is in the appendix.

The effects on earnings quality differ significantly for the three risk components. The reason does not rest in the operations themselves, but in the different information that is available about the cash flow components: The accounting system reports accounting earnings that are informative about $\tilde{\varepsilon}$; the manager obtains private information about $\tilde{\delta}$ that is not included in the accounting earnings; and there is no additional information available on $\tilde{\omega}$. The key accounting variable is $W_1(t)$, which carries information about $\tilde{\varepsilon}$ and, indirectly, about $\tilde{\delta}$ through the equilibrium bias.

Generally, a higher variance of cash flows makes accounting information about that component more useful, and the impact on earnings quality should be higher. However, this intuition is incomplete. First, note that the residual volatility of the future cash flow, measured by the variance σ_ω^2, does not affect earnings quality. The reason simply is that reported earnings contain no information about $\tilde{\omega}$, so there is no decrease in the variance of future cash flows. This independence result makes it simple to generate predictions about keeping the total variance of the cash flows constant, but simultaneously changing either σ_ε^2 or σ_δ^2 and σ_ω^2 by the same amount in the reverse direction. All results we state in the proposition (i) and (ii) are preserved.

The manager learns about the second cash flow component $\tilde{\delta}$ from privately observing $z(t) = \tilde{\delta} + \tilde{u}$. In the second period of tenure, the manager does not base the bias on $z(t+1)$, so W_2 does not carry information about $\tilde{\delta}$ of the concurrent project; therefore, EQ_2 is unaffected. This is different from the first period, where $z(t)$ is embedded in b_1^*. The information content of $z(t)$ depends on the relative risks of the two components. Fixing some σ_u^2, an increase in the volatility of the cash flow component σ_δ^2 increases the information content of $z(t)$, which induces an increase in earnings quality.

Finally, consider a variation of the risk of the cash flow component $\tilde{\varepsilon}$. A logic similar to the one above applies for EQ_2, so EQ_2 increases in σ_ε^2. However, the effects of varying σ_ε^2 on EQ_1 are more complicated because σ_ε^2 affects the information content of $y_1(t)$ and of the bias b_1^*. The proposition states that EQ_1 strictly increases in σ_ε^2 for high σ_ε^2, but can strictly decrease in σ_ε^2 for low σ_ε^2. The proof provides a sufficient condition for a strict increase over all σ_ε^2, which is

$$\left(\sigma_\delta^2\right)^2 - \frac{r}{2s}\sigma_n^2\sigma_\delta^2 - \frac{r}{2s}\sigma_n^2\sigma_u^2 < 0.$$

This condition holds, for example, if σ_δ^2 is low, if σ_n^2 is large, if *r* is high, or if *s* is low. The converse of this condition is a necessary condition for a decrease of EQ_1 in σ_ε^2. EQ_1 can only decrease for low σ_ε^2 and will always increase for sufficiently high σ_ε^2. Intuitively, an increase in σ_ε^2 can significantly increase the variance of reported earnings, thus, lowering the precision of these earnings and diluting the information that can be inferred about the signals $y_1(t)$ and $z(t)$. This negative effect can overcompensate the benefit of conveying more information about $\tilde{\varepsilon}$. This result mirrors that for a variation of *k*.

5. Value relevance and earnings quality

The analysis has established how a variation of individual characteristics of the accounting system affects earnings quality, defined as information content of reported earnings. Since earnings quality is not directly observable, the empirical accounting literature uses several measures as proxies for earnings quality (see, e.g., Francis, Olsson, and Schipper 2006 and Dechow, Ge, and Schrand 2010). In this section, we study how one common measure, value relevance, is related to information content. We select value relevance as it is closely related to our rational expectations capital market model.[19] Value relevance is used in many studies, but is a controversial measure for earnings quality (see, e.g., Holthausen and Watts 2001).

Value relevance captures the notion that earnings are of high quality if they are capable to explain the firm's market price and/or market returns. Most common are two measures: the earnings response coefficient, which is the coefficient of the earnings variable in a regression of the market price and/or return on earnings; and the R^2 from the price-earnings regression. Our analysis provides theoretical insights into the question how well these value relevance measures perform as proxies for earnings quality. We also address the question under what circumstances one of the two measures is a preferable proxy.

An empirical study uses reported earnings at face value and relates them to market prices (or price changes), whereas investors perform several adjustments when they interpret

[19] Accounting-based measures are examined in Ewert and Wagenhofer (2013).

reported earnings and other information available (book values and prior period earnings) in order to elicit the information about the interesting signals $\tilde{y}$ and $\tilde{z}$. While one may argue if this is a fair comparison, this in fact is what empirical studies usually do.[20] Hence, we do not expect that these proxies perfectly trace earnings quality.

We focus on the first period of a manager's tenure as it is more representative of firms in a cross-sectional analysis. While the second period has a strong effect on earnings quality (as stated in proposition 2 in comparison to proposition 3), the horizon effect is weaker in this analysis as total reported earnings always include two active projects. Formally, in the first period investors base their pricing decisions on the working capital accruals $W_1(t)$ rather than on the reported earnings M_t, where

$$\tilde{M}_t = \tilde{y}_2(t-1) - \frac{p_2\beta_2 + g_2}{r} - I + \underbrace{\tilde{y}_1(t) + \tilde{b}_1^*}_{=\tilde{W}_1(t)}.$$

It is easy to see that $\tilde{M}_t$ depends on the period t earnings of the projects initiated in t and t–1. Therefore, the statistical relationships of prices with $\tilde{M}_t$ are different from those with $\tilde{W}_1(t)$, which implies that the behavior of value relevance measures also differs from that of actual earnings quality. To eliminate a "mechanistic" co-variation of earnings and prices that depends on the cash position from the project that ends at the end of the period, we use ex dividend prices in the subsequent analysis.[21]

5.1. Earnings response coefficient

The earnings response coefficient (*ERC*) is the market price reaction to (unadjusted) reported earnings. In our model, it is the direct equivalent to the capital market reaction β on reported earnings and is defined as

[20] While more variables are often included in a price-earnings regression, we believe that there will still be differences due to the structural assumptions underlying linear regressions.

[21] This is consistent with empirical value relevance studies based on level regressions, which typically use prices lagging a few months after the fiscal year end. Using cum dividend prices would add terms to the covariance and variance terms in the analysis that do not vary with the information content of reported earnings, but may affect the results.

$$ERC_1 = \frac{\operatorname{Cov}\left(\tilde{P}_t, \tilde{M}_t\right)}{\operatorname{Var}\left(\tilde{M}_t\right)}.$$

This formulation captures a cross-sectional analysis of a sample of firms drawn from our stochastic setting. To calculate this term, note that the equilibrium price depends only on $\tilde{y}_1(t)$ and $\tilde{z}(t)$ because investors infer and correct for the earnings of the "inherited" project, $\tilde{y}_2(t-1)$. However, the accruals $\tilde{W}_1(t)$ include $-Z\tilde{y}_2(t-1)$ through the bias (see equation (14)), which is eliminated by using the following price equation,

$$\tilde{P}_t = \alpha_1 + \beta_1\left(\tilde{W}_1(t) + Z\tilde{y}_2(t-1)\right).$$

Note that this modification applies for $\tilde{P}_t$ but not for $\tilde{M}_t$. The earnings response coefficient then is

$$\begin{aligned}
ERC_1 &= \frac{\operatorname{Cov}\left(\tilde{P}_t, \tilde{M}_t\right)}{\operatorname{Var}\left(\tilde{M}_t\right)} = \frac{\operatorname{Cov}\left(\beta_1\left(\tilde{W}_1(t) + Z\tilde{y}_2(t-1)\right), \tilde{M}_t\right)}{\operatorname{Var}\left(\tilde{M}_t\right)} \\
&= \beta_1 \frac{\operatorname{Cov}\left(\tilde{W}_1(t) + Z\tilde{y}_2(t-1), \tilde{W}_1(t) + \tilde{y}_2(t-1)\right)}{\operatorname{Var}\left(\tilde{M}_t\right)} \\
&= \beta_1 \frac{\operatorname{Cov}\left(\left(Q\tilde{y}_1(t) + ZH\tilde{z}(t)\right), \left(Q\tilde{y}_1(t) + ZH\tilde{z}(t)\right)\right)}{\operatorname{Var}\left(\tilde{M}_t\right)} \\
&= \beta_1 \frac{\operatorname{Var}\left(Q\tilde{y}_1(t) + ZH\tilde{z}(t)\right)}{\operatorname{Var}\left(\tilde{M}_t\right)} = \frac{\operatorname{Cov}\left(\tilde{x}(t), \tilde{W}_1(t)\right)}{\operatorname{Var}\left(\tilde{M}_t\right)} \\
&= \frac{Qk\sigma_\varepsilon^2 + ZH\sigma_\delta^2}{Q^2\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + Z^2H\sigma_\delta^2 + (1-Z)^2\left(\sigma_\varepsilon^2(1-k)^2 + \sigma_n^2 + \sigma_\delta^2 + \sigma_\omega^2\right)}.
\end{aligned} \tag{22}$$

It is apparent that ERC_1 and β_1 are closely related, and they are scaled by a factor that depends on the variances of the informative part of earnings and the reported earnings.

Proposition 5: The earnings response coefficient ERC_1

(i) strictly increases in the accrual parameter *k* for low *k*; for high *k* it can decrease (a necessary condition is $k^2 > \sigma_n^2/\sigma_\varepsilon^2$);

(ii) strictly increases in accounting precision $(1/\sigma_n^2)$;

(iii) strictly increases in the precision of non-financial information $(1/\sigma_u^2)$;

(iv) strictly decreases in the cost r of earnings management if r is sufficiently large; it can increase for low r.

The proof is in the appendix.

The result for a variation of the accrual parameter k is structurally similar to that of earnings quality EQ_1. Increasing k increases ERC_1 if k is low (sufficient condition). If k is large there are parameter constellations in which ERC_1 decreases. The underlying reason is similar to that for EQ_1, but the condition for which the effect reverses differs. In Proposition 3, the necessary condition is

$$k^2 > \frac{4s}{r}\frac{\sigma_n}{\sigma_\varepsilon} - \frac{\sigma_n^2}{\sigma_\varepsilon^2}$$

whereas here it is $k^2 > \sigma_n^2 / \sigma_\varepsilon^2$, which does not depend on s and r. Therefore, using ERC_1 as a measure for a change in earnings quality can lead to wrong conclusions; the error can be in either direction. For example, if σ_n^2 is equal to σ_ε^2, then ERC_1 always increases for any k, whereas earnings quality can decrease if $k > 4s/r$, which holds for high r although this is commonly considered an indicator of a high-quality reporting system.

Increasing the precision of the accounting signal (decreasing the variance σ_n^2) or increasing the precision of the non-financial information (decreasing the variance σ_u^2) unambiguously increases the ERC_1. These results are similar to earnings quality EQ_1, therefore, the earnings response coefficient is a useful measure of earnings quality if there is a variation in the precision of the internal management accounting system.

An increase in the cost r of earnings management can lead to different effects on ERC_1 and EQ_1. While EQ_1 unambiguously decreases in r, this is not necessarily true for ERC_1 as it can increase if r is small until it attains a maximum and decreases afterwards. This behavior is inconsistent with that of EQ_1 for small r, e.g., for imprecise accounting standards or low auditing and enforcement standards.

To gain more intuition, consider the boundary case with $r = 0$, so that biasing is costless and unlimited smoothing occurs. For $k > 0.5$, the built-in smoothing of the accounting system with respect to the risk component $\tilde{\varepsilon}$ is “too large” from the manager’s point of view, hence, a negative bias is preferable. Costless smoothing implies that a relatively large portion of the variability of $\tilde{\varepsilon}$ is smoothed away, leading to a lower covariance between the reported portion of $\tilde{\varepsilon}$ and future cash flows than would occur if $k\tilde{\varepsilon}$ was reported without bias. In addition, if there is large accounting noise, the desire to negatively bias the signal $y_1(t)$ is even stronger, which further reduces the covariance. An increase of r (starting from $r = 0$) makes smoothing more expensive and has two effects on the covariance: First, the weight with which $z(t)$ enters reported earnings diminishes, thus lowering the covariance term in the numerator of ERC_1. Second, the manager reduces the smoothing of $y_1(t)$ which increases the covariance term under the assumed setting of parameters. If the variance σ_δ^2 of the risk component δ and/or the precision of the manager's signal $z(t)$ are low, the first effect is relatively minor, resulting in a net increase of the covariance term.

Of course, the impact on ERC_1 depends on the ratio of the covariance and the variance of total reported earnings which increases by increasing r, but the effect on the numerator may dominate, so that ERC_1 can increase in r for small r. As shown in the proof, this does not hold for all r. If the risk σ_δ^2 is large, then the negative impact of an increase of r on the weight of $z(t)$ in the earnings report dominates, implying a lower covariance for larger r. Then ERC_1 decreases for increasing r.

The behavior of the earnings response coefficient in the second period, ERC_2, is similar to that of ERC_1, hence, we do not report it here.

In summary, the *ERC* is generally a good proxy for earnings quality, particularly if the variations relate to changes in the precision of the accounting system or non-financial information. However, there are settings in which *ERC* leads to results opposite to those for earnings quality for variations in the accrual parameter and the cost of earnings management.

5.2. Correlation coefficient

The R^2 of a price-earnings regression translates into the correlation coefficient (*CORR*) in our model. It is defined as

$$CORR_1 = \frac{\mathrm{Cov}(\tilde{P}_t, \tilde{M}_t)}{\mathrm{Std}(\tilde{P}_t)\mathrm{Std}(\tilde{M}_t)},$$

which is equal to (shown in the appendix)

$$CORR_1 = \sqrt{\frac{Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2H\sigma_\delta^2}{Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2H\sigma_\delta^2+\left(1-Z\right)^2\left(\sigma_\varepsilon^2\left(1-k\right)^2+\sigma_n^2+\sigma_\delta^2+\sigma_\omega^2\right)}}. \qquad (23)$$

This measure differs from the ERC_1 only in that the denominator includes $\text{Std}(\tilde{P}_t)$ instead of $\text{Std}(\tilde{M}_t)$. Despite this close relationship, the effects of a variation of the accounting characteristics are significantly different.

Proposition 6: The correlation $CORR_1$

(i) strictly increases in the accrual parameter k;

(ii) can strictly increase or decrease in accounting precision $(1/\sigma_n^2)$ depending on parameters;

in particular, if $k < 4s/r$, then $CORR_1$ strictly decreases in σ_n^2 for small σ_n^2 and can increase for large σ_n^2;

(iii) strictly increases in the precision of non-financial information $(1/\sigma_u^2)$;

(iv) can strictly decrease or increase in the cost r of earnings management.

The proof is in the appendix.

The correlation strictly increases in the accrual parameter k, which is in contrast to the behavior of EQ_1. Recall that EQ_1 can decrease for high k under certain conditions.

A decrease in accounting precision (a higher variance σ_n^2) either decreases or increases the correlation. This result is also in contrast to EQ_1, which always decreases in σ_n^2. To see why $CORR_1$ can increase in σ_n^2 assume $k > 4s/r$. In the appendix, we show that if the residual risk σ_ω^2 is large enough, then $CORR_1$ increases in σ_n^2 for all levels of σ_n^2. Intuitively, if $k > 4s/r$ then the impact of the incentive to smooth earnings is small, so that a larger accounting noise increases the variance of both the informative part of earnings $\tilde{W}_1(t)$ (the working capital accruals) and reported total earnings $\tilde{M}_t$. While the absolute increase of the variance of total earnings (the denominator of $CORR_1$) is larger than that of the informative part of earnings (affecting the numerator), the relative change is stronger for the numerator if there is large risk in total earnings (e.g., for a high σ_ω^2), yielding an increase in the correlation coefficient. Conversely, $k < 4s/r$ implies that the smoothing incentive is relatively strong, inducing the manager to dampen the impact of a larger accounting noise by smoothing. Then the variance of the informative part of earnings decreases by introducing accounting noise. This leads to a reduction of $CORR_1$ for small accounting noise, but this behavior may change for larger σ_n^2.

$CORR_1$ strictly increases in the precision of non-financial information (decreases in the variance σ_u^2), which is consistent with the behavior of EQ_1.

Finally, $CORR_1$ can strictly decrease or increase in a variation of the cost r of earnings management, depending on the parameters. The reason lies in the impact that the cost of smoothing has on the variances of the informative part of earnings $\tilde{W}_1(t)$ and reported total earnings $\tilde{M}_t$. To illustrate the possible effects, consider the boundary case $r = 0$ in which the manager's bias is determined only by smoothing considerations, so the manager chooses the maximum amount of smoothing (i.e., $Z = 0.5$). Here the manager incorporates $z(t)$ with the maximum weight into reported earnings, which has a strong impact on the variance of $\tilde{W}_1(t)$. An increase in r now dampens the weight of $z(t)$ and leads to a reduction of the earnings variance. At the same time, the higher cost also dampens the incentive to smooth the other risks, and depending on the relative size of these risks (accounting noise and operating risks), the ratio of the two variances in $CORR_1$ can increase or decrease. Note that EQ_1 always decreases in r, thus the behavior of the correlation coefficient differs from that of earnings quality.

A similar analysis of the second-period correlation $CORR_2$ yields similar results.

Taken together, comparing the behavior of $CORR_1$ and ERC_1 with that of earnings quality EQ_1 shows that neither of the two proxies perfectly traces EQ_1 in a simple price-earnings regression. ERC_1 is generally a more reliable proxy than $CORR_1$ because its behavior is better aligned with that of EQ_1 in more situations. We should note, though, that the correlation is often used in regressions with more than one independent variable, such as changes in earnings or net assets. In that case, the *ERC* cannot capture the total effect and may perform worse. Therefore, an empirical design choice faces a trade-off between these two effects.

5.3. Forward changes of parameters

The analysis so far has implicitly assumed that a change in one parameter is effective over the entire time the firm exists. For example, changing *k* essentially compares two different regimes, one in which *k* is unaffected and the other in which *k* is different. Such an analysis is helpful to assess the earnings quality of different accounting systems. In an empirical study, it is particularly applicable to international, cross-country studies.

An alternative research question is, what are the immediate effects of a change of a parameter effective at a certain date? In this case, the equilibrium strategies before that date are governed by the original parameter value, and afterwards they are based on the changed parameter value. Indeed, many empirical studies examine the effect of a change of the accounting regime in a jurisdiction. Formally, the effect of a change does not alter the past, but only the future, assuming that managers do not anticipate the change.

Note that the equilibrium effects of a variation of the parameters on earnings quality *EQ* are unaffected by this forward change because investors base their price reaction on the inferred working capital accruals $\tilde{W}_j(t)$, where

$$W_1(t) = y_1(t) + b_1 \text{ and } W_2(t) = y_1(t) + b_2.$$

Therefore, $\tilde{W}_j(t)$ captures only forward-looking information. Past effects are included in the fixed term α_t of the market pricing equation and do not affect earnings quality.

In contrast, value relevance is based on reported total earnings $\tilde{M}_t$, which aggregate earnings from two overlapping projects in the following way:

$$\tilde{M}_t = \tilde{y}_2\left(t-1\right) - b_2^* - I + \underbrace{\tilde{y}_1\left(t\right) + \tilde{b}_1^*}_{\tilde{W}_1(t)}.$$

A forward change of a parameter does not affect the distribution of $\tilde{y}_2(t-1)$ because it is governed by the previous parameter value, but it affects only the distribution of those parts of

$\tilde{W}_j(t)$ that are independent from $\tilde{y}_2(t-1)$. Therefore, the effects of a forward change on ERC_1 and $CORR_1$ differ from those presented in Propositions 5 and 6.

For the sake of brevity, we do not repeat the analysis for this situation and just note that some of the results are unchanged, others show even stronger deviations from those for EQ_1, and still others are mitigated. To illustrate the latter effect, consider the effect of an increase in accounting risk σ_n^2 on $CORR_1$. If only forward changes are considered, then both the numerator and the denominator of $CORR_1$ increase by the same amount, implying a strict increase in $CORR_1$. This eliminates the ambiguous effect recorded in Proposition 6 (ii) and aligns the behavior of $CORR_1$ with that of EQ_1 for a variation of σ_n^2.

6. Conclusions

This paper shows that some intuitively plausible effects of a change in accounting standards and the institutional financial reporting setting on earnings quality are not generally valid. In particular, we identify earnings smoothing incentives by management as crucial that earnings management incorporates some of the manager's private information. A main result of the analysis is that making accounting earnings more informative reduces earnings quality under certain conditions. Changing the information content also affects the weights with which financial and non-financial signals enter the bias, hence, a more informative accounting signal can drive out the non-financial information in reported earnings. Collectively, this change can generate lower-quality earnings in the capital market. Another main result is that an increase in the cost of earnings management (e.g., less discretion, higher audit and enforcement quality) reduces earnings quality. This change diminishes the ability of the bias, and reported earnings, to convey private information. We derive these effects in a rational expectations equilibrium that features a steady state firm with investment in every period, an accounting system, non-financial information, and earnings management.

We also examine how accurately value relevance measures, the earnings response coefficient and the correlation of a price-earnings regression, trace the changes in the information content of earnings induced by a variation of the characteristics. They perform well for a variation of some characteristics, but may lead to wrong conclusions for other characteristics. We also show that the effects differ whether one compares firms following different accounting standards or one studies a change in accounting standards for the same firms.

The model features several key ingredients of accounting standards, including the recognition of accruals and the precision of measurement in an ongoing firm. However, we use several simplifying assumptions to keep the analysis tractable, which may affect the generality of our findings. We emphasize three of them. One assumption is that, although we study a production setting with periodic investment decisions, we do not explicitly model decisions by management that affect the future cash flows (see Kanodia (2006) for such models). While the cost of earnings management is a “real” cost, for simplicity we assume this cost arises as private disutility of the manager and is not mirrored in the firm’s cash flows. Moreover, if investment affects future expected cash flows, as long as shareholders obtain sufficient information about this investment, our results would not qualitatively change. Our analysis is useful for broader models for at least three reasons: (i) It provides insights if the production effect is not of first-order interest; (ii) it is descriptive of situations in which the information affects production decisions later, if these decisions are not directly linked with the utility of the manager; and (iii) it examines a second step, after production decisions have been taken, perhaps even based on the anticipated reporting strategy.

Our analysis examines ceteris paribus changes of individual characteristics of the accounting system. For example, we assume the manager’s incentives are exogenously fixed and are not adjusted to a change in the respective characteristic. In a more general setting, incentive contracts would determine the weights endogenously. Further, the market discount factor may depend on the amount of information about the firm. Our analysis states that earnings smoothing, for whatever reason it obtains, can have unintended consequences in standard setting, and we believe that this result does not disappear in a more inclusive model.

Finally, we note that earnings management is informationally not harmful in our setting; investors are able to elicit the essential decision-useful information in equilibrium on average. It is only the cost of earnings management that makes it undesirable. This is consistent with the view that earnings management signals private information (which drives our results), while opportunistic elements are correctly recognized by investors in interpreting the earnings report. Extending the analysis to allow for additional economic forces provide fruitful opportunities for further research.

References

Christensen, P.O., G.A. Feltham, and F. Şabac (2005). A Contracting Perspective on Earnings Quality. *Journal of Accounting and Economics* 39: 265-294.

Christensen, J., and H. Frimor (2007). Fair Value, Accounting Aggregation and Multiple Sources of Information. In: R. Antle, F. Gjesdal, and P.J. Liang. *Essays in Accounting Theory in Honour of Joel S. Demski*, New York: Springer: 35-51.

Christensen, P.O., H. Frimor, and F. Şabac (2013). The Stewardship Role of Analyst Forecasts, and Discretionary Versus Non-discretionary Accruals. *European Accounting Review* 22: 257-296.

Cho, K., and D.M. Kreps (1987). Signaling Games and Stable Equilibria. *Quarterly Journal of Economics* 102: 179-221.

De Jong, A., G. Mertens, van der Poel, M., and van Dijk, R. (2012). How Does Earnings Management Influence Investors' Perceptions of Firm Value? Survey Evidence from Financial Analysts. Unpublished working paper, Erasmus University.

Dechow, P.M., W. Ge, and C. Schrand (2010). Understanding Earnings Quality: A Review of the Proxies, Their Determinants and Their Consequences. *Journal of Accounting and Economics* 50: 344-401.

Demski, J.S. (1998). Performance Measure Manipulation. *Contemporary Accounting Research* 15: 261-285.

Dichev, I., J. Graham, C.R. Harvey, and S. Rajgopal (2013). Earnings Quality: Evidence from the Field. *Journal of Accounting and Economics* (forthcoming).

Drymiotes, G., and T. Hemmer (2013). On the Stewardship and Valuation Implications of Accrual Accounting Systems. *Journal of Accounting Research* 51: 281-334.

Dye, R.A. (1988). Earnings Management in an Overlapping Generations Model. *Journal of Accounting Research* 26: 195-235.

Dye, R.A., and S.S. Sridhar (2004). Reliability-Relevance Trade-offs and the Efficiency of Aggregation. *Journal of Accounting Research* 42: 51-88.

Dutta, S. and S. Reichelstein (2005). Stock Price, Earnings, and Book Value in Managerial Performance Measures. *The Accounting Review* 80: 1069-1100.

Dutta, S. (2007). Dynamic Performance Measurement. *Foundations and Trends in Accounting* 2: 175-240.

Einhorn, E., and A. Ziv (2012). Biased Voluntary Disclosure. *Review of Accounting Studies* 17: 420-442..

Ewert, R., and A. Wagenhofer (2013). Earnings Quality Metrics and What They Measure. Working Paper, University of Graz.

Fischer, P.E., and P.C. Stocken (2004). Effect of Investor Speculation on Earnings Management. *Journal of Accounting Research* 42: 843-870.

Fischer, P.E., and R.E. Verrecchia (2000). Reporting Bias. *The Accounting Review* 75: 229-245.

Francis, J., P. Olsson, and K. Schipper (2006). Earnings Quality. *Foundations and Trends in Accounting* 1(4): 259-340 (1-85).

Graham, J.R., C.R. Harvey, and S. Rajgopal (2005). The Economic Implications of Corporate Financial Reporting. *Journal of Accounting and Economics* 40: 3-73.

Guttman, I., O. Kadan, and E. Kandel (2006). A Rational Expectations Theory of Kinks in Financial Reporting. *The Accounting Review* 81: 811-848.

Holthausen, R.W., and R.L. Watts (2001). The Relevance of the Value-relevance Literature for Financial Accounting Standard Setting. *Journal of Accounting and Economics* 31: 3-75.

Huson, M.R., Y. Tian, C.I. Wiedman, and H.A. Wier (2012). Compensation Committees' Treatment of Earnings Components in CEO's Terminal Years. *The Accounting Review* 87: 231-259.

Kanodia, C. (2006). Accounting Disclosure and Real Effects. *Foundations and Trends in Accounting* 1(3): 1-95.

Kirschenheiter, M., and N.D. Melumad (2002). Can "Big Bath" and Earnings Smoothing Co-exist as Equilibrium Reporting Strategies? *Journal of Accounting Research* 40: 761-796.

Marinovic, I. (2013). Internal Control System, Earnings Quality, and the Dynamics of Financial Reporting. *Rand Journal of Economics* 44: 145-167.

Sankar, M.R., and K.R. Subramanyam (2001). Reporting Discretion and Private Information Communication through Earnings. *Journal of Accounting Research* 39: 365-386.

Stocken, P.C., and R. Verrecchia (2004). Financial Reporting System Choice and Disclosure Management. *Review of Accounting Studies* 79: 1181-1203.

Trueman, B., and S. Titman (1988). An Explanation for Income Smoothing. *Journal of Accounting Research* 26, Supplement: 127-139.

Tucker, J.W., and P.A. Zarowin (2006). Does Income Smoothing Improve Earnings Informativeness? *The Accounting Review* 81: 251-270.

Appendix

Proof of proposition 3

The partial derivative of EQ_1 with respect to any parameter i has the following form:

$$\frac{\partial EQ_1}{\partial i} = \frac{\partial \beta_1}{\partial i}\operatorname{Cov}\left(\tilde{x}(t),\tilde{W}_1(t)\right) + \beta_1 \frac{\partial Cov\left(\tilde{x}(t),\tilde{W}_1(t)\right)}{\partial i}$$

$$= \frac{\dfrac{\partial \operatorname{Cov}\left(\tilde{x}(t),\tilde{W}_1(t)\right)}{\partial i}\operatorname{Var}\left(\tilde{W}_1(t)\right) - \operatorname{Cov}\left(\tilde{x}(t),\tilde{W}_1(t)\right)\dfrac{\partial \operatorname{Var}\left(\tilde{W}_1(t)\right)}{\partial i}}{\operatorname{Var}\left(\tilde{W}_1(t)\right)^2}\operatorname{Cov}\left(\tilde{x}(t),\tilde{W}_1(t)\right)$$

$$+ \beta_1 \frac{\partial \operatorname{Cov}\left(\tilde{x}(t),\tilde{W}_1(t)\right)}{\partial i}$$

$$= \frac{\beta_1}{\operatorname{Var}\left(\tilde{W}_1(t)\right)}\underbrace{\left(2\frac{\partial \operatorname{Cov}\left(\tilde{x}(t),\tilde{W}_1(t)\right)}{\partial i}\operatorname{Var}\left(\tilde{W}_1(t)\right) - \operatorname{Cov}\left(\tilde{x}(t),\tilde{W}_1(t)\right)\frac{\partial \operatorname{Var}\left(\tilde{W}_1(t)\right)}{\partial i}\right)}_{\equiv B} \qquad \text{(A1)}$$

Since β_1 and $\operatorname{Var}\left(\tilde{W}_1(t)\right)$ are positive, the sign of the derivative of EQ_1 is the same as the sign of B as defined in (A1). For the rest of the proof it useful to collect the following derivatives:

$$\frac{\partial Z}{\partial s} = 4rR^2, \quad \frac{\partial Z}{\partial r} = -4sR^2, \quad \frac{\partial (rR)}{\partial s} = -8rR^2, \quad \frac{\partial (rR)}{\partial r} = 8sR^2$$

$$\frac{\partial Q}{\partial s} = 4rR^2\left(\frac{k\sigma_\varepsilon^2}{k^2\sigma_\varepsilon^2 + \sigma_n^2}\right) - 8rR^2 = 4rR^2\left(\beta_2 - 2\right), \quad \frac{\partial Q}{\partial r} = -4sR^2\left(\frac{k\sigma_\varepsilon^2}{k^2\sigma_\varepsilon^2 + \sigma_n^2}\right) + 8sR^2 = 4sR^2\left(2 - \beta_2\right).$$

(i) Accrual parameter k

The two components of the partial derivative with respect to k are (using $Q' = \partial Q / \partial k$)

$$2\frac{\partial \operatorname{Cov}\left(\tilde{x}(t),\tilde{W}_1(t)\right)}{\partial k} = 2\left(Q'k\sigma_\varepsilon^2 + Q\sigma_\varepsilon^2\right)$$

$$\frac{\partial \operatorname{Var}\left(\tilde{W}_1(t)\right)}{\partial k} = 2QQ'\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + 2Q^2k\sigma_\varepsilon^2$$

Using these expressions,

$$B = 2\left(Q'k\sigma_\varepsilon^2 + Q\sigma_\varepsilon^2\right)\left(Q^2\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + Z^2H\sigma_\delta^2\right)$$
$$-\left(Qk\sigma_\varepsilon^2 + ZH\sigma_\delta^2\right)\left(2QQ'\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + 2Q^2k\sigma_\varepsilon^2\right).$$

Multiplying out, cancelling common terms and rearranging yields $B = B_1 + B_2$, where

$$B_1 = 2\sigma_\varepsilon^2\sigma_n^2 Q^3$$
$$B_2 = 2ZH\sigma_\delta^2\left(\sigma_\varepsilon^2\left(Q'k+Q\right)\left(Z-kQ\right)-QQ'\sigma_n^2\right)$$

We note that $B_1 > 0$. To determine the sign of B_2 we start with expanding the difference $Z - kQ$ in B_2 which yields:

$$Z-kQ = Z-k\left(Z\frac{k\sigma_\varepsilon^2}{k^2\sigma_\varepsilon^2+\sigma_n^2}+rR\right) = Z-Z\frac{k^2\sigma_\varepsilon^2}{k^2\sigma_\varepsilon^2+\sigma_n^2}-krR = Z\left(1-\beta_2 k\right)-krR$$

Next we expand $(Q'k+Q)$ and compute $QQ'\sigma_n^2$:

$$\begin{aligned}
Q'k+Q &= Z\frac{\partial\beta_2}{\partial k}k+Z\beta_2+rR = Zk\left(\frac{\sigma_\varepsilon^2\sigma_n^2-k^2\left(\sigma_\varepsilon^2\right)^2}{\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)^2}\right)+Z\beta_2+rR\\
&= Z\beta_2\frac{\sigma_n^2}{\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)}-Z\beta_2^2 k+Z\beta_2+rR = Z\beta_2\left(1-\beta_2 k-\beta_2 k\right)+Z\beta_2+rR\\
&= 2Z\beta_2\left(1-\beta_2 k\right)+rR>0
\end{aligned}$$

$$\begin{aligned}
Q\left[Q'\sigma_n^2\right] &= Q\left[\sigma_n^2 Z\left(\frac{\sigma_\varepsilon^2\sigma_n^2-k^2\left(\sigma_\varepsilon^2\right)^2}{\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)^2}\right)\right] = Q\left[Z\sigma_\varepsilon^2\left(\left(1-\beta_2 k\right)^2-\beta_2\left(1-\beta_2 k\right)k\right)\right]\\
&= \left(Z\beta_2+rR\right)\left[Z\sigma_\varepsilon^2\left(\left(1-\beta_2 k\right)^2-\beta_2\left(1-\beta_2 k\right)k\right)\right]\\
&= \sigma_\varepsilon^2\left(Z^2\beta_2\left(1-\beta_2 k\right)^2-Z^2\beta_2^2\left(1-\beta_2 k\right)k+\left(rR\right)Z\left(1-\beta_2 k\right)^2-\left(rR\right)Z\beta_2\left(1-\beta_2 k\right)k\right)\\
&= \sigma_\varepsilon^2\left(Z^2\beta_2\left(1-\beta_2 k\right)\left(1-2\beta_2 k\right)+\left(rR\right)Z\left(1-\beta_2 k\right)\left(1-2\beta_2 k\right)\right)
\end{aligned}$$

The difference of these terms is

$$\begin{aligned}
&\sigma_\varepsilon^2\left(Q'k+Q\right)\left(Z-kQ\right)-QQ'\sigma_n^2\\
&\quad= \sigma_\varepsilon^2\left(Z\beta_2\left(1-2\beta_2 k\right)+Z\beta_2+rR\right)\left(Z\left(1-\beta_2 k\right)-k\left(rR\right)\right)\\
&\quad -\sigma_\varepsilon^2\left(Z^2\beta_2\left(1-\beta_2 k\right)\left(1-2\beta_2 k\right)+\left(rR\right)Z\left(1-\beta_2 k\right)\left(1-2\beta_2 k\right)\right)\\
&\quad= \sigma_\varepsilon^2\begin{bmatrix} Z^2\beta_2\left(1-2\beta_2 k\right)\left(1-\beta_2 k\right)-k\left(rR\right)Z\beta_2\left(1-2\beta_2 k\right)\\ +Z^2\beta_2\left(1-\beta_2 k\right)-k\left(rR\right)Z\beta_2+\left(rR\right)Z\left(1-\beta_2 k\right)-k\left(rR\right)^2\\ -Z^2\beta_2\left(1-\beta_2 k\right)\left(1-2\beta_2 k\right)-\left(rR\right)Z\left(1-\beta_2 k\right)\left(1-2\beta_2 k\right)\end{bmatrix}\\
&\quad= \sigma_\varepsilon^2\begin{bmatrix} -2k\left(rR\right)Z\beta_2\left(1-\beta_2 k\right)+Z^2\beta_2\left(1-\beta_2 k\right)+\left(rR\right)Z\left(1-\beta_2 k\right)-k\left(rR\right)^2\\ -\left(rR\right)Z\left(1-\beta_2 k\right)\left(1-2\beta_2 k\right)\end{bmatrix}\\
&\quad= \sigma_\varepsilon^2\left(Z^2\beta_2\left(1-\beta_2 k\right)-k\left(rR\right)^2\right)
\end{aligned}$$

Taken together, B becomes:

$$B = B_1 + B_2 = 2\sigma_\varepsilon^2 \left(\sigma_n^2 Q^3 + ZH\sigma_\delta^2 \underbrace{\left(Z^2\beta_2\left(1-\beta_2 k\right) - k\left(rR\right)^2 \right)}_{\equiv C} \right)$$

If $C \geq 0$ then B is positive and EQ_1 increases unambiguously in k. Otherwise, B can always become negative because σ_δ^2 can be set high enough to result in $B < 0$. C is negative if

$$Z^2\beta_2\left(1-\beta_2 k\right) < k\left(rR\right)^2 \Leftrightarrow Z^2 \frac{k\sigma_\varepsilon^2\sigma_n^2}{\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)^2} < k\left(rR\right)^2$$

$$\Leftrightarrow \frac{4s}{r}\frac{\sigma_n}{\sigma_\varepsilon} - \frac{\sigma_n^2}{\sigma_\varepsilon^2} < k^2$$

Thus, there exists a threshold $\Gamma \geq 0$ such that $C < 0$ iff $k > \Gamma$, where

$$\Gamma = \begin{cases} 0 & \text{if } \frac{4s}{r}\frac{\sigma_n}{\sigma_\varepsilon} - \frac{\sigma_n^2}{\sigma_\varepsilon^2} < 0 \\ \sqrt{\left(\frac{4s}{r}\right)\frac{\sigma_n}{\sigma_\varepsilon} - \frac{\sigma_n^2}{\sigma_\varepsilon^2}} & \text{otherwise} \end{cases}$$

Notice that even if $\Gamma = 0$ and thus $C < 0$ for all positive k, the derivative of EQ_1 with respect to k is always positive for $k = 0$ because then $\beta_2 = 0$, $C = 0$, $Q = rR$ and therefore

$$B\left(k=0\right) = 2\sigma_\varepsilon^2\sigma_n^2\left(rR\right)^3 > 0.$$

Therefore, if the threshold $\Gamma \geq 1$, then EQ_1 is strictly increasing in k; if $\Gamma < 1$, then EQ_1 may reach a maximum with respect to k for some $k < 1$ depending on the parameters.

Proof of the special case of $\sigma_n^2 = 0$: If $\sigma_n^2 = 0$ then $B_1 = 0$, and

$$\beta_2\left(\sigma_n^2 = 0\right) = \frac{k\sigma_\varepsilon^2}{k^2\sigma_\varepsilon^2} = \frac{1}{k}$$

$$Q = Z\beta_2 + rR = \frac{Z}{k} + rR$$

$$\mathrm{Cov}\left(\tilde{x}(t), \tilde{W}_1(t) \middle| \sigma_n^2 = 0\right) = Qk\sigma_\varepsilon^2 + ZH\sigma_\delta^2 = Z\sigma_\varepsilon^2 + \left(rR\right)k\sigma_\varepsilon^2 + ZH\sigma_\delta^2$$

$$\mathrm{Var}\left(\tilde{W}_1(t) \middle| \sigma_n^2 = 0\right) = Q^2\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + Z^2H\sigma_\delta^2 = Z^2\sigma_\varepsilon^2 + 2Z\left(rR\right)k\sigma_\varepsilon^2 + \left(rR\right)^2k^2\sigma_\varepsilon^2 + Z^2H\sigma_\delta^2$$

The derivatives are

$$2\frac{\partial \operatorname{Cov}\left(\tilde{x}(t),\tilde{W}_1(t)\right)}{\partial k}=2rR\sigma_\varepsilon^2$$

$$\frac{\partial \operatorname{Var}\left(\tilde{W}_1(t)\right)}{\partial k}=2ZrR\sigma_\varepsilon^2+2\left(rR\right)^2 k\sigma_\varepsilon^2$$

Inserting $1-\beta_2 k=\dfrac{\sigma_n^2}{k^2\sigma_\varepsilon^2+\sigma_n^2}=0$ into B yields

$$B\left(\sigma_n^2=0\right)=-2Z\left(rR\right)^2 kH\sigma_\delta^2\sigma_\varepsilon^2<0 \text{ for } k>0 \ .$$

(ii) Variance σ_n^2

Next, we turn to the effect of σ_n^2 on EQ_1.

$$2\frac{\partial \operatorname{Cov}\left(\tilde{x}(t),\tilde{W}_1(t)\right)}{\partial \sigma_n^2}=2k\sigma_\varepsilon^2\frac{\partial Q}{\partial \sigma_n^2}=-2k\sigma_\varepsilon^2 Z\frac{\beta_2}{k^2\sigma_\varepsilon^2+\sigma_n^2}=-2Z\beta_2^2<0$$

Next rewrite the variance as

$$\begin{aligned}\operatorname{Var}\left(\tilde{W}_1(t)\right)&=Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2H\sigma_\delta^2\\&=Z^2\beta_2 k\sigma_\varepsilon^2+2ZrRk\sigma_\varepsilon^2+\left(rR\right)^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2H\sigma_\delta^2\end{aligned}$$

The derivative is

$$\frac{\partial \operatorname{Var}\left(\tilde{W}_1(t)\right)}{\partial \sigma_n^2}=Z^2k\sigma_\varepsilon^2\frac{\partial \beta_2}{\partial \sigma_n^2}+\left(rR\right)^2=-Z^2k\sigma_\varepsilon^2\frac{\beta_2}{k^2\sigma_\varepsilon^2+\sigma_n^2}+\left(rR\right)^2=-Z^2\beta_2^2+\left(rR\right)^2$$

Inserting these expressions in B yields

$$B=-2Z\beta_2^2\left(Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2H\sigma_\delta^2\right)-\left(\left(rR\right)^2-Z^2\beta_2^2\right)\left(Qk\sigma_\varepsilon^2+ZH\sigma_\delta^2\right)$$

The sign of B depends on the sign of the difference

$$\Delta=-2Z\beta_2^2Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2\beta_2^2Qk\sigma_\varepsilon^2$$

because the other terms in B are unambiguously negative.

$$\begin{aligned}\Delta&=Z\beta_2^2Q\left(Zk\sigma_\varepsilon^2-2Q\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)\right)\\&=Z\beta_2^2Q\left(Zk\sigma_\varepsilon^2-2Zk\sigma_\varepsilon^2-2\left(rR\right)\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)\right)\\&=Z\beta_2^2Q\left(-Zk\sigma_\varepsilon^2-2\left(rR\right)\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)\right)<0.\end{aligned}$$

Taken together, this proves $\dfrac{\partial EQ_1}{\partial \sigma_n^2}<0$.

(iii) Variance σ_u^2

The partial derivative of EQ_1 with respect to σ_u^2 uses the following derivatives:

$$2\frac{\partial \operatorname{Cov}\left(\tilde{x}(t),\tilde{W}_1(t)\right)}{\partial \sigma_u^2} = 2Z\sigma_\delta^2 \frac{\partial H}{\partial \sigma_u^2} = 2Z\sigma_\delta^2\left(-\frac{\sigma_\delta^2}{\left(\sigma_\delta^2+\sigma_u^2\right)^2}\right) = -2ZH^2$$

$$\frac{\partial \operatorname{Var}\left(\tilde{W}_1(t)\right)}{\partial \sigma_u^2} = Z^2\sigma_\delta^2 \frac{\partial H}{\partial \sigma_\delta^2} = Z^2\sigma_\delta^2\left(-\frac{\sigma_\delta^2}{\left(\sigma_\delta^2+\sigma_u^2\right)^2}\right) = -Z^2H^2$$

Inserting these expressions in B yields

$$\begin{aligned}
B &= -2ZH^2\left(Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2H\sigma_\delta^2\right)+Z^2H^2\left(Qk\sigma_\varepsilon^2+ZH\sigma_\delta^2\right)\\
&= H^2\left(-Z^3H\sigma_\delta^2-2ZQ^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2Qk\sigma_\varepsilon^2\right)\\
&= H^2\left(-Z^3H\sigma_\delta^2-2ZQ\left(Zk\sigma_\varepsilon^2+rR\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)\right)+Z^2Qk\sigma_\varepsilon^2\right)\\
&= H^2\left(-Z^3H\sigma_\delta^2-Z^2Qk\sigma_\varepsilon^2-2ZQ\left(rR\right)\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)\right)<0.
\end{aligned}$$

(iv) Cost r

The partial derivatives of Z and Q with respect to r and s are

$$\frac{\partial Z}{\partial r} = -\frac{s}{r}\frac{\partial Z}{\partial s} \text{ and } \frac{\partial Q}{\partial r} = -\frac{s}{r}\frac{\partial Q}{\partial s}.$$

Therefore, $\frac{\partial EQ_1}{\partial r} = -\frac{s}{r}\frac{\partial EQ_1}{\partial s}$. The partial derivative of EQ_1 with respect to s uses the following partial derivatives:

$$2\frac{\partial \operatorname{Cov}\left(\tilde{x}(t),\tilde{W}_1(t)\right)}{\partial s} = 2\left(4rR^2\left(\left(\beta_2-2\right)k\sigma_\varepsilon^2+H\sigma_\delta^2\right)\right) = 8rR^2\left(\left(\beta_2-2\right)k\sigma_\varepsilon^2+H\sigma_\delta^2\right)$$

$$\begin{aligned}
\frac{\partial \operatorname{Var}\left(\tilde{W}_1(t)\right)}{\partial s} &= 2Q4rR^2\left(\beta_2-2\right)\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+2Z4rR^2H\sigma_\delta^2\\
&= 8rR^2\left(Q(\beta_2-2)(k^2\sigma_\varepsilon^2+\sigma_n^2)+ZH\sigma_\delta^2\right).
\end{aligned}$$

Inserting into the expression for B yields

$$\begin{aligned}
B &= 8rR^2\left(\left(\beta_2-2\right)k\sigma_\varepsilon^2+H\sigma_\delta^2\right)\left(Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2H\sigma_\delta^2\right)\\
&\qquad -8rR^2\left(Q\left(\beta_2-2\right)\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+ZH\sigma_\delta^2\right)\left(Qk\sigma_\varepsilon^2+ZH\sigma_\delta^2\right)\\
&= 8rR^2\Big[\left(\beta_2-2\right)k\sigma_\varepsilon^2Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+\left(\beta_2-2\right)k\sigma_\varepsilon^2Z^2H\sigma_\delta^2\\
&\qquad +H\sigma_\delta^2Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2H^2\left(\sigma_\delta^2\right)^2-ZH\sigma_\delta^2Qk\sigma_\varepsilon^2-Z^2H^2\left(\sigma_\delta^2\right)^2\\
&\qquad -\left(\beta_2-2\right)k\sigma_\varepsilon^2Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)-Q\left(\beta_2-2\right)\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)ZH\sigma_\delta^2\Big]
\end{aligned}$$

and after rearranging

$$\begin{aligned} B &= 8r^2R^3\left(2-\beta_2\right)ZH\sigma_\delta^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+8r^2R^3QH\sigma_\delta^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right) \\ &= 8r^2R^3H\sigma_\delta^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)\left(\left(2-\beta_2\right)Z+Q\right) \\ &= 8r^2R^3H\sigma_\delta^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)>0. \end{aligned}$$

Therefore, $\dfrac{\partial EQ_1}{\partial s}>0$ and, hence, $\dfrac{\partial EQ_1}{\partial r}<0$, which completes the proof.

Q.E.D.

Proof of proposition 4

The proof follows the same steps as those used to prove Proposition 3. The sign of the partial derivative of EQ_1 and EQ_2 depends on the sign of the term B, which is defined in (A1).

(i) Operating risk σ_ε^2

The partial derivatives that determine B with respect to σ_ε^2 are

$$2\frac{\partial \operatorname{Cov}\left(\tilde{x}(t),\tilde{W}_1(t)\right)}{\partial\sigma_\varepsilon^2}=2k\left(Q'\sigma_\varepsilon^2+Q\right)$$

$$\frac{\partial \operatorname{Var}\left(\tilde{W}_1(t)\right)}{\partial\sigma_\varepsilon^2}=2QQ'\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Q^2k^2$$

where $Q'=\partial Q/\partial\sigma_\varepsilon^2$. Inserting into B yields

$$B=2k\left(Q'\sigma_\varepsilon^2+Q\right)\left(Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2H\sigma_\delta^2\right)-\left(2QQ'\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Q^2k^2\right)\left(Qk\sigma_\varepsilon^2+ZH\sigma_\delta^2\right)$$

After rearranging, B consists of three terms,

$$B=\underbrace{Q^3k\left(k^2\sigma_\varepsilon^2+2\sigma_n^2\right)}_{\equiv B_1}+\underbrace{kZH\sigma_\delta^2\left(Z^2\beta_2\left(2-\beta_2\right)-\left(rR\right)^2\right)}_{\equiv B_2}+\underbrace{2k\left(rR\right)Z^2H\sigma_\delta^2\beta_2\left(k-1\right)}_{\equiv B_3}.$$

B_1 is strictly positive, B_3 is non-positive (and strictly negative for $k<1$), and the sign of B_2 depends on the parameters.

To prove the proposition, consider first $\sigma_\varepsilon^2=0$. Then $\beta_2=0$ and

$$\begin{aligned} B(\sigma_\varepsilon^2=0) &= 2\left(rR\right)^3k\sigma_n^2-kZH\sigma_\delta^2\left(rR\right)^2 \\ &= k\left(rR\right)^2\left(2\left(rR\right)\sigma_n^2-ZH\sigma_\delta^2\right) \end{aligned}$$

This expression is negative if and only if

$$2(rR)\sigma_n^2 - ZH\sigma_\delta^2 < 0 \Leftrightarrow \left(\sigma_\delta^2\right)^2 - \frac{r}{2s}\sigma_n^2\sigma_\delta^2 - \frac{r}{2s}\sigma_n^2\sigma_u^2 > 0$$

It follows that, starting from $\sigma_\varepsilon^2 = 0$, an increase in σ_ε^2 first reduces EQ_1 provided $B(\sigma_\varepsilon^2 = 0) < 0$. However, inspection of B shows that B approaches infinity if $\sigma_\varepsilon^2 \to \infty$ because B_1 approaches infinity. Therefore, even if $B\left(\sigma_\varepsilon^2 = 0\right) < 0$ for a certain setting, the derivative must change signs for larger values of σ_ε^2.

To show that B changes signs only once, we show that each stationary point of EQ_1 with respect to σ_ε^2 is a minimum. The second derivative is

$$\frac{\partial^2 EQ_1}{\partial\left(\sigma_\varepsilon^2\right)^2} = \frac{\partial\left(\frac{\beta_1}{\operatorname{Var}\left(\tilde{W}_1(t)\right)}\underbrace{\left(2\frac{\partial \operatorname{Cov}\left(\tilde{x}(t),\tilde{W}_1(t)\right)}{\partial\sigma_\varepsilon^2}\operatorname{Var}\left(\tilde{W}_1(t)\right) - \operatorname{Cov}\left(\tilde{x}(t),\tilde{W}_1(t)\right)\frac{\partial \operatorname{Var}\left(\tilde{W}_1(t)\right)}{\partial\sigma_\varepsilon^2}\right)}_{\equiv B}\right)}{\partial\left(\sigma_\varepsilon^2\right)^2}$$

$$= \frac{\partial B}{\partial\sigma_\varepsilon^2}\frac{\beta_1}{\operatorname{Var}\left(\tilde{W}_1(t)\right)} + B\frac{\partial\left(\frac{\beta_1}{\operatorname{Var}\left(\tilde{W}_1(t)\right)}\right)}{\partial\sigma_\varepsilon^2}.$$

At a stationary point, $B = 0$, and it suffices to consider the sign of

$$\frac{\partial B}{\partial\sigma_\varepsilon^2} = 2\frac{\partial^2 \operatorname{Cov}\left(\tilde{x}(t),\tilde{W}_1(t)\right)}{\partial\left(\sigma_\varepsilon^2\right)^2}\operatorname{Var}\left(\tilde{W}_1(t)\right) - \operatorname{Cov}\left(\tilde{x}(t),\tilde{W}_1(t)\right)\frac{\partial^2 \operatorname{Var}\left(\tilde{W}_1(t)\right)}{\left(\partial\sigma_\varepsilon^2\right)^2}$$
$$+ \frac{\partial \operatorname{Cov}\left(\tilde{x}(t),\tilde{W}_1(t)\right)}{\partial\sigma_\varepsilon^2}\frac{\partial \operatorname{Var}\left(\tilde{W}_1(t)\right)}{\partial\sigma_\varepsilon^2}.$$

To sign this expression, it is helpful to rewrite the covariance and variance as follows:

$$\operatorname{Cov}\left(\tilde{x}(t),\tilde{W}_1(t)\right) = Qk\sigma_\varepsilon^2 + ZH\sigma_\delta^2 = Z\frac{k^2\sigma_\varepsilon^2}{k^2\sigma_\varepsilon^2 + \sigma_n^2}\sigma_\varepsilon^2 + rRk\sigma_\varepsilon^2 + ZH\sigma_\delta^2$$
$$= Z \cdot EQ_2 + rRk\sigma_\varepsilon^2 + ZH\sigma_\delta^2$$

$$\operatorname{Var}\left(\tilde{W}_1(t)\right) = Q^2\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + Z^2H\sigma_\delta^2$$
$$= Z^2\beta_2^2\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + 2ZrR\beta_2\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + (rR)^2\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + Z^2H\sigma_\delta^2$$
$$= Z^2\frac{k^2\sigma_\varepsilon^2}{\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right)}\sigma_\varepsilon^2 + 2ZrRk\sigma_\varepsilon^2 + (rR)^2\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + Z^2H\sigma_\delta^2$$
$$= Z^2 \cdot EQ_2 + 2ZrRk\sigma_\varepsilon^2 + (rR)^2\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + Z^2H\sigma_\delta^2$$

Let $EQ_2' = \partial EQ_2 / \partial \sigma_\varepsilon^2$ and $EQ_2'' = \partial^2 EQ_2 / \partial \left(\sigma_\varepsilon^2\right)^2$. Then

$$\frac{\partial \operatorname{Cov}\left(\tilde{x}(t), \tilde{W}_1(t)\right)}{\partial \sigma_\varepsilon^2} = Z \cdot EQ_2' + rRk > 0 \text{ and } \frac{\partial^2 \operatorname{Cov}\left(\tilde{x}(t), \tilde{W}_1(t)\right)}{\partial \left(\sigma_\varepsilon^2\right)^2} = Z \cdot EQ_2'' > 0$$

$$\frac{\partial \operatorname{Var}\left(\tilde{W}_1(t)\right)}{\partial \sigma_\varepsilon^2} = Z^2 \cdot EQ_2' + 2Z\left(rR\right)k + \left(rR\right)^2 k^2 > 0 \text{ and } \frac{\partial^2 \operatorname{Var}\left(\tilde{W}_1(t)\right)}{\partial \left(\sigma_\varepsilon^2\right)^2} = Z^2 \cdot EQ_2'' > 0$$

Inserting these terms results in

$$\begin{aligned}\Delta &= 2\frac{\partial^2 \operatorname{Cov}\left(\tilde{x}(t), \tilde{W}_1(t)\right)}{\partial \left(\sigma_\varepsilon^2\right)^2} \operatorname{Var}\left(\tilde{W}_1(t)\right) - \operatorname{Cov}\left(\tilde{x}(t), \tilde{W}_1(t)\right) \frac{\partial^2 \operatorname{Var}\left(\tilde{W}_1(t)\right)}{\left(\partial \sigma_\varepsilon^2\right)^2} \\ &= Z^3 \cdot EQ_2'' \cdot EQ_2 + 3Z^2 \cdot EQ_2'' \cdot rRk\sigma_\varepsilon^2 + 2Z \cdot EQ_2'' \cdot \left(rR\right)^2 \left(k^2\sigma_\varepsilon^2 + \sigma_\delta^2\right) + Z^3 \cdot EQ_2'' \cdot H\sigma_\delta^2 > 0.\end{aligned}$$

Hence, $\dfrac{\partial B}{\partial \sigma_\varepsilon^2} = \Delta + \dfrac{\partial \operatorname{Cov}\left(\tilde{x}(t), \tilde{W}_1(t)\right)}{\partial \sigma_\varepsilon^2} \dfrac{\partial \operatorname{Var}\left(\tilde{W}_1(t)\right)}{\sigma_\varepsilon^2} > 0$, which implies $\left.\dfrac{\partial^2 EQ_1}{\partial \left(\sigma_\varepsilon^2\right)^2}\right|_{\partial EQ_1/\partial \sigma_\varepsilon^2 = 0} > 0$.

This completes the proof of the effect of varying σ_ε^2 on EQ_1.

(ii) Operating risk σ_δ^2

To prove that EQ_1 increases in σ_δ^2, consider the partial derivatives in B,

$$2\frac{\partial \operatorname{Cov}\left(\tilde{x}(t), \tilde{W}_1(t)\right)}{\partial \sigma_\delta^2} = 2Z\left(H + H'\sigma_\delta^2\right)$$

$$\frac{\partial \operatorname{Var}\left(\tilde{W}_1(t)\right)}{\partial \sigma_\delta^2} = Z^2\left(H + H'\sigma_\delta^2\right)$$

where $H' = \partial H / \partial \sigma_\delta^2 > 0$. Inserting these expressions in B yields

$$\begin{aligned}B &= 2Z\left(H + H'\sigma_\delta^2\right)\left(Q^2\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + Z^2 H\sigma_\delta^2\right) - Z^2\left(H + H'\sigma_\delta^2\right)\left(Qk\sigma_\varepsilon^2 + ZH\sigma_\delta^2\right) \\ &= \left(H + H'\sigma_\delta^2\right)\left(Z^3 H\sigma_\delta^2 + 2ZQ^2\left(k^2\sigma_\varepsilon^2 + \sigma_\eta^2\right) - Z^2 Qk\sigma_\varepsilon^2\right) \\ &= \left(H + H'\sigma_\delta^2\right)\left(Z^3 H\sigma_\delta^2 + Z^2 Qk\sigma_\varepsilon^2 + 2ZQrR\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right)\right) > 0.\end{aligned}$$

Therefore, EQ_1 strictly increases in σ_δ^2.

(iii) Operating risk σ_ω^2

Finally, to prove that EQ_1 is unaffected by a change in σ_ω^2 note that

$$EQ_1 = \frac{\mathrm{Cov}\left(\tilde{x}(t), \tilde{W}_1(t)\right)^2}{\mathrm{Var}\left(\tilde{W}_1(t)\right)} = \frac{\left(Qk\sigma_\varepsilon^2 + ZH\sigma_\delta^2\right)^2}{Q^2\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + Z^2 H\sigma_\delta^2}.$$

is independent of σ_ω^2.

Q.E.D.

Proof of proposition 5

The proof follows the same steps as those used in the proof of Proposition 3. The general expression for the partial derivative of ERC_1 with respect to parameter i is

$$\frac{\partial ERC_1}{\partial i} = \frac{1}{\mathrm{Var}(\tilde{M}_t)^2}\underbrace{\left(\frac{\partial \mathrm{Cov}\left(\tilde{x}(t), \tilde{W}_1(t)\right)}{\partial i}\mathrm{Var}\left(\tilde{M}_t\right) - \mathrm{Cov}\left(\tilde{x}(t), \tilde{W}_1(t)\right)\frac{\partial \mathrm{Var}(\tilde{M}_t)}{\partial i}\right)}_{\equiv B} \quad \text{(A2)}$$

For brevity, we provide only a sketch of the main steps of the proof.[22]

(i) Accrual parameter k

$$\frac{\partial \mathrm{Cov}\left(\tilde{x}(t), \tilde{W}_1(t)\right)}{\partial k} = Q'k\sigma_\varepsilon^2 + Q\sigma_\varepsilon^2$$

$$\frac{\partial \mathrm{Var}\left(\tilde{M}_t\right)}{\partial k} = 2QQ'\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + 2Q^2 k\sigma_\varepsilon^2 - 2\left(1-Z\right)^2\left(1-k\right)\sigma_\varepsilon^2$$

B consists of several terms, some of which have positive and others negative signs. After several rearrangements, it turns out that a sufficient condition for $B > 0$ is

$$k \le \hat{k} = \sqrt{\frac{\sigma_n^2}{\sigma_\varepsilon^2}}.$$

If $k > \hat{k}$, then B can change signs. To see this, consider the limiting case with $\sigma_\delta^2 = \sigma_\omega^2 = 0$ and $k = 1$. If the accounting noise σ_n^2 is small enough, then B becomes negative. Since B is continuous, there is always a non-zero set of parameters such that B is negative.

[22] The full proof is available from the authors.

(ii) Variance σ_n^2

$$\frac{\partial \operatorname{Cov}\left(\tilde{x}(t), \tilde{W}_1(t)\right)}{\partial \sigma_n^2} = Q'k\sigma_\varepsilon^2$$

$$\frac{\partial \operatorname{Var}\left(\tilde{M}_t\right)}{\partial \sigma_n^2} = Q^2 + 2QQ'\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + \left(1 - Z\right)^2$$

Inserting these expressions in B gives

$$\begin{aligned} B &= Q'k\sigma_\varepsilon^2\left(Q^2\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + Z^2H\sigma_\delta^2 + \left(1-Z\right)^2\left(\sigma_\varepsilon^2\left(1-k\right)^2 + \sigma_n^2 + \sigma_\delta^2 + \sigma_\omega^2\right)\right) \\ &\quad - \left(Qk\sigma_\varepsilon^2 + ZH\sigma_\delta^2\right)\left(Q^2 + 2QQ'\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + \left(1-Z\right)^2\right) \\ &= -Z\beta_2^2 QrR\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) - Z\beta_2^2\left(1-Z\right)^2\left(\sigma_\varepsilon^2\left(1-k\right)^2 + \sigma_n^2 + \sigma_\delta^2 + \sigma_\omega^2\right) \\ &\quad - \left(Qk\sigma_\varepsilon^2 + Zh\sigma_\delta^2\right)\left(\left(rR\right)^2 + \left(1-Z\right)^2\right) < 0 \end{aligned}$$

Therefore, $\dfrac{\partial ERC_1}{\partial \sigma_n^2} < 0.$

(iii) Variance σ_u^2

$$\frac{\partial \operatorname{Cov}\left(\tilde{x}(t), \tilde{W}_t(t)\right)}{\partial \sigma_u^2} = -ZH^2$$

$$\frac{\partial \operatorname{Var}\left(\tilde{M}_t\right)}{\partial \sigma_u^2} = -Z^2H^2$$

Substituting these expressions in B gives

$$B = -ZH^2\left(1-Z\right)^2\left(\sigma_\varepsilon^2\left(1-k\right)^2 + \sigma_n^2 + \sigma_\delta^2 + \sigma_\omega^2\right) - ZH^2QrR\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) < 0.$$

Therefore, $\dfrac{\partial ERC_1}{\partial \sigma_u^2} < 0.$

(iv) Cost r

The sign of the partial derivative of ERC_1 with respect to r is the reverse of that of the derivative with respect to s because

$$\frac{\partial Z}{\partial r} = -\frac{s}{r}\frac{\partial Z}{\partial s}.$$

The derivative of the terms of B with respect to s results in

$$\frac{\partial \operatorname{Cov}\left(\tilde{x}(t), \tilde{W}_1(t)\right)}{\partial s} = Q'k\sigma_\varepsilon^2 + Z'H\sigma_\delta^2$$
$$\frac{\partial \operatorname{Var}\left(\tilde{M}_t\right)}{\partial s} = 2QQ'\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + 2ZZ'H\sigma_\delta^2 - 2\left(1-Z\right)Z'\left(\sigma_\varepsilon^2\left(1-k\right)^2 + \sigma_n^2 + \sigma_\delta^2 + \sigma_\omega^2\right)$$

Then

$$\begin{aligned} B = &\left(Q'k\sigma_\varepsilon^2 + Z'H\sigma_\delta^2\right)\left(Q^2\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + Z^2H\sigma_\delta^2 + \left(1-Z\right)^2\left(\sigma_\varepsilon^2\left(1-k\right)^2 + \sigma_n^2 + \sigma_\delta^2 + \sigma_\omega^2\right)\right) \\ &-\left(Qk\sigma_\varepsilon^2 + ZH\sigma_\delta^2\right)\left(2QQ'\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + 2ZZ'H\sigma_\delta^2 - 2\left(1-Z\right)Z'\left(\sigma_\varepsilon^2\left(1-k\right)^2 + \sigma_n^2 + \sigma_\delta^2 + \sigma_\omega^2\right)\right)\end{aligned}$$

One can prove that $B(s = 0)$ is nonnegative. For $s > 0$, if ERC_1 attains a stationary point, then it is a maximum, implying that the maximum is unique. That is, $\left.\frac{\partial^2 B}{\partial s^2}\right|_{\frac{\partial B}{\partial s}=0} < 0.$ Collecting these results proves that $\frac{\partial ERC_1}{\partial r} < 0$ for large r.

Q.E.D.

Proof of proposition 6

The covariance and variance terms in the definition of $CORR_1$ are

$$\operatorname{Cov}(\tilde{P}_t, \tilde{M}_t) = \beta_1 \operatorname{Var}\left(Q\tilde{y}_1\left(t\right) + ZH\tilde{z}\left(t\right)\right).$$

$$\begin{aligned}\operatorname{Var}\left(\tilde{M}_t\right) &= \operatorname{Var}\left(\tilde{y}_2\left(t-1\right) + \tilde{y}_1\left(t\right) + \tilde{b}_1^*\right) \\ &= \operatorname{Var}\left(Q\tilde{y}_1\left(t\right) + ZH\tilde{z}\left(t\right) + \left(1-Z\right)\tilde{y}_2\left(t-1\right)\right) \\ &= Q^2\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + Z^2H\sigma_\delta^2 + \left(1-Z\right)^2\left(\sigma_\varepsilon^2\left(1-k\right)^2 + \sigma_n^2 + \sigma_\delta^2 + \sigma_\omega^2\right) \\ &= \left(Q^2k^2 + \left(1-k\right)^2\left(1-Z\right)^2\right)\sigma_\varepsilon^2 + \left(Q^2 + \left(1-Z\right)^2\right)\sigma_n^2 + \left(Z^2H + \left(1-Z\right)^2\right)\sigma_\delta^2 + \left(1-Z\right)^2\sigma_\omega^2.\end{aligned}$$

Substituting into $CORR_1$ yields

$$\begin{aligned} CORR_1 &= \frac{\operatorname{Cov}(\tilde{P}_t, \tilde{M}_t)}{\operatorname{Std}(\tilde{P}_t)\operatorname{Std}(\tilde{M}_t)} \\ &= \frac{\beta_1 \operatorname{Var}\left(Q\tilde{y}_1\left(t\right) + ZH\tilde{z}\left(t\right)\right)}{\beta_1 \operatorname{Std}\left(Q\tilde{y}_1\left(t\right) + ZH\tilde{z}\left(t\right)\right)\operatorname{Std}\left(\tilde{M}_t\right)} = \sqrt{\frac{\operatorname{Var}\left(Q\tilde{y}_1\left(t\right) + ZH\tilde{z}\left(t\right)\right)}{\operatorname{Var}\left(\tilde{M}_t\right)}} \\ &= \sqrt{\frac{Q^2\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + Z^2H\sigma_\delta^2}{Q^2\left(k^2\sigma_\varepsilon^2 + \sigma_n^2\right) + Z^2H\sigma_\delta^2 + \left(1-Z\right)^2\left(\sigma_\varepsilon^2\left(1-k\right)^2 + \sigma_n^2 + \sigma_\delta^2 + \sigma_\omega^2\right)}}.\end{aligned}$$

(i) Accrual parameter k

For the ensuing analysis, it is sufficient to consider the term under the square root. Its derivative is

$$\frac{\partial\left(\frac{\operatorname{Var}\left(Q\tilde{y}_1(t)+ZH\tilde{z}(t)\right)}{\operatorname{Var}\left(\tilde{M}_t\right)}\right)}{\partial k} = \frac{1}{\operatorname{Var}\left(\tilde{M}_t\right)^2}\underbrace{\left(\frac{\partial\operatorname{Var}\left(Q\tilde{y}_1(t)+ZH\tilde{z}(t)\right)}{\partial k}\cdot\operatorname{Var}\left(\tilde{M}_t\right)-\operatorname{Var}\left(Q\tilde{y}_1(t)+ZH\tilde{z}(t)\right)\cdot\frac{\partial\operatorname{Var}\left(\tilde{M}_t\right)}{\partial k}\right)}_{\equiv B} \tag{A3}$$

where

$$\frac{\partial\operatorname{Var}\left(Q\tilde{y}_1(t)+ZH\tilde{z}(t)\right)}{\partial k} = 2QQ'\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+2Q^2k\sigma_\varepsilon^2$$

$$\frac{\partial\operatorname{Var}\left(\tilde{M}_t\right)}{\partial k} = 2QQ'\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+2Q^2k\sigma_\varepsilon^2-2(1-Z)^2(1-k)\sigma_\varepsilon^2$$

Substituting into B yields

$$\begin{aligned}B &= \left(2QQ'\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+2Q^2k\sigma_\varepsilon^2\right)\left(Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2H\sigma_\delta^2+(1-Z)^2\left(\sigma_\varepsilon^2(1-k)^2+\sigma_n^2+\sigma_\delta^2+\sigma_\omega^2\right)\right)\\ &\quad -\left(Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2H\sigma_\delta^2\right)\left(2QQ'\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+2Q^2k\sigma_\varepsilon^2-2(1-Z)^2(1-k)\sigma_\varepsilon^2\right)\\ &= 2Q\left(Q'\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Qk\sigma_\varepsilon^2\right)\left((1-Z)^2\left(\sigma_\varepsilon^2(1-k)^2+\sigma_n^2+\sigma_\delta^2+\sigma_\omega^2\right)\right)\\ &\quad +\left(Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2H\sigma_\delta^2\right)\left(2(1-Z)^2(1-k)\sigma_\varepsilon^2\right).\end{aligned}$$

The sign of B depends on the sign of $Q'\left(k^2\sigma_\varepsilon^2+\sigma_\eta^2\right)+Qk\sigma_\varepsilon^2$.

$$\begin{aligned}Q'\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Qk\sigma_\varepsilon^2 &= Z\frac{\sigma_\varepsilon^2\sigma_n^2-k^2\left(\sigma_\varepsilon^2\right)^2}{k^2\sigma_\varepsilon^2+\sigma_n^2}+\left(Z\frac{k\sigma_\varepsilon^2}{k^2\sigma_\varepsilon^2+\sigma_n^2}+rR\right)k\sigma_\varepsilon^2\\ &= Z\frac{\sigma_\varepsilon^2\sigma_n^2}{k^2\sigma_\varepsilon^2+\sigma_n^2}+rRk\sigma_\varepsilon^2>0.\end{aligned}$$

Therefore, $\frac{\partial CORR_1}{\partial k}>0$.

(ii) Variance σ_n^2

The partial derivative is constructed similar to (A3). The relevant terms in *B* are

$$\frac{\partial \operatorname{Var}\left(Q\tilde{y}_1(t)+ZH\tilde{z}(t)\right)}{\partial \sigma_n^2}=Q^2+2QQ'\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)=-Z^2\beta_2^2+\left(rR\right)^2$$

$$\frac{\partial \operatorname{Var}\left(\tilde{M}_t\right)}{\partial \sigma_n^2}=Q^2+2QQ'\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+\left(1-Z\right)^2$$

$$\begin{aligned}B=&\left(-Z^2\beta_2^2+\left(rR\right)^2\right)\left(Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2H\sigma_\delta^2+\left(1-Z\right)^2\left(\sigma_\varepsilon^2\left(1-k\right)^2+\sigma_n^2+\sigma_\delta^2+\sigma_\omega^2\right)\right)\\&-\left(Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2H\sigma_\delta^2\right)\left(Q^2+2QQ'\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+\left(1-Z\right)^2\right)\\=&\left(-Z^2\beta_2^2+\left(rR\right)^2\right)\left(\left(1-Z\right)^2\left(\sigma_\varepsilon^2\left(1-k\right)^2+\sigma_n^2+\sigma_\delta^2+\sigma_\omega^2\right)\right)-\left(Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2H\sigma_\delta^2\right)\left(1-Z\right)^2\end{aligned}$$

Further,

$$\begin{aligned}\frac{\partial B}{\partial \sigma_n^2}&=\left(1-Z\right)^2\left(-2Z^2\beta_2\frac{\partial \beta_2}{\partial \sigma_n^2}\left(\sigma_\varepsilon^2\left(1-k\right)^2+\sigma_n^2+\sigma_\delta^2+\sigma_\omega^2\right)+\left(\left(rR\right)^2-Z^2\beta_2^2\right)-\left(\left(rR\right)^2-Z^2\beta_2^2\right)\right)\\&=\left(1-Z\right)^2\left(-2Z^2\beta_2\frac{\partial \beta_2}{\partial \sigma_n^2}\left(\sigma_\varepsilon^2\left(1-k\right)^2+\sigma_n^2+\sigma_\delta^2+\sigma_\omega^2\right)\right)>0.\end{aligned}$$

Therefore, if $CORR_1$ increases in the accounting noise ($B > 0$) at some $\hat{\sigma}_n^2$, it increases for all $\sigma_n^2>\hat{\sigma}_n^2$; furthermore, if B achieves a stationary point, then it is a minimum.

Now consider the expression for B and observe that its sign depends on the difference $\left(rR\right)^2-Z^2\beta_2^2$. In particular, if $rR<Z\beta_2 \Leftrightarrow 1-2Z<Z\beta_2 \Leftrightarrow 1<Z\beta_2+2Z$, then $B < 0$. Inserting for β_2 yields

$$1<Z\frac{k\sigma_\varepsilon^2}{k^2\sigma_\varepsilon^2+\sigma_n^2}+2Z.$$

At $\sigma_n^2=0$ this reduces to $1<\frac{Z}{k}+2Z$, so for any $s > 0$ there exist $k<\frac{Z}{1-2Z}=\frac{4s}{r}$ such that $B < 0$.

(iii) Variance σ_u^2

The relevant terms for B (with derivatives taken over σ_u^2) are

$$\frac{\partial \operatorname{Var}\left(Q\tilde{y}_1(t)+ZH\tilde{z}(t)\right)}{\partial \sigma_u^2}=-Z^2\sigma_\delta^2\frac{\sigma_\delta^2}{\left(\sigma_\delta^2+\sigma_u^2\right)^2}=-Z^2H^2$$

$$\frac{\partial \operatorname{Var}\left(\tilde{M}_t\right)}{\partial \sigma_u^2}=-Z^2H^2$$

$$B = -Z^2H^2\left(Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2H\sigma_\delta^2+\left(1-Z\right)^2\left(\sigma_\varepsilon^2\left(1-k\right)^2+\sigma_n^2+\sigma_\delta^2+\sigma_\omega^2\right)\right)$$
$$+\left(Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2H\sigma_\delta^2\right)Z^2H^2$$
$$= -Z^2H^2\left(\left(1-Z\right)^2\left(\sigma_\varepsilon^2\left(1-k\right)^2+\sigma_n^2+\sigma_\delta^2+\sigma_\omega^2\right)\right)<0.$$

Therefore, $\frac{\partial CORR_1}{\partial \sigma_u^2}<0.$

(iv) Cost r

Using the fact that

$$\frac{\partial Z}{\partial r}=-\frac{s}{r}\frac{\partial Z}{\partial s},\frac{\partial\left(rR\right)}{\partial r}=-\frac{s}{r}\frac{\partial\left(rR\right)}{\partial s}\text{ and }\frac{\partial Q}{\partial r}=-\frac{s}{r}\frac{\partial Q}{s},$$

the sign of the partial derivative of $CORR_1$ with respect to r is simply the reverse of the derivative with respect to s. Therefore, the terms in B in the derivative with respect to s are

$$\frac{\partial\operatorname{Var}\left(Q\tilde{y}_1\left(t\right)+ZH\tilde{z}\left(t\right)\right)}{\partial s}=2QQ'\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+2ZZ'H\sigma_\delta^2$$

$$\frac{\partial\operatorname{Var}\left(\tilde{M}_t\right)}{\partial s}=2QQ'\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+2ZZ'H\sigma_\delta^2-2\left(1-Z\right)Z'\left(\sigma_\varepsilon^2\left(1-k\right)^2+\sigma_n^2+\sigma_\delta^2+\sigma_\omega^2\right)$$

$$B=\left(2QQ'\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+2ZZ'H\sigma_\delta^2\right)\left(Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2H\sigma_\delta^2+\left(1-Z\right)^2\left(\sigma_\varepsilon^2\left(1-k\right)^2+\sigma_n^2+\sigma_\delta^2+\sigma_\omega^2\right)\right)$$
$$-\left(Q^2\left(k^2\sigma_\varepsilon^2+\sigma_\eta^2\right)+Z^2H\sigma_\delta^2\right)\left(2QQ'\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+2ZZ'H\sigma_\delta^2-2\left(1-Z\right)Z'\left(\sigma_\varepsilon^2\left(1-k\right)^2+\sigma_n^2+\sigma_\delta^2+\sigma_\omega^2\right)\right)$$
$$=2\left(1-Z\right)4rR^2\left(\sigma_\varepsilon^2\left(1-k\right)^2+\sigma_n^2+\sigma_\delta^2+\sigma_\omega^2\right)\left[\left(1-Z\right)\left(\left(\beta_2-2\right)Q\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+ZH\sigma_\delta^2\right)\right.$$
$$\left.+\left(Q^2\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)+Z^2H\sigma_\delta^2\right)\right]$$

Rearranging yields

$$B=2\left(1-Z\right)4rR^2\left(\sigma_\varepsilon^2\left(1-k\right)^2+\sigma_n^2+\sigma_\delta^2+\sigma_\omega^2\right)\left(Q\left(k\left(1-k\right)\sigma_\varepsilon^2-\sigma_n^2\right)+ZH\sigma_\delta^2\right).$$

The sign of B depends on the sign of $Q\left(k\left(1-k\right)\sigma_\varepsilon^2-\sigma_n^2\right)$. If this term is greater zero, then $B>0$, otherwise B can become negative, which can occur particularly for large σ_n^2. Note that

$$\frac{\partial B}{\partial s}|_{B=0}=2\left(1-Z\right)8rR^2\left(\sigma_\varepsilon^2\left(1-k\right)^2+\sigma_n^2+\sigma_\delta^2+\sigma_\omega^2\right)\left(\left(k^2\sigma_\varepsilon^2+\sigma_n^2\right)\left(\beta_2-2\right)\left(\beta_2-1\right)+H\sigma_\delta^2\right)>0$$

if $\beta_2 < 1$. That is, B is then U-shaped and there can be regions where the $CORR_1$ increases or decreases. The same applies for a variation of r, but in the reverse direction.

Q.E.D.

Fig. 1: Sequence of events

This table describes the sequence of events over the tenure of a manager (*t* and *t*+1). *y* is the (unmanaged) earnings signal, *z* is non-financial information by the manager, *M* is the (total) reported earnings in a period, *P* is the market price of the firm, and *U* is the manager's utility. The variables are defined in detail in the text.

Begin of period t			End of period t
New manager is hired. Manager invests in project t.	Accounting system reports earnings $y_2(t-1)$ and $y_1(t)$ of projects $t-1$ and t. Manager obtains private information $z(t)$ about earnings $y_2(t)$ of project t.	Manager determines bias b_1 and issues earnings report M_t to the market.	Market price adjusts to P_t, contingent on financial report. Manager receives utility U_1.

Begin of period t+1			End of period t+1	Begin of period t+2
Manager invests in project t+1.	Accounting system reports earnings $y_2(t)$ and $y_1(t+1)$ of projects t and t+1 Manager obtains private information $z(t+1)$ about earnings $y_2(t+1)$ of project t+1.	Manager determines bias b_2 and issues earnings report M_{t+1} to the market.	Market price adjusts to P_{t+1}, contingent on financial report. Manager receives utility U_2. Manager leaves the firm.	New manager is hired. …

Fig. 2: The financial reports

This table depicts the cash flows, the reported earnings and end-of-period assets for each project and for the tenure of the incumbent manager. Note that it does not include cash flows from financing. *I* is the investment cost, *x* is the cash flow from the project, *m* is the reported earnings, *y* is the (unmanaged) earnings signal, *d* is depreciation, and *b* the bias in reported earnings. The variables are defined in detail in the text.

	Previous manager $t–1$	Current manager t	Current manager $t+1$	New manager $t+2$
Project $t–1$:				
Cash flows:	$–I$	$x(t–1)$		
Reported earnings $m_\tau(t–1)$:	$y_1(t–1) – d_1 + b_2(t–1)$	$y_2(t–1) – d_2 – b_2(t–1)$		
End-of-period assets $A_\tau(t–1)$:	$I + m_1(t–1)$	$x(t–1)$		
Project t:				
Cash flows:		$–I$	$x(t)$	
Reported earnings $m_\tau(t)$:		$y_1(t) – d_1 + b_1(t)$	$y_2(t) – d_2 – b_1(t)$	
End-of-period assets $A_\tau(t)$:		$I + m_1(t)$	$x(t)$	
Project $t+1$:				
Cash flows:			$–I$	$x(t+1)$
Reported earnings $m_\tau(t+1)$:			$y_1(t+1) – d_1 + b_2(t+1)$	$y_2(t+1) – d_2 – b_2(t+1)$
End-of-period assets $A_\tau(t+1)$:			$I + m_1(t+1)$	$x(t+1)$
Financial report for periods t and $t+1$:				
Cash flows:		$x(t–1) – I$	$x(t) – I$	
Reported earnings:		$M_t = m_2(t–1) + m_1(t)$	$M_{t+1} = m_2(t) + m_1(t+1)$	
End-of-period assets:		$x(t–1) + I + m_1(t)$	$x(t) + I + m_1(t+1)$	