UTS UNIVERSITY OF TECHNOLOGY SYDNEY

UTS Business School – Department of Economics
Dr Chau Chak Wing Building
14 – 28 Ultimo Road, Ultimo NSW 2007

T: +61 2 9514 7713
economics@uts.edu.au

Department of Economics
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'Efficiently Imprecise Contracts'

Toru Suzuki[1]

[1] University of Technology Sydney

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Efficiently Imprecise Contracts

Toru Suzuki[*†]

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Abstract

Actual contracts are often written in an imprecise manner. This paper introduces a formal writing cost framework in which the language of a contract, i.e., natural language, is explicitly modeled with predicate logic. It is shown that even if any obligation is contractible and describable by the language, the equilibrium contract can exhibit two kinds of impreciseness: (i) descriptive impreciseness, i.e., a contract leaves some relevant detail of the duty unmentioned, and (ii) semantic impreciseness, i.e., a contract uses some imprecise words leaving room for interpretation. Contractual impreciseness can persist even under a vanishingly small writing cost. Some novel comparative statics and other economic applications are provided.

Keywords: A foundation of incomplete contract, contractual impreciseness, writing cost, predicate logic

[*]University of Technology Sydney, PO Box 123, Broadway NSW 2007, Australia. Email address: toru.suzuki@uts.edu.au

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The limits of my language mean the limits of my world. Wittgenstein [1922]

1 Introduction

As Tirole [1999] points out, "many contracts are vague or silent on a number of key features." In fact, if actual contracts describe contractual duties precisely, the job description in most employment contracts should be longer than the computer program for a factory robot, and legal practitioners do not need to spend substantial time to interpret contract clauses. The purpose of this paper is to provide a formal writing cost framework that helps us to apprehend why actual contracts are imprecise and how an imprecise contract can function effectively.

The approach of the current paper is based on the simple observation; in practice, a principal needs to describe a contractual obligation with the help of natural language. The current paper then employs predicate logic, which captures some essence of natural language, to model contractual language. The explicit model of contractual language allows us to analyze how a contract can be written and interpreted rigorously. This paper provides two formal notions of contractual impreciseness: (i) descriptive impreciseness, i.e., a contract leaves some relevant detail of the duty unmentioned, and (ii) semantic impreciseness, i.e., a contract uses imprecise words leaving room for interpretation. It is shown that any equilibrium contract exhibits both kinds of contractual impreciseness if the service space, the set of predicates, and its semantic structure are rich enough. Moreover, on the contrary to typical writing cost models in which the non-standard feature disappears as the writing cost approaches zero, both kinds of impreciseness persist even under a vanishingly small writing cost. Furthermore, when the writing cost is high, the current model captures not only the distortion studied in the existing literature but also a new kind that is relevant to understand other economic phenomena.

In order to focus on the main idea, the current paper considers a simple contract problem in the deterministic environment.[1] There is a principal (she) and an agent (he). There is a set of elementary actions, and service is defined as a set of elementary actions. There is a conflict of interest between two parties, and the principal writes a contract to receive some service from the agent. The principal needs to describe a service obligation with a pre-existing language. Specifically, the principal describes a service by composing a compound predicate formula from a set of feasible predicates. Writing a contract with more predicates is assumed to be more costly. Moreover, to

[1] As will be shown in Section 5-1, the basic results can be preserved even if the model is extended to accommodate contingent contracts.

rule out contractual impreciseness caused by a contractual constraint, all actions are assumed to be contractible. There are two kinds of predicates: (i) elementary predicate, which describes a specific elementary action and (ii) conceptual predicate, which describes a set of elementary actions indirectly as a "concept." It is assumed that every action has the elementary predicate so that every obligation can be described precisely by a set of elementary predicates. A conceptual predicate has room for interpretation; specifically, there is a set of possible interpretations for conceptual predicates, and the meaning of a conceptual predicate is determined by choice of interpretation.

In this paper, the description of a contract is determined by the equilibrium of the following contract game. A contract consists of a transfer and a service description. The principal offers a contract to maximize her profit. The agent then decides whether to accept the contract, and he provides a service if he accepts the contract. The principal then decides whether to accept the provided service. If she accepts the service, the contractual relationship ends; if she rejects the service, the court enforces the service based on the most efficient interpretation among those that respect the description of the contract. This paper then analyzes the subgame perfect equilibrium of the contract game.

In this game, a service description does not bind the agent's behavior directly, but it affects the agent's behavior through the off-equilibrium enforcement by the court who respects the description. Knowing the court could enforce a service according to the description, the principal can describe a service *as if* she has direct control over the agent's behavior. Moreover, since there is no information asymmetry, the equilibrium transfer is chosen to extract all the surplus. Consequently, the principal's problem can be reduced to the efficient description problem. The efficient description problem can be quite complex since the solution depends not only on the economic environment but also on the set of available predicates. Nevertheless, some general properties of the equilibrium description can be obtained. It is shown that even though disjunction "or" is essential for a fuller description of a service, and negation "not" can be useful to clarify the meaning of conceptual predicates, any equilibrium description can be written as a logical and cost-equivalent predicate formula that uses neither disjunction nor negation. This result suggests that whenever the equilibrium description contains a conceptual predicate, the role of an elementary predicate is either (i) supplementation, which adds obligations that cannot be obliged under any interpretation of conceptual predicates or (ii) clarification, which manages the interpretation of conceptual predicates. In other words, an equilibrium contract uses conceptual predicates to describe the base of the equilibrium service, while elementary predicates adjust the detail.

The main interest of this paper is in contractual impreciseness. This paper introduces two formal notions of contractual impreciseness. The first kind is descriptive impreciseness; a service description is descriptively imprecise if there is more than one service that can satisfy the service description. Put differently, a descriptively imprecise contract leaves some relevant detail of the duty unmentioned as we often find in actual contracts. Impreciseness of the second kind is semantic; a semantically imprecise description does not fully clarify the meaning of some words, leaving room for interpretation. Given the fact that one of the major tasks for courts and contract lawyers is to interpret clauses, it is quite common for actual contracts to be semantically imprecise. Descriptive and semantic impreciseness are distinct concepts; a service description can be descriptively precise but semantically imprecise, and vice versa. When the writing cost is not negligible, both kinds of impreciseness can distort the economic outcome. First, the effect of descriptive impreciseness is straightforward; if some relevant actions are unmentioned because of the writing cost, the agent chooses not to perform those actions as he can still satisfy the contract. Second, the effect of semantic impreciseness is subtle; if the meaning of a conceptual predicate is not clarified because of the writing cost, the court's interpretation of the conceptual predicate can be different from that with some clarification clauses. Thus, the writing cost can affect the equilibrium service by influencing the court's interpretation of the conceptual predicate.

When writing is highly costly, it might not be so surprising to have a descriptively imprecise contract. Thus, a question is whether the equilibrium contract can exhibit impreciseness of the first kind under a small writing cost. Since the set of predicates is rich enough to write a precise contract in this paper, one might speculate that the equilibrium contract becomes descriptively precise under a sufficiently small writing cost. However, it is shown that whenever the service space is sufficiently rich, the equilibrium description is descriptively imprecise even under a vanishingly small writing cost. To illustrate the basic idea, consider a simple contract problem for a school. Suppose there are only three elementary actions: "giving lectures," "giving the final exam," and "having meetings with parents." All actions are beneficial for the school but costly for the teacher. The principal then describes a contractual duty with elementary predicates of the above three actions. If a contract states "the duty of this position is to give lectures and the final exam," the contract is descriptively imprecise; since the relevant action "meeting parents" is neither requested nor prohibited, the description provides discretion in this matter. Now, suppose that the wage that covers all three actions is too high for the principal given the benefit, and the school's optimal service is giving lectures and the final exam without meeting parents. Note if the school provides discretion in meeting parents, the teacher will choose not to meet parents. Then, as long as the writing cost is non zero, the school prefers the imprecise description "giving lectures and the exam" rather than the precise description that prohibits meetings explicitly. It can be shown that the basic idea of this example can be extended even if the principal can use conceptual predicates, and the choice of a description can affect how the court interprets conceptual predicates.

Turning to semantic impreciseness, note that while a conceptual predicate is inherently imprecise, a contract with a conceptual predicate can be semantically precise as long as the principal clarifies the meaning with elementary predicates. If the principal could clarify a description, one might imagine that a sufficiently low writing cost would eliminate semantic impreciseness. Nevertheless, it is shown that any equilibrium description exhibits semantic impreciseness when the service space, the set of conceptual predicates, and its semantic structure are rich enough. To see the basic idea, consider the earlier example with three elementary actions. Suppose, in addition to elementary predicates for those three actions, there is a conceptual predicate ".. is a teaching position." For simplicity, assume the concept "teaching position" has only two interpretations as a duty; "giving lectures and exams" and "giving lectures, giving exams, and meeting parents." If a contract states "this is a teaching position, and a teacher is expected to meet parents," the description is semantically precise; the obligation is the same under any interpretation of "teaching" since the conceptual predicate is clarified by the elementary predicate. Now, suppose that a principal who runs an elementary school writes a contract that states "this is a teaching position." Moreover, suppose that meeting parents is an essential service for elementary schools, and the court's socially efficient interpretation of "teaching position" is giving lectures and exams in addition to meeting parents. Then, as long as the writing cost is non-zero, the principal would save her writing cost by omitting the clarification clause "meeting parents" given the prospect of the court's interpretation. In the general setting, the equilibrium description can contain some clarification clauses for conceptual predicates. However, it can be shown that except for some pathological cases, the use of clarification never eliminates semantic impreciseness completely.

The current model also provides novel comparative statics. First, it is shown that, even though contractual impreciseness can persist under any non-zero writing cost, the equilibrium service coincides with the first best service under a sufficiently small writing cost. Second, while a higher writing cost reduces efficiency, the effect on the value of the equilibrium service is not always monotonic. Third, this paper also analyzes how a change in the set of available predicates can affect efficiency. It is found that while a larger set of predicates can enhance efficiency, a refinement of predicates can increase or decrease efficiency. Moreover, while having a set of conceptual predicates with precise definitions seems desirable, it is shown that there is no benefit of eliminating room for interpretation. Furthermore, when the contractual language is used to write contracts for various services, eliminating room for interpretation is socially inefficient.

The rest of this paper proceeds as follows. In Section 2, the model is introduced. Section 3 provides general properties of the equilibrium contract and analyzes contractual impreciseness. Some comparative statics are studied in Section 4. Section 5 discusses the extension to accommodate contingent contracts, the robustness of the main results, and how the current framework can be applied to study other economic phenomena. Section 6 concludes the paper.

Related literature. The current paper can be categorized into the literature on "foundations for incomplete contracts."[2] In particular, this paper contributes to the literature that treats writing costs as a foundation of contractual incompleteness.[3] [4] Dye [1985] introduces the simple writing cost model in which the writing cost is increasing in the number of contingencies. Anderlini and Felli [1994] propose an alternative approach based on the notion of computability. They show that the optimal contract is coarser than the first best if the selection procedure of a contract is required to be computable.[5] Battigalli and Maggi [2002] enrich Dye's approach by providing a foundation for the writing cost. Specifically, in order to define the writing cost of a contract rigorously, they model contractual language with propositional logic. The rich writing cost structure allows them to analyze whether to simplify or omit a certain contract clause. They then obtain two kinds of contractual incompleteness: rigidity and discretion. A contract with rigidity has a contingent instruction that is coarser than the first best, whereas a contract with discretion lacks instructions for some relevant actions.

Since the current paper models contractual language explicitly, this paper is closest to Battigalli and Maggi [2002]. However, the angle of the current paper is different from theirs; they analyze rigidity and discretion in a contingent instruction, whereas the current paper focuses on descriptive and semantic impreciseness of contractual obligation. While discretion and descriptive

[2]As Tirole [1999] puts it, "incomplete contracts are not members of a well-circumscribed family." While impreciseness is not a textbook example of incompleteness, Tirole [1999] uses "loosely described objectives" of ministries and agencies as an example of contractual incompleteness.

[3]Tirole [1999] lists writing costs as one of the major causes of contractual incompleteness and seeks a new framework; "while there is no arguing that writing down detailed contracts is very costly, we have no good paradigm in which to apprehend such costs."

[4]There are also other approaches to endogenous contractual incompleteness. Just to name a few, asymmetric information causes contractual incompleteness in Allen and Gale [1992] and Spier [1992]; Mukerji [1998] shows ambiguity aversion could make a contract rigid; Boot et al. [1993] and Bernheim and Whinston [1998] argue non-verifiability of some contingency/action could produce contractual incompleteness; Segal [1999] and Hart and Moore [1999] show that a null contract can be optimal in a setting that allows renegotiation. Lipman [1992], Tirole [2009], and Bolton and Faure-Grimaud [2010] explain contractual incompleteness based on bounded rationality.

[5]Anderlini and Felli [1999] extend their earlier approach by incorporating the complexity cost of computable contracts, and show that optimal contracts with complexity costs are constrained efficient.

impreciseness are closely related concepts, there is no room for semantic impreciseness in their model. In order to accommodate room for contractual interpretation, the current paper employs the language of predicate logic, which has a richer semantic structure than propositional logic. In the current paper, a contract affects the agent's behavior indirectly through the off-equilibrium enforcement by the court who interprets a contract. This is contrary to their model in which the agent's behavior is directly restricted by a precise contract. Regardless of the difference in the setting, the current model can also produce rigidity and discretion as will be shown in Section 5-1. Thus, the current model can be considered as a generalization of Battigalli and Maggi [2002].

The current paper also contributes to the literature that studies the role of contractual interpretation when writing a contract is costly. Posner [2004] argues that a contract writer's effort to make a contract less incomplete depends on how a court interprets gaps and ambiguous expressions. Shavell [2006] provides a model in which a court interprets an incomplete contract based on an interpretation rule. He then studies the optimal interpretation rule that maximizes welfare. Heller and Spiegler [2008] extend the idea of Shavell [2006] to investigate the use of contradiction in contracts. The current paper is also built on the idea that the principal, who foresees how a court interprets a contract, simplifies a contract to save the writing cost. However, the nature of interpretation in this paper is different from that in the existing literature. The role of interpretation in the existing literature is to specify the obligation for missing, coarse, or contradictory contingencies. Consequently, when the writing cost is small enough to write a complete contract, there is no role of interpretation. In contrast, the current paper considers the interpretation of predicates, i.e., "words," that describe an obligation given a context. Since the equilibrium contract can use conceptual predicates even under a vanishingly small writing cost, contractual interpretation always plays an essential role regardless of the level of the writing cost.

One of the typical concerns about the writing cost approach is that the writing cost, which is often considered as negligible by some economists, must be sufficiently high to produce a nonstandard property. The distinctive feature of the current model is that contractual impreciseness can persist even under a vanishingly small writing cost. Another appealing aspect of this paper is that contractual impreciseness can be obtained without any contractual constraint, i.e., any obligation is describable and contractible in the current model.

2 Model

2.1 Basics

There is a principal (she) and an agent (he). The principal wishes to receive a service from the agent. Let A be a finite set of **elementary actions**. A **service** consists of elementary actions. Formally, let $S \subset A$ be a service. A service S is feasible if, for any $a', a'' \in S$, a' and a'' are not mutually exclusive actions.[6] Let $\mathcal{S} \subset P(A)$ be the set of feasible services. The value of a service S for the principal is $v(S)$ where $v : P(A) \to [0, \infty)$ and $v(\emptyset) = 0$. The agent's cost of providing a service S is $c(S)$ where $c : P(A) \to [0, \infty)$ and $c(\emptyset) = 0$.[7]

There are some assumptions on v and c. The first assumption states that performing an additional elementary action is always costly.

Assumption 1 (Monotonicity) If $S'' \supsetneq S'$, then $c(S'') > c(S')$.

The next assumption states that if service S' is more (less) costly than service S'', then service S' is more (less) valuable than service S''. That is, as in the standard principal-agent model, there is a conflict of interest between two parties.

Assumption 2 (Conflict of interest) $c(S') \geq c(S'')$ if and only if $v(S') \geq v(S'')$.

Finally, assume that all services are strictly ranked in terms of cost and value. This assumption simplifies the exposition.

Assumption 3 (No indifference) If $S' \neq S''$, then $c(S') \neq c(S'')$ and $v(S') \neq v(S'')$.

The current setting accommodates various environments.

Example 1. Suppose the agent is a dry-cleaner. Then, an elementary action a is "dry-clean a," and a service S is dry-cleaning for a set of clothes S. Suppose that the value of dry-cleaning one item does not depend on whether other items are dry-cleaned or not. Let v_a be the value of dry-cleaning a, and let c_a be the cost of dry-cleaning a. Then, the value and the cost of S are

$$v(S) = \sum_{a \in S} v_a,$$

[6] For example, "start working from 8 am" and "start working from 9 am" are mutually exclusive actions.

[7] The domain of v and c are $P(A)$ instead of $\mathcal{S}$ for the following reason. If a' and a'' are mutually exclusive actions, there is no $S \in \mathcal{S}$ that contains both a' and a''. However, in the current model, there can be a situation in which a court requests the agent to perform a'' when the agent has already performed a' earlier. Thus, v and c need to be defined so that the value and the cost of a sequence of mutually exclusive actions can be evaluated.

$$c(S) = \sum_{a \in S} c_a.$$

In Example 1, the value of a service is additively separable. However, performing a certain combination of elementary actions can be crucial in some services.

Example 2. Consider an electronic product that consists of N components. Assume that a component n works properly only if all elementary actions in $A_n \subset A$ are performed by the agent. Let $v_n > 0$ be the value of a well-functioning component n. The value of a service S is then

$$v(S) = \sum_{n=1}^{N} v_n \mathbf{1}_{\{S':S' \supset A_n\}}(S) + v_0(S)$$

where $\mathbf{1}_{\{S':S' \supset A_n\}}(S)$ is an indicator function, and $v_0(S)$ is a strictly monotonic function.

The current setting can also accommodate the monotonic environment, which is common in the standard principal-agent model.

Example 3. Let $A = \{a_n\}_{n=1}^N$ be the set of feasible investment levels where $a_n \in [0, \infty)$ and $a_n < a_{n+1}$. Then, since two investment levels cannot be chosen at the same time, $\mathcal{S} = \{\emptyset, \{a_n\}\}_{n=1}^N$. That is, every $S \in \mathcal{S}$ is a singleton. Then, $v(S)$ is the return from S, which is increasing in a_n, whereas $c(S)$ is the cost from S, which is also increasing in a_n.

2.2 Language

Suppose there is no mutually recognized name or code for S with which the principal can directly refer to S and convey the exact content of S to the agent.[8] The principal then needs to *describe* S with a pre-existing language.[9] The current paper employs the language of predicate logic as the model of contractual language. There are three benefits to use predicate logic. First, a predicate can describe a property of a variable such as service S. Thus, when the obligation is described with predicate logic, we can systematically evaluate whether a provided service satisfies the description.[10] Second, most contracts are mainly written in natural language. Since a predicate

[8]For example, if the principal develops a code system that assigns a unique binary string with the length of $|A|$ to each $S \in \mathcal{S}$, she can directly refer to each service in $\mathcal{S}$ with the code system. However, unless the agent is already familiar with such an artificial language, the code itself cannot convey the content of the obligation to the agent.

[9]Rubinstein [1996] points out one of the major functions of natural language is to describe an object that has no mutually recognized name.

[10]In contrast, propositional logic, which directly deals with elementary statements as the input, is not equipped to evaluate a variable, i.e., it is zeroth-order logic.

describes an object analogously to natural language, the number of predicates in a contract is a good proxy for the writing cost. Finally, the rich semantic structure of predicate logic allows us to model contractual interpretation formally.[11]

Let Φ be a finite set of feasible predicates. A **predicate** $\phi \in \Phi$ describes a service the principal wishes to receive from the agent. Formally, a predicate ϕ is a Boolean-valued function $\phi : \mathcal{S} \to \{0,1\}$ where $\phi(S) = 1(0)$ can be read as "the service S does (not) have the property ϕ" or " the service S does (not) satisfy the description ϕ." Since the current paper wishes to accommodate room for contractual interpretation by a court, the set of predicates is endowed with a set of possible interpretations. Formally, let Θ be the finite set of **interpretation types**. Then, let $\phi(S;\theta)$ be a predicate ϕ with an interpretation type θ.[12] Specifically,

$$\phi(S;\theta) = \begin{cases} 1 & \text{if } S \supset S_\phi(\theta) \\ 0 & \text{if } S \not\supset S_\phi(\theta) \end{cases}$$

where $S_\phi(\theta) \subset A$ is the set of elementary actions that are essential to satisfy ϕ under θ. It might be worth clarifying that even though θ is called a type, it is not a trait or characteristics of a player; θ is called an interpretation type since it determines not only the interpretation of a particular predicate but also that of other predicates.[13]

It is convenient to categorize predicates into two classes. An **elementary predicate** describes a specific elementary action a in A. An elementary predicate thus has no room for interpretation. Formally, an elementary predicate, denoted by ϕ_a, has $S_{\phi_a}(\theta) = \{a\}$ for all θ. Any other predicate is a **conceptual predicate**, which describes a set of elementary actions indirectly as a "concept." Since specific elements are not described directly, a conceptual predicate can have room for interpretation. Formally, a conceptual predicate, denoted by ϕ_m, has $S_{\phi_m}(\theta)$ such that $|S_{\phi_m}(\theta)| > 1$ for some θ. Let M be a set of existing concepts.[14]

Example 4. Suppose $A = \{a', a''\}$ where a' is the action "giving lectures" and a'' is the action "giv-

[11]Propositional logic treats elementary statements, whose truth-values are predetermined, as the inputs. Thus, the semantic structure is not rich enough to deal with multiple interpretations of a clause formally.

[12]Formally, an interpretation type θ determines the *structure* $\langle \mathcal{S}, \{\neg, \vee, \wedge\}, \{\phi(.;\theta)\}_{\phi\in\Phi} \rangle$, which defines the semantics of predicates. For more details about the concept of structure in predicate logic, see Chiswell and Hodges [2007].

[13]As we will see later, the court's interpretation of a contract is modeled as the choice of θ.

[14]While formal concepts in mathematics have the precise definitions, most concepts written in natural language lack precise definitions. This paper considers concepts in the latter sense. For comprehensive discussions on the notion of concepts, see Margolis and Laurence [1999].

ing exams." Let $\phi_{a'}$ and $\phi_{a''}$ be the elementary predicates. There is a conceptual predicate $\phi_{teaching}$, and suppose there are only two interpretation types $\Theta = \{\theta_1, \theta_2\}$. Suppose that $S_{\phi_{teaching}}(\theta_1) = \{a'\}$ and $S_{\phi_{teaching}}(\theta_2) = \{a', a''\}$.

Suppose that the agent provides $\{a'\}$. Whether the service satisfies an elementary predicate does not depend on θ.

$$\phi_{a'}(\{a'\}; \theta_1) = \phi_{a'}(\{a'\}; \theta_2) = 1,$$

$$\phi_{a''}(\{a'\}; \theta_1) = \phi_{a''}(\{a'\}; \theta_2) = 0.$$

In contrast,

$$\phi_{teaching}(\{a'\}; \theta_1) = 1, \phi_{teaching}(\{a'\}; \theta_2) = 0.$$

That is, when the agent only gives lectures, he satisfies the service description "teaching" under the interpretation type θ_1, whereas he does not satisfy the description "teaching" under the interpretation type θ_2.

Suppose that every elementary action has an elementary predicate. The set of available predicates is then

$$\Phi = \{\phi_a\}_{a \in A} \cup \{\phi_m\}_{m \in M}.$$

The principal can compose a compound predicate ψ by combining predicates in Φ with the help of logical connectives $\neg$ "not," $\wedge$ "and," and $\vee$ "or." Let $\Psi(\Phi)$ be the set of all predicate formulas that can be composed by combining predicates in Φ with connectives $\{\neg, \wedge, \vee\}$.[15] The value of $\psi(S; \theta)$ is then deductively determined by the truth table given θ.[16] We say a service S **satisfies** a service description ψ under θ if $\psi(S; \theta) = 1$. Note that since $\phi_a \in \Phi$ for all $a \in A$, the set of predicates Φ is rich enough to describe any service in $\mathcal{S}$.

Example 5. Let $A = \{a', a'', a'''\}$ and $\mathcal{S} = P(A)$. First, consider a service description

$$\psi' = \phi_{a'} \wedge \phi_{a''}.$$

If the agent provides the service $\{a', a''\}$,

[15] In this paper, the principal does not use $\longleftrightarrow$ "bi-conditional" and $\rightarrow$ "implication." However, the current setting is without loss of generality; it is known that any formula with bi-conditional and implication can be written only with $\{\neg, \wedge, \vee\}$, and, there is no benefit of using bi-conditional and implication in the current setting.

[16] (i) Negation: $\neg\phi(S; \theta) = 1$ iff $\phi(S; \theta) = 0$. (ii) Conjunction: Let $\psi' = \phi' \wedge \phi''$. Then, $\psi'(S; \theta) = 1$ iff $\phi'(S; \theta) = \phi''(S; \theta) = 1$. (iii) Disjunction: Let $\psi'' = \phi' \vee \phi''$. Then, $\psi''(S; \theta) = 1$ iff it is at least $\phi'(S; \theta) = 1$ or $\phi''(S; \theta) = 1$.

$$\phi_{a'}(\{a', a''\}; \theta) = \phi_{a''}(\{a', a''\}; \theta) = 1.$$

for all θ. Thus, $\psi'(\{a', a''\}; \theta) = 1$ for all θ. Similarly, if the agent provides the service $\{a', a'', a'''\}$, then $\psi'(\{a', a'', a'''\}; \theta) = 1$ for all θ. In contrast, if the agent provides the service $\{a'\}$,

$$\phi_{a'}(\{a'\}; \theta) = 1, \phi_{a''}(\{a'\}; \theta) = 0$$

for all θ. Thus, $\psi'(\{a'\}; \theta) = 0$ for all θ.

Second, consider the following description

$$\psi'' = (\phi_{a'} \vee \phi_{a''}) \wedge \neg \phi_{a'''}.$$

If the agent provides $\{a'\}$,

$$\phi_{a'}(\{a'\}; \theta) = \neg \phi_{a'''}(\{a'\}; \theta) = 1, \phi_{a''}(\{a'\}; \theta) = 0.$$

for all θ. Thus, $\psi''(\{a'\}; \theta) = 1$ for all θ. Similarly, $\psi''(\{a''\}; \theta) = 1$ and $\psi''(\{a', a''\}; \theta) = 1$ for all θ. In contrast, if the agent provides $\{a', a'''\}$, then

$$\phi_{a'}(\{a', a'''\}; \theta) = 1, \phi_{a''}(\{a', a'''\}; \theta) = \neg \phi_{a'''}(\{a', a'''\}; \theta) = 0.$$

for all θ. Thus, $\psi''(\{a', a'''\}; \theta) = 0$ for all θ. Similarly, $\psi''(\{a'', a'''\}; \theta) = 0$ and $\psi''(\{a', a'', a'''\}; \theta) = 0$ for all θ.

In this paper, writing is treated as a costly economic activity. Since the number of predicates in ψ is a good proxy for the length of writing in a contract, the current paper assumes that a description ψ with more predicates is more costly. Formally, let $\kappa(\psi)$ be the writing cost of ψ, and let $n(\psi)$ be the number of predicates in ψ.

Assumption 4. *For any ψ such that $n(\psi) > 0$, $\kappa(\psi) > 0$. Moreover, $\kappa(\psi') > \kappa(\psi'')$ if $n(\psi') > n(\psi'')$.*

The idea that the cost of writing is increasing in the number of formulas is the same as that in Battigalli and Maggi [2002]. However, there is a notable difference between the two models. Since Battigalli and Maggi [2002] use propositional logic that describes each elementary action, the writing cost is always higher when the principal requests a larger number of actions. In contrast, requesting a larger number of actions does not always increase the writing cost in the current paper; if a certain set of actions can be described by one conceptual predicate but a smaller set of actions have no conceptual predicate, describing the former set of actions can be cheaper than describing the latter. Put differently, when one describes some standard service that already has an established conceptual predicate, describing the standard service can be cheaper than a novel service that consists of a smaller number of actions but has no established conceptual predicate.

The specification of the writing cost in this paper is consistent with the assumption that the principal uses a preexisting language to write a contract. To see this, suppose that the principal tries to introduce a new conceptual predicate, which is not in Φ. Since it is not in Φ, the new predicate needs to be defined in a contract. However, since defining a new predicate requires the principal to describe the set of elementary actions explicitly, the cost of introducing a new conceptual predicate is at least as costly as describing those elementary predicates. Consequently, the principal has no incentive to introduce a new conceptual predicate in the current setting.

2.3 Contract game

A **contract** is a pair of a service description $\psi \in \Psi(\Phi)$ and a transfer $t \in T = [0, \infty)$.[17] In the current paper, the principal's choice of a contract is determined by the equilibrium of the following sequential game.

Period-1 The principal offers a contract (ψ, t). The agent then chooses whether to accept the offer. Let $x \in X = \{0, 1\}$ be the agent's choice where $x = 1$ is "accept" and $x = 0$ is "reject." Formally, the agent's **acceptance strategy** is $\rho(\psi, t)$ where

$$\rho : \Psi \times T \to X.$$

If the agent rejects the offer, the game ends. Then, since the principal has already written ψ, her payoff is $-\kappa(\psi)$, whereas the agent gets his reservation payoff 0. If the agent accepts the offer, the game proceeds to the next period.

Period-2 Given (ψ, t), the agent provides a service $S \in \mathcal{S}$. Formally, the agent's **service provision strategy** is $f(\psi, t)$ where

$$f : \Psi \times T \to \mathcal{S}.$$

[17] If we consider transfers as the principal's actions and add elementary predicates for transfers, a transfer can also be described as a part of contractual obligations. However, since such an extension adds almost no insight, we treat t separately.

The principal then chooses whether to accept the provided service S as a valid service. Let $y \in Y = \{0, 1\}$ be the principal's acceptance decision; $y = 1$ if she accepts S as a valid service, whereas $y = 0$ if she rejects S as an invalid service. The principal's **acceptance strategy** is $\sigma(S)$ where

$$\sigma : \mathcal{S} \to Y.$$

If $y = 1$, the game ends. The principal's payoff is then $v(S) - t - \kappa(\psi)$, whereas the agent's payoff is $t - c(S)$. If $y = 0$, the principal requests the court to enforce the contract. The game then proceeds to the next period.

Period-3 The court interprets the description ψ and requests the agent to provide the service based on the interpretation type θ_ψ. Specifically, let $S_\psi(\theta) \in \mathcal{S}$ be the most economical service that satisfies ψ under θ.[18] That is,

$$S_\psi(\theta) \in \arg \min_{S \in \{S' : \psi(S';\theta)=1\}} c(S).$$

Then, θ_ψ solves

$$\max_{\theta \in \Theta} \{v(S_\psi(\theta)) - c(S_\psi(\theta))\}.$$

That is, the court interprets ψ so that $S_\psi(\theta)$ is socially efficient given ψ. Given a service S provided in period 2, the court orders the agent to provide the unfulfilled obligation $S_\psi(\theta_\psi) \backslash S$. Thus, given a service S provided in period 2, the principal's payoff from the enforcement is $v(S_\psi(\theta_\psi) \cup S) - t - \kappa(\psi)$. The agent incurs an additional cost $\gamma > 0$ if $y = 0$.[19] Then, given a service S provided in period 2, the agent's payoff from the enforcement is $t - c(S_\psi(\theta_\psi) \cup S) - \gamma$.

The current setting of contractual interpretation accommodates two major aspects of contractual interpretation in practice. First, contractual interpretation exhibits a certain degree of textualism; even if a contract can be interpreted by a court, the explicit writing determines the major content of a contract. This aspect is captured by the current setting that the court can enforce a service S only if S can be provided by the agent who respects ψ under some θ. That is, if ψ has no predicate that describes a' under any θ, a' is never requested by the court. The second aspect is contextualism; contractual interpretation takes into account the commercial context of a contract. In the current setting, the court's interpretation of ψ depends on v and c, which characterizes a

[18] $S_\psi(\theta)$ is equivalent to the service that the agent would provide if he could respect ψ according to θ.

[19] Needless to say, an actual legal procedure can be much more complex and various across countries. The current model aims to capture the basic idea that there is some penalty from a breach of contract.

commercial context.[20]

This paper analyzes the **subgame perfect equilibrium (SPE)** of the contract game. Formally, let $U_1(\psi, t, y, x, S)$ be the principal's expected payoff from (ψ, t, y, x, S), and let $U_2(\psi, t, y, x, S)$ be the agent's expected payoff from (ψ, t, y, x, S). A strategy profile $(\psi^*, t^*, \sigma^*, \rho^*, f^*)$ is an SPE if the following conditions are satisfied:

(i) the principal optimally chooses whether to accept S given $(\psi, t, 1, S)$;

$$\sigma^*(S) \in \arg\max_{y \in Y} U_1(\psi, t, y, 1, S)$$

(ii) the agent optimally provides a service S given $(\psi, t, \sigma^*(S), 1)$;

$$f^*(\psi, t) \in \arg\max_{S \in \mathcal{S}} U_2(\psi, t, \sigma^*(S), 1, S)$$

(iii) the agent optimally decides whether to accept an offer (ψ, t) given $(\psi, t, \sigma^*(f^*(\psi, t)), f^*(\psi, t))$;

$$\rho^*(\psi, t) \in \arg\max_{x \in X} U_2(\psi, t, \sigma^*(f^*(\psi, t)), x, f^*(\psi, t))$$

(iv) the principal optimally chooses a contract given $(\sigma^*(f^*(\psi, t)), \rho^*(\psi, t), f^*(\psi, t))$;

$$(\psi^*, t^*) \in \arg\max_{(\psi, t) \in \Psi \times T} U_1(\psi, t, \sigma^*(f^*(\psi, t)), \rho^*(\psi, t), f^*(\psi, t)).$$

3 Analysis

This section provides the properties of the equilibrium contract and the analysis of contractual impreciseness.

3.1 Preliminary analysis

In period 3, neither the principal nor the agent has a decision to make. Thus, consider the principal's problem in period 2. Given a history $(\psi, t, 1, S)$, the principal's expected payoff from accepting S is

$$v(S) - t - \kappa(\psi),$$

[20] See Gilson et al. [2014] for a comprehensive discussion on contextualism and textualism.

whereas her expected payoff from rejecting S is

$$v(S_\psi(\theta_\psi) \cup S) - t - \kappa(\psi).$$

Thus, the principal's optimal acceptance strategy is

$$\sigma^*(\psi, t, 1, S) = \begin{cases} 1 & \text{if } v(S_\psi(\theta_\psi) \cup S) \leq v(S) \\ 0 & \text{if } v(S_\psi(\theta_\psi) \cup S) > v(S) \end{cases}.$$

Observe that whenever the game reaches period 2, the agent accepts a contract in period 1, i.e., $x = 1$. Moreover, the principal's optimal acceptance strategy $\sigma^*(\psi, t, 1, S)$ is constant in t. Thus, henceforth, we write the optimal acceptance rule as $\sigma^*(\psi, S)$ instead of $\sigma^*(\psi, t, 1, S)$.

Turning to the agent's service provision problem in period 2, if the agent wishes to be accepted by the principal, the agent's optimal service solves

$$\max_{S \in \{S' : \sigma^*(\psi, S') = 1\}} \{t - c(S)\}.$$

That is, the agent provides the most economical acceptable service given σ^*. Let $\zeta(\psi)$ be the solution of the above problem, which is unique from Assumption 3. If the agent wishes to be rejected by the principal, the agent's optimal service solves

$$\max_{S \in \{S' : \sigma^*(\psi, S') = 0\}} \{t - c(S_\psi(\theta_\psi) \cup S) - \gamma\}.$$

Let $\xi(\psi)$ be a solution of the above problem. Then, the agent's optimal service provision strategy is

$$f^*(\psi, t, 1) = \begin{cases} \zeta(\psi) & \text{if } c(\zeta(\psi)) \leq c(S_\psi(\theta_\psi) \cup \xi(\psi)) + \gamma \\ \xi(\psi) & \text{if } c(\zeta(\psi)) > c(S_\psi(\theta_\psi) \cup \xi(\psi)) + \gamma \end{cases}.$$

Lemma 1. $f^*(\psi, t, 1) = S_\psi(\theta_\psi)$.

Lemma 1 states that the agent always chooses the most economical service that satisfies a description ψ under the interpretation type θ_ψ. Henceforth, we write the optimal service provision rule as $f^*(\psi)$ instead of $f^*(\psi, t, 1)$.

To state the next lemma, let

$$\Psi^- = \{\psi \in \Psi : v(S_\psi(\theta_\psi)) - c(S_\psi(\theta_\psi)) < 0\}.$$

Lemma 2. *Given $\psi \in \Psi \backslash \Psi^-$, the optimal transfer level is $c(S_\psi(\theta_\psi))$. Given $\psi \in \Psi^-$, the optimal transfer level is any value in $[0, c(S_\psi(\theta_\psi)))$.*

Lemma 2 states that the principal extracts the entire surplus whenever a description ψ induces a service that generates a positive surplus.

The following result states that the equilibrium service description can be obtained as a solution of the efficient description problem.

Proposition 1. *Any equilibrium description ψ^* solves*

$$\max_{\psi \in \Psi} \{v(S_\psi(\theta_\psi)) - c(S_\psi(\theta_\psi)) - \kappa(\psi)\}.$$

Since there can be more than one efficient description, the uniqueness of the equilibrium is not guaranteed. However, all equilibria are payoff-equivalent.

3.2 Properties of equilibrium contract

The principal composes a description ψ by combining a set of predicates with logical connectives $\{\neg, \wedge, \vee\}$. The following example illustrates how logical connectives can be utilized in a description.

Example 6. Suppose $A = \{a_n\}_{n=1}^9$, $\mathcal{S} = P(A)$, $M = \{m\}$, $\Theta = \{\theta_1, \theta_2\}$, $S_{\phi_m}(\theta_1) = \{a_1, a_2, a_3, a_4\}$, and $S_{\phi_m}(\theta_2) = \{a_1, a_2, a_6, a_7\}$. If the principal wants the agent to perform the service $\{a_1, a_2, a_3, a_4, a_8\}$, the following description can induce the service she wishes.

$$\psi' = \phi_m \wedge \phi_8 \wedge \neg\phi_9 \wedge \neg(\phi_6 \vee \phi_7).$$

The role of ϕ_m is to describe the "base" of the service. The predicate ϕ_8 describes a specific duty, which is never requested by ϕ_m under any θ. The predicate $\neg\phi_9$ prohibits a_9. Finally, the formula $\neg(\phi_6 \vee \phi_7)$ clarifies how ϕ_m should be interpreted. To see how it works, note that $\neg(\phi_6 \vee \phi_7) = \neg\phi_6 \wedge \neg\phi_7$ by De Morgan's law. That is, $\neg(\phi_6 \vee \phi_7)$ requests not to perform $\{a_6, a_7\}$. Then, since $S_{\phi_m}(\theta_2) = \{a_1, a_2, a_6, a_7\}$, if the court chooses θ_2 as the interpretation of ψ', $S_{\psi'}(\theta_2) = \emptyset$. In contrast, if the court chooses θ_1, $S_{\psi'}(\theta_1) = \{a_1, a_2, a_3, a_4, a_8\}$. Thus, whenever $\{a_1, a_2, a_3, a_4, a_8\}$ produces a positive surplus, $\neg(\phi_6 \vee \phi_7)$ makes sure that the court chooses θ_1 as the interpretation of ψ'.

To study the general properties of equilibrium descriptions, it is useful to find the general form.

The following concept is useful to represent the general form; a *literal* l is a predicate ϕ or its negation $\neg\phi$. Let

$$L(\Phi) = \Phi \cup \{\neg\phi : \phi \in \Phi\}.$$

That is, $L(\Phi)$ is the set of literals generated by a set of predicates Φ.

Lemma 3. *Given any equilibrium description ψ^*, there exists a set of literals $L^* \subset L(\Phi)$ such that*

$$\bigwedge_{l \in L^*} l = \psi^*$$

and $|L^*| = n(\psi^*)$.

Lemma 3 states that any equilibrium description ψ^* can be written as a conjunction of literals that is not only logically equivalent but also cost-equivalent to ψ^*. Thus, Lemma 3 allows us to restrict our attention to the simple class of descriptions without loss of generality.

To illustrate why a disjunction of predicates does not get along with an equilibrium description, consider the simplest case. If an equilibrium description is $\psi^* = \phi' \vee \phi''$, the agent chooses the service that satisfies only one of the predicates under θ_{ψ^*} since his cost function is monotonic in S. As a result, only one of the predicates should be binding under θ_{ψ^*}. Suppose that ϕ' is the binding predicate, and consider the alternative description $\psi' = \phi'$. Clearly, $S_{\psi'}(\theta_{\psi^*}) = S_{\psi^*}(\theta_{\psi^*})$ and $\kappa(\psi') < \kappa(\psi^*)$. Thus, if $\theta_{\psi'} = \theta_{\psi^*}$, the principal strictly prefers ψ' to ψ^*, contradicting the optimality of ψ^*. Moreover, if $\theta_{\psi'} \neq \theta_{\psi^*}$, the surplus from $S_{\psi'}(\theta_{\psi'})$ has to be at least as high as that from $S_{\psi'}(\theta_{\psi^*})$ and $S_{\psi^*}(\theta_{\psi^*})$. Then, since $\kappa(\psi') < \kappa(\psi^*)$, ψ' is more profitable than ψ^*. Hence, ψ^* cannot be an equilibrium description.

Note that Lemma 3 does not rule out the use of disjunction; if ψ^* contains a subformula $\neg(\phi' \vee \phi'')$, it can still be written as a conjunction of literals $\neg\phi' \wedge \neg\phi''$.

Lemma 4. *Given an equilibrium description ψ^*, let L^* be the set of literals in Lemma 3. Then, there exists $\Phi^* \subset \Phi$ such that $|\Phi^*| = |L^*|$ and*

$$\bigwedge_{l \in L^*} l = \bigwedge_{\phi \in \Phi^*} \phi.$$

If a duty requests to perform a certain action, it is called a positive duty; if a duty demands not to perform a certain action, it is called a negative duty. Lemma 4 states that any equilibrium description can be written as a list of positive duties.[21] [22]

To explain the basic idea behind Lemma 4, consider the setting in Example 6, and suppose that ψ', which contains negations, is an equilibrium description. Then, $\theta_{\psi'} = \theta_1$ and $S_{\psi'}(\theta_{\psi'}) = \{a_1, a_2, a_3, a_4, a_8\}$. By De Morgan's law, we can write

$$\psi' = \phi_m \wedge \phi_8 \wedge \neg\phi_6 \wedge \neg\phi_7 \wedge \neg\phi_9.$$

By eliminating $\neg\phi_9$ from ψ', we get the alternative description

$$\psi'' = \phi_m \wedge \phi_8 \wedge \neg\phi_6 \wedge \neg\phi_7.$$

Since $S_{\phi_m}(\theta_1) = \{a_1, a_2, a_3, a_4\}$,

$$S_{\psi''}(\theta_1) = S_{\psi'}(\theta_1) = \{a_1, a_2, a_3, a_4, a_8\},$$

and $\kappa(\psi'') < \kappa(\psi')$. Thus, if $\theta_{\psi''} = \theta_1$, the principal strictly prefers ψ'' to ψ', contradicting the optimality of ψ'. Moreover, if $\theta_{\psi''} = \theta_2$, $S_{\psi''}(\theta_2) = \emptyset$. Since $\theta_{\psi''} = \theta_2$, the surplus from $S_{\psi''}(\theta_2)$ has to be at least as high as that from $S_{\psi''}(\theta_1)$ and $S_{\psi'}(\theta_{\psi'})$. Then, since $\kappa(\psi'') < \kappa(\psi')$, ψ'' is more profitable than ψ', contradicting the optimality of ψ'. Therefore, ψ' cannot be an equilibrium description.

The next lemma clarifies the role of elementary predicates in equilibrium descriptions.

Lemma 5. *Given an equilibrium description ψ^*, let Φ^* be the set of predicates in Lemma 4. If Φ^* contains some conceptual predicate and $\phi_{a'} \in \Phi^*$, then*

$$a' \notin \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta_\psi).$$

Lemma 5 suggests that whenever an equilibrium description uses a conceptual predicate, there

[21] Actual contracts often contain negative duties that prohibit some damaging actions. If we add the set of elementary actions that are damaging for the principal but beneficial for the agent, e.g., selling classified information, the equilibrium description can contain negative duties.

[22] Technically speaking, Lemma 4 does not rule out the use of negation in an equilibrium description since the principal can still use double negation to write a description that is cost and logical-equivalent to $\bigwedge_{\phi \in \Phi^*} \phi$.

are two possible roles of elementary predicates. The first role is *supplementation*; if $\phi_{a'} \in \Phi^*$ and

$$a' \notin \bigcup_{\theta} \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta),$$

then the principal uses $\phi_{a'}$ to add the request a' that cannot be performed under any interpretation of conceptual predicates in Φ^*. In Example 6, ϕ_8 in ψ' is supplementation.

Another role of an elementary predicate is *clarification*; if $\phi_{a'} \in \Phi^*$ and

$$a' \in \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta),$$

for some $\theta \neq \theta_\psi$, the role of $\phi_{a'}$ is to influence the court's interpretation. To see how clarification works, suppose Φ^* contains ϕ_m such that $S_{\phi_m}(\theta_1) = \{a_1, a_2\}$ and $S_{\phi_m}(\theta_2) = \{a_1, a_2, a_3, a_4, a_5\}$. If the socially efficient service is $\{a_1, a_2, a_3, a_4\}$, the court's interpretation of $\psi = \phi_m$ can be θ_2 since $S_{\phi_m}(\theta_2)$ is closer to $\{a_1, a_2, a_3, a_4\}$. If the principal uses $\psi' = \phi_m \wedge \phi_{a_3} \wedge \phi_{a_4}$ by adding ϕ_{a_3} and ϕ_{a_4} to ϕ_m, the court's interpretation of ψ' can be changed to θ_1 since $S_{\psi'}(\theta_1) = \{a_1, a_2, a_3, a_4\}$.

3.3 Contractual impreciseness

The main interest of this paper is in contractual impreciseness. The current paper considers two kinds of contractual impreciseness. The first kind is about description.

Definition 1. A description ψ is **descriptively imprecise** if $\#\{S : \psi(S; \theta_\psi) = 1\} > 1$.

In words, a service description ψ is descriptively imprecise if there exists more than one service that satisfies the description under the court's interpretation θ_ψ. The definition of descriptive impreciseness might remind some readers of *discretion* in Battigalli and Maggi [2002]. In fact, one can consider descriptive impreciseness as an analogue of discretion in the current setting; both discretion and descriptive impreciseness are caused by relevant actions left unmentioned in a contract.[23]

Fact 1. *Suppose there is no writing cost, violating Assumption 4. Then, there exists an equilibrium description that is descriptively precise.*

[23] Battigalli and Maggi [2002] consider an additively separable environment where each action is independent of other actions. Thus, when a contract does not provide any instruction about a certain action, the lack of instruction can be considered as discretion. In contrast, a contract describes service as a whole, and each action is a constituent of service in the current paper. Hence, when a contract does not mention some relevant action, it is an imprecise description of service rather than discretion.

The following proposition suggests that *any* equilibrium contract is descriptively imprecise when a service space $\mathcal{S}$ is rich enough given v and c as long as the writing cost is non-zero.

Proposition 2. *An equilibrium description ψ^* is descriptively imprecise if and only if there exists $S' \in \mathcal{S}$ such that $S' \supsetneq S_{\psi^*}(\theta_{\psi^*})$.*

Proposition 2 exploits Lemma 4. To see the idea, suppose there exists $S' \in \mathcal{S}$ such that $S' \supsetneq S_{\psi^*}(\theta_{\psi^*})$. Then, the description ψ^* is descriptively precise only if it explicitly prohibits actions in $S' \backslash S_{\psi^*}(\theta_{\psi^*})$ with negated predicates. However, from Lemma 4, we know ψ^* consists entirely of positive duties.

Corollary 1. *If $\mathcal{S} = P(A)$ and $v(A) - c(A) < 0$, any equilibrium description ψ^* is descriptively imprecise.*

The current model also captures the fact that a certain kind of contract is fairly precise in reality. Consider the monotonic environment in Example 3.

Corollary 2. *If $\mathcal{S} = \{\emptyset, \{a\}_{a \in A}\}$, any equilibrium description is descriptively precise.*

Corollary 2 suggests that when we consider a simple service that only needs to describe the quantity of a simple commodity, the equilibrium contract can be descriptively precise.

Comment 1. In the standard model, an obligation in a contract would simply be modeled as $S \in \mathcal{S}$. Under this modeling approach, an obligation in a contract is always precise; if S' is an obligation, the agent violates the contract whenever he provides $S \neq S'$. However, Proposition 2 suggests that such a modeling approach presupposes the availability of a mutually recognized name or code for each S in $\mathcal{S}$ whenever the writing is costly and $\mathcal{S}$ is rich; if there is no mutually recognized name or code for service, the principal needs to describe the content of an obligation with a preexisting language, which leads to the efficient description problem in the current paper.

Comment 2. In Battigalli and Maggi [2002], when the writing cost is high, the benefit of providing instruction for some action does not justify the writing cost, resulting in discretion. The nature of distortion that can be created by descriptive impreciseness is analogous to that of discretion; in the current paper, if the set of predicates consists entirely of elementary predicates, a higher writing cost can reduce the number of specifications in the equilibrium contract as in Battigalli and Maggi [2002]. However, if the set of predicates contains some conceptual predicates, higher writing cost does not always make the obligation less demanding as we will see in Section 4.

When the writing cost is sufficiently small, discretion disappears in Battigalli and Maggi [2002], whereas descriptive impreciseness can persist in the current paper. This difference is due to the difference in the payoff structure rather than that in the language. In the current paper, requesting too many actions can be suboptimal for the principal even if the writing cost is zero. In contrast, whenever an action creates a positive value, the cost of implementing such action is always justified under a small writing cost in Battigalli and Maggi [2002]. This is because they consider a linear structure such that if π is the principal's payoff and t is a transfer, the agent's payoff is $t - \delta\pi$ where $\delta \in (0, 1)$. Thus, when the writing cost is sufficiently small, the principal requests all productive actions to be performed, resulting in a lack of discretion. Thus, if their model had a less stylized payoff setting in which performing some productive actions can be excessive, the optimal contract would exhibit discretion even under a vanishingly small writing cost.

The second kind of impreciseness is semantic.

Definition 2. A service description ψ is **semantically imprecise** if there exists a pair of interpretation types $\theta', \theta'' \in \Theta$ such that $\psi(S; \theta') \neq \psi(S; \theta'')$ for some $S \in \mathcal{S}$.

When a description ψ is semantically imprecise, whether some service satisfies the description or not depends on interpretation type θ. While the notion of vagueness can be considered as one form of semantic impreciseness, semantic impreciseness is a more general concept in the sense that some conceptual predicate can be semantically imprecise without having borderline cases.[24] It might be also worth noting that semantic and descriptive impreciseness are two distinct concepts; a semantically precise description can be descriptively imprecise, whereas a descriptively imprecise description can be semantically precise.

Clearly, a description is semantically imprecise only if it contains some conceptual predicate. Consider Example 3 in which any service can be described by one elementary predicate.

Fact 2. *If $\mathcal{S} = \{\emptyset, \{a\}_{a \in A}\}$, any equilibrium description is semantically precise.*

However, the use of a conceptual predicate does not always make description semantically imprecise.

[24] In the field of semantics, a predicate is vague if it has borderline cases. For example, the predicate "x is a heap of sand" is vague since when we keep taking one sand from a heap, there is no clear cut point at which we have to stop calling the mass of sand as a heap, i.e., Sorites paradox. In the current model, S' is considered as a borderline case for a conceptual predicate ϕ if there exist θ' and θ'' such that $\phi(S'; \theta') = 1$ and $\psi(S'; \theta'') = 0$.

Example 7. Suppose $M = \{m', m''\}$, $\Theta = \{\theta_1, \theta_2\}$, $S_{\phi_{m'}}(\theta_1) = \{a_1, a_2\}$, $S_{\phi_{m'}}(\theta_2) = \{a_1, a_3\}$, $S_{\phi_{m''}}(\theta_1) = \{a_3, a_4\}$, and $S_{\phi_{m''}}(\theta_2) = \{a_2, a_4\}$. Then, the description $\phi_{m'} \wedge \phi_{m''}$ is semantically precise even though it contains conceptual predicates.

The following result suggests that when an equilibrium description uses some conceptual predicates, the description is semantically imprecise as long as the semantic structure of conceptual predicates is rich enough to rule out pathological cases.

Proposition 3. *Given an equilibrium description ψ^*, let Φ^* be the set of predicates in Lemma 4. The equilibrium description ψ^* is semantically imprecise if and only if there exists θ such that*

$$\bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta) \neq \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta_{\psi^*}).$$

When the condition in Proposition 3 is satisfied, there is a gap between the meaning of the equilibrium description under the court's interpretation and that under another interpretation. This gap itself, however, does not guarantee semantic impreciseness since the principal could use a set of elementary predicates to clarify the meaning of the description. However, from Lemma 4 and 5, it can be shown that the optimal clarification never eliminates the semantic gap completely. Note that, as in the case of descriptive impreciseness, an equilibrium contract can be semantically imprecise even if the writing cost is arbitrarily small.

The following lemma provides the condition that guarantees the use of a conceptual predicate in the equilibrium contract. Let

$$\hat{S} \in \arg\max_{S \in \mathcal{S}} \{v(S) - c(S) - \kappa(|S|)\}.$$

Lemma 6. *If there exists ϕ'_m such that $\hat{S} \supset S_{\phi'_m}(\theta)$ and $|S_{\phi'_m}(\theta)| > 1$ for some θ, then the equilibrium description ψ^* uses some conceptual predicate.*

Lemma 6 states that if there is a conceptual predicate that can describe a part of $\hat{S}$ under some interpretation type, the equilibrium description always utilizes some conceptual predicate.

The following result is almost immediate from Proposition 3 and Lemma 6.

Proposition 4. *Suppose, given any θ' and $\Phi' \subset \Phi$ such that $\Phi' \cap \{\phi_m\}_{m \in M} \neq \emptyset$, there exists θ'' such that*

$$\bigcup_{\phi_m \in \Phi'} S_{\phi_m}(\theta'') \neq \bigcup_{\phi_m \in \Phi'} S_{\phi_m}(\theta').$$

If there exists ϕ'_m such that $\hat{S} \supset S_{\phi'_m}(\theta)$ and $|S_{\phi'_m}(\theta)| > 1$ for some θ, then any equilibrium description ψ^* is semantically imprecise.

This result suggests that *any* equilibrium description is semantically imprecise if the service space, the set of conceptual predicates, and its semantic structure are rich enough.

Comment 3. If the writing cost is sufficiently small, semantic impreciseness does not create any distortion. However, when the writing cost is not negligible, it can distort the equilibrium outcome. As shown in Section 3-2, one of the roles of elementary predicates in the efficient description is clarification, which manages how the court interprets a semantically imprecise contract. Thus, when a contract is semantically imprecise, the writing cost, which affects the number of clarification clauses, can affect the equilibrium service by influencing the court's interpretation of conceptual predicates in the contract.

Finally, it might be worth clarifying that an equilibrium description can be semantically precise even if the conditions in Proposition 4 are satisfied when the writing is costless.

Fact 3. *Suppose there is no writing cost, violating Assumption 4. Then, there exists an equilibrium description that is semantically precise.*

4 Comparative statics

This section studies how a change in the writing cost and the set of predicates affect the equilibrium outcome.

4.1 Writing cost

For the comparative statics, assume $\kappa(\psi) = \alpha k(n(\psi))$ where $k(n)$ is a strictly increasing function, and $\alpha > 0$ is the parameter that measures the importance of the writing cost. To state the next result, let S^{1st} be the solution of $\max_{S \in \mathcal{S}}\{v(S) - c(S)\}$. That is, S^{1st} is the first best service.

Lemma 7. *There exists $\alpha^* > 0$ such that $f^*(\psi^*) = S^{1st}$ under any $\alpha \in (0, \alpha^*)$.*

Lemma 7 states that the distortion in the equilibrium service disappears under a small writing cost. This result is contrary to the fact that contractual impreciseness can persist even under a vanishingly small writing cost.

Proposition 5. *A higher α decreases efficiency, whereas a higher α can increase or decrease the value of the equilibrium service.*

The negative effect of a higher writing cost on efficiency is intuitive; when the writing cost increases, the set of profitable descriptions shrinks. Then, from Proposition 1, a higher writing cost decreases the principal's equilibrium payoff.

The effect of a higher writing cost on the value of the equilibrium service can be complex. First, to see the negative effect, suppose that the set of predicates consists entirely of elementary predicates. If the writing cost is small, the equilibrium service is the first best from Lemma 7. When the writing cost gets higher, the principal uses a smaller number of predicates in the equilibrium. The requested service should then have a lower value than the first best service. Turning to the positive effect, suppose that some conceptual predicates are available but they are too broad to describe the first best service. From Lemma 7, if the writing cost is sufficiently small, the principal induces the first best service. Thus, if the writing cost is small, the equilibrium description consists entirely of elementary predicates. When the writing cost gets higher, a description that only uses elementary predicates becomes too expensive. As a result, the principal could prefer to use a coarse conceptual predicate to save her writing cost, requesting an excessive service.

4.2 Set of conceptual predicates

While there are many ways to consider a change in the available predicates, the current paper considers the following three. First, a set of conceptual predicates M' is an **enrichment** of a set of conceptual predicates M if $M' \supset M$. It is worth clarifying that any enrichment does not increase the descriptive power of the original Φ since Φ is already rich enough to describe any service in the current paper.

Second, a set of conceptual predicates M' is a **refinement** of a set of conceptual predicates M if (i) each $m \in M$ has $N_m \subset M'$ such that

$$S_{\phi_m}(\theta) = \bigcup_{m \in N_m} S_{\phi_m}(\theta)$$

for all θ; (ii) each $m' \in M'$ has $m \in M$ such that $S_{\phi_{m'}}(\theta) \subsetneq S_{\phi_m}(\theta)$ for all θ. In short, if a set of conceptual predicates is refined, the principal can describe a service more accurately with a conceptual predicate.

Finally, θ-**formalization** of a set of conceptual predicates M is to fix the interpretation type to θ, eliminating room for interpretation.

Proposition 6. *Any enrichment of M weakly improves efficiency, whereas a refinement of M can increase or decrease efficiency. Finally, no formalization improves efficiency.*

The idea behind the positive effect of enrichment is simple; since any description that is available under M is still available under the enriched predicates, the result immediately follows from Proposition 1. The positive effect of enrichment suggests that the efficiency loss from the writing cost can be higher when a principal writes a contract for a novel service that lacks established conceptual predicates to save the writing cost.

The effect of refinement is subtle. First, to see how a refinement of M can improve efficiency, consider an environment in which all conceptual predicates in M are too broad, and the principal needs to write the equilibrium description only with elementary predicates. If some conceptual predicate in a set of refined predicates M' is concise enough, the principal could describe the same service by combining conceptual predicates and some elementary predicates. Then, since the description induces the same service with a lower writing cost, the set of refined predicates improves efficiency. To illustrate how a refinement of M can reduce efficiency, suppose that there is a conceptual predicate in M, which is concise enough to use in the equilibrium description. Under a refinement, it could be the case that the principal needs to combine refined conceptual predicates to induce the same service. Then, since the writing cost is higher while inducing the same service, the refinement reduces efficiency.

Turning to the ineffectiveness of formalization, recall that when there is no formalization, the court interprets a contract to maximize social surplus. Thus, if some formalization can improve efficiency, it must help the principal to write a more profitable description than the equilibrium ψ^* under no formalization. Suppose that θ'-formalization induces $\psi' \neq \psi^*$. Note that the principal's payoff from ψ' under θ' cannot be higher than that from ψ' under the court's efficient interpretation $\theta_{\psi'}$. Then, since ψ^* is optimal under the court's efficient interpretation, the principal's payoff from ψ' cannot be higher than that from ψ^*.

In the current paper, there is no cost of interpretation. If we take into account the cost of interpretation, a formalization of conceptual predicates can be beneficial for the court. However, a formalization forces the principal to use more elementary predicates to fill out the detail. That is, writing the same contract becomes more costly. Thus, it is not clear whether a formalization improves efficiency even if we take into account the cost of interpretation.

Finally, there is a substantial cost of formalization that is not reflected in the current comparative statics. Suppose that there is a set of (v, c), and nature draws (v, c) according to some distribution. In this extended setting, a formalization of predicates can reduce efficiency since the efficient interpretation of a predicate under some (v, c) can be suboptimal under some (v, c). Thus, when conceptual predicates are used to write various contracts, keeping room for interpretation is, in fact, efficient.

5 Discussion

5.1 Contingent contract

Until now, we consider a contract in the deterministic environment. The basic approach of this paper can also be applied to contingent contracts. As a service S is founded on elementary actions in the basic setting, we consider a state ω that is founded on **elementary events**. Specifically, let E be a finite set of elementary events E. A state ω is then determined by the set of elementary events that occur. Let $\omega \subset E$ be a state. For example, we have the state $\omega = \{e', e''\}$ when elementary events e' and e'' happen but any $e \in E \backslash \{e', e''\}$ does not happen. Thus, ω never contains any pair of mutually exclusive events. Let Ω be the set of possible states. The value and cost functions are then extended to $v(S, \omega)$ and $c(S, \omega)$.

A **realization** $Z \subset A \cup E$ is a collection of elementary actions and events.[25] A realization Z is possible if $Z \cap E \in \Omega$ and $Z \cap A \in \mathcal{S}$. Let $\mathcal{Z}$ be the set of possible realizations, which is the domain of discourse for predicates. A predicate in the extended model is defined as

$$\phi(Z; \theta) = \begin{cases} 1 & \text{if } Z \supset Z_\phi(\theta) \\ 0 & \text{if } Z \not\supset Z_\phi(\theta) \end{cases}.$$

A realization Z then satisfies ϕ under θ if $\phi(Z; \theta) = 1$. We enrich the set of predicates by adding elementary predicates for each elementary event ϕ_e. Assume that conceptual predicates are only for describing a service, i.e., $Z_{\phi_m}(\theta) \subset A$ for all $m \in M$.[26] The set of extended feasible predicates

[25] We do not endow Z with a product structure since a contract is written with unary predicates whose domain of discourse is the set of possible realizations.

[26] If we also accommodate conceptual predicates that describe events or/and some combination of events and actions, the notion of impreciseness needs to be extended. Since the purpose of this subsection is to illustrate how the basic model can be extended to accommodate contingent contracts, we focus on the simple case.

is then

$$\hat{\Phi} = \Phi \cup \{\phi_e\}_{e \in E}.$$

The principal describes a contingent obligation with predicates from $\hat{\Phi}$ and connectives $\{\neg, \wedge, \vee\}$.[27]

In the current setting, since a transfer level only affects the agent's participation decision, there is no gain from a state-dependent transfer.[28] Thus, consider a constant transfer. A contract is then defined as (ψ, t) as in the basic setting. The extended contract game is the same as the basic game except that the state ω is observed by both players in Period 2, and strategies f and σ can depend on ω. Assume that the court can verify the true state in period 3. Let $S_\psi(\theta, \omega)$ be the most economical service that satisfies ψ under θ at ω. Given ω, the court chooses θ to solve

$$\max_{\theta \in \Theta} \{v(S_\psi(\theta, \omega), \omega) - c(S_\psi(\theta, \omega), \omega)\}.$$

Let $\theta_\psi(\omega)$ be the court's interpretation type at ω. Note that, in the extended model, the court's interpretation is state-dependent, reflecting the commercial context at each state.

Observe that once ω is fixed, the extended model is essentially the same as the basic model. Thus, the major change in the analysis is the choice of a contract (ψ, t), which takes place before ω is realized. Given ψ, the equilibrium transfer is simply determined by the ex ante expected cost of the optimal service given ψ. While the equilibrium ψ can still be obtained as a solution of the efficient description problem, ψ needs to describe contingencies in addition to obligations in the extended setting.

To illustrate how we can describe a contingent contract with predicates, suppose that $A = \{a_1, a_2\}$, $E = \{e_1, e_2\}$, $\mathcal{S} = P(A)$, and $\Omega = P(E)$. Suppose v and c are such that the following state-contingent obligation maximizes social surplus.

$$\begin{cases} a_1 & \text{if } \omega = \emptyset \\ a_2 & \text{if } \omega = \{e_1\} \\ a_1 & \text{if } \omega = \{e_2\} \\ a_2 & \text{if } \omega = \{e_1, e_2\} \end{cases}$$

[27] We could also consider a formula that uses $\rightarrow$ "implication." However, a formula that only uses logical connectives $\{\neg, \wedge, \vee\}$ can still describe a contingent contract as efficient as a formula with $\{\neg, \wedge, \vee, \rightarrow\}$.

[28] Since an obligation can depend on contingency, the description of contingencies is still relevant.

With predicates, a precise description of the above state-contingent obligation is

$$\begin{aligned}
& (\phi_{a_1} \wedge \neg\phi_{a_2} \wedge \neg\phi_{e_1} \wedge \neg\phi_{e_2}) \vee (\neg\phi_{a_1} \wedge \phi_{a_2} \wedge \phi_{e_1} \wedge \neg\phi_{e_2}) \\
\vee \quad & (\phi_{a_1} \wedge \neg\phi_{a_2} \wedge \phi_{e_2} \wedge \neg\phi_{e_1}) \vee (\neg\phi_{a_1} \wedge \phi_{a_2} \wedge \phi_{e_1} \wedge \phi_{e_2}).
\end{aligned}$$

To see how the above predicate formula works, note that when neither e_1 nor e_2 occurs, only the first term $\phi_{a_1} \wedge \neg\phi_{a_2} \wedge \neg\phi_{e_1} \wedge \neg\phi_{e_2}$ can be satisfied. Thus, $\phi_{a_1} \wedge \neg\phi_{a_2}$ is the instruction for $\omega = \emptyset$, i.e., "perform only a_1." When only e_1 occurs, only the second term $\neg\phi_{a_1} \wedge \phi_{a_2} \wedge \phi_{e_1} \wedge \neg\phi_{e_2}$ can be satisfied. Then, the obligation at $\omega = \{e_1\}$ is to perform only a_2. Similarly, the obligation at $\omega = \{e_2\}$ is to perform only a_1, and the obligation at $\omega = \{e_1, e_2\}$ is to perform only a_2.

Clearly, the above description is not written efficiently. First, as in the basic setting, the principal could just write ϕ_{a_1} instead of $\phi_{a_1} \wedge \neg\phi_{a_2}$ since the agent would choose a_1 when both a_1 and $\{a_1, a_2\}$ can satisfy the description. Second, if the principal wants the agent to perform a_1 at $\omega = \emptyset$ and $\{e_1\}$, the instruction could simply be written as "perform a_1 if e_2 does not occur." Similarly, for $\omega = \{e_1, e_2\}$ and $\{e_2\}$, the instruction could be "perform a_2 if e_2 occurs." Then, if the writing cost is sufficiently small, the equilibrium description is

$$(\phi_{a_1} \wedge \neg\phi_{e_1}) \vee (\phi_{a_2} \wedge \phi_{e_1}).$$

Note that the above description exhibits descriptive impreciseness as in the basic setting.[29]

To see how a higher writing cost could affect the above description, suppose that the elementary event e_1 almost always happens. Then, under a higher writing cost, the principal would further simplify the description to ϕ_{a_2} or $\phi_{a_2} \wedge \phi_{e_1}$. The former description exhibits rigidity since the obligation is state-independent; the latter description exhibits discretion since the agent has no obligation when e_1 does not happen. If performing a_2 at $\omega = \emptyset$ and $\{e_2\}$ is harmless, the principal prefers the description with rigidity, i.e., ϕ_{a_2}. In contrast, if performing a_2 at $\omega = \emptyset$ and $\{e_2\}$ is harmful than doing nothing, the principal can prefer the description with discretion $\phi_{a_2} \wedge \phi_{e_1}$; this is because the agent chooses $S = \emptyset$ rather than a_2 if no action is requested at $\omega = \emptyset$ and $\{e_2\}$.

As we can see from the above example, the equilibrium description in the extended game could use all logical connectives $\{\neg, \vee, \wedge\}$ because of contingencies. Hence, the equilibrium description

[29] One might suggest the formula with the logical connective $\rightarrow$, i.e., $(\neg\phi_{e_1} \rightarrow \phi_{a_1}) \wedge (\phi_{e_1} \rightarrow \phi_{a_2})$, is a more natural description as a contingent instruction. However, they are logically equivalent; it is known that $\neg\phi_{e_1} \rightarrow \phi_{a_1} \vdash \phi_{e_1} \vee \phi_{a_1}$ and $\phi_{e_1} \rightarrow \phi_{a_2} \vdash \neg\phi_{e_1} \vee \phi_{a_2}$. Thus, the above formula with $\rightarrow$ is logically equivalent to $(\phi_{a_1} \wedge \neg\phi_{e_1}) \vee (\phi_{a_2} \wedge \phi_{e_1}) \vee (\phi_{e_1} \wedge \neg\phi_{e_1}) \vee (\phi_{a_2} \wedge \phi_{a_1})$, which can be simplified to $(\phi_{a_1} \wedge \neg\phi_{e_1}) \vee (\phi_{a_2} \wedge \phi_{e_1})$.

can be a complex compound formula, which has a large number of logically equivalent expressions. Consequently, there is no simple general form for the equilibrium description.[30] Nevertheless, since the complication comes from the description of contingencies rather than that of services, the basic properties of the equilibrium service description can be preserved. That is, the principal gets no benefit from adding negative duties to eliminate descriptive impreciseness; moreover, it is redundant to add elementary predicates to eliminate semantic impreciseness. Thus, as in the basic model, if the service space, the set of conceptual predicates, and its semantic structure are sufficiently rich, the equilibrium description of the service for each contingency is descriptively and semantically imprecise.

As the above example illustrates, the extended model can generate rigidity and discretion as in Battigalli and Maggi [2002]. However, a fuller analysis of the efficient contingent description is beyond the scope of this paper. The efficient contingent description depends not only on the structure of the environment but also on the probability distribution of states. Hence, unless we restrict our attention to a stylized setting as in Battigalli and Maggi [2002] or a certain class of ψ, it is hard to obtain sharp results.

5.2 Designing contractual language

The current paper treats contractual language as a preliminary of the model. One question is how a social planner can design contractual language to improve efficiency. The answer is obvious when a planner can design a language without any constraint; a planner would create a name or code for every possible service in $\mathcal{S}$ so that it can directly refer to the exact service without any description of the content. However, actual contracts are rarely written with such an ideal language. One of the primary reasons might be the prohibitively large cost of learning or using such an artificial language. Note that the cost of introducing new codes is not only that of creating them but also that of making them recognizable by all users. Arrow [1974] observes that when a firm introduces organizational codes to improve communication, one of the major costs is the irreversible investment to learn the new language.[31] Such a cost could be small if codes are designed for routine communication, which requires a small number of codes. However, the number of codes for contractual obligations would be prohibitively large since codes have to be created not only for

[30]For instance, the disjunctive normal form $(\phi_{a_1} \wedge \neg\phi_{e_1}) \vee (\phi_{a_2} \wedge \phi_{e_1})$ in our example can be an equilibrium description. However, in a more complex environment, the equilibrium description does not always take a disjunctive normal form.

[31]Cremer et al. [2007] provide a formal analysis of organizational codes.

the standard services but also for novel services. Consequently, the cost of describing an obligation with a familiar pre-existing language can be much lower than the cost of learning or using an unfamiliar artificial language.

A practical improvement of contractual language might be found in the development of new predicates. That is, rather than a precise code system, one could develop new conceptual predicates that can describe a major building block of service. To see the idea, consider Example 2. If some electronic products share a common component that requires a set of elementary actions $A_n \subset A$. A new predicate that describes A_n or some core components of A_n could be created so that subsequent contracts can be written concisely.

Finally, as analyzed in Section 4-2, a formalization of conceptual predicates, i.e., eliminating room for interpretation, does not improve efficiency, and it can be even inefficient when we use the contractual language to write various contracts. This observation might explain why contractual language does not always evolve to be more precise in practice.

5.3 Robustness

The current paper considers a simple setting to focus on the main idea. As we already discussed in 5.1, the basic message of this paper can be preserved even if we extend the model to accommodate contingent contracts. This subsection provides further discussions on the robustness of contractual impreciseness.

First, the current paper assumes that the agent knows the principal's value function v. If the agent does not know v, he has to infer θ_ψ conditional on (ψ, t). Consequently, in some situations, the principal needs to use a less semantically imprecise contract by adding some elementary predicates as clarification. However, such clarification clauses do not eliminate semantic impreciseness in general.

Second, in the current setting, all actions in A is productive for the principal and costly for the agent as in the standard principal-agent model. In reality, there can be some actions that are harmful to the principal but beneficial for the agent. Let A^- be a set of such actions. As mentioned in Section 3-2, the equilibrium description can contain negative duties if the model is extended with $A \cup A^-$. Nevertheless, as long as the equilibrium contract in the model with A exhibits descriptive and semantic impreciseness, the result is preserved in the extended model with $A \cup A^-$.

Third, the court respects the literal meaning of a contract in the current setting; if ψ has no predicate that describes a' under any interpretation, the court never expects the agent to perform a' in the current setting. One could also consider a more radical interpretation rule under which the court could request any service beyond the meaning of predicates as long as it does not violate ψ under some θ. Not surprisingly, the alternative rule does not discourage contractual impreciseness.

Finally, the current paper considers an environment with symmetric information. If the agent has private information, the principal might use a menu contract to extract the agent's information. Since a menu contract consists of a set of contracts, and the obligation for each contract needs to be described as in the current paper, the basic insight should be preserved in the asymmetric information setting.

5.4 Other applications

While the main purpose of this paper is to introduce a formal framework to comprehend contractual impreciseness, the framework can also be applied to analyze other economic questions. One of the potential applications can be found in the analysis of organizational forms. Casual observation suggests that while the corporate form is common in the manufacturing industry, partnerships have been prominent in professional services such as consulting. The current framework can provide fresh insight into the difference in organizational forms across businesses.

Suppose that the principal can choose how she works with the agent; specifically, she can select either an employment relationship or a partnership. In the former case, the principal writes a contract as in the current paper. In the latter case, the principal shares the revenue equally with her partner. Since there is no conflict of interest in the partnership, the principal can convey the content of the work to her partner via cheap talk, which is costless. Moreover, since the partnership contract does not need to specify the work detail but 50-50 split of the revenue, the writing cost of the partnership contract can be negligible. Then, for simplicity, assume that the writing cost of the partnership contract is zero.

Suppose that the value of a service with n elementary actions is Rn where $R > 0$, and the cost of performing n elementary actions is $\frac{n^2}{2}$. For the writing cost, consider the following reduced form. Suppose that when the principal describes a service with n elementary actions, the number of predicates in the efficient description is $\frac{n}{\beta}$. That is, the parameter β reflects the number of elementary predicates saved by conceptual predicates in the efficient description. Thus, if there is a richer set of conceptual predicates for the service, β is higher. The cost of writing the efficient description for n elementary actions is then $\alpha\frac{n}{\beta}$ where α is the cost of writing one predicate.

First, consider the employment relationship with a contract. Since the principal's payoff from the service with n actions is $Rn - \frac{n^2}{2} - \alpha\frac{n}{\beta}$, the optimal n is $R - \frac{\alpha}{\beta}$. The principal's payoff from the employment relationship is then $\frac{(R-\frac{\alpha}{\beta})^2}{2}$. Turning to the partnership case, let n_1 be the number of actions by "principal," and let n_2 be the number of actions by her partner. Since the revenue is shared, the principal's payoff from n_1 and n_2 is $\frac{1}{2}R(n_1 + n_2) - \frac{n_1^2}{2}$. The optimal number of actions for each individual is then $\frac{R}{2}$. Thus, the payoff for "principal" is $\frac{1}{2}R^2 - \frac{(R/2)^2}{2} = \frac{3}{8}R^2$. Then, the principal prefers the employment relationship (the partnership) if $\frac{(R-\frac{\alpha}{\beta})^2}{2} > (<)\frac{3}{8}R^2$ or $\frac{\alpha}{\beta} < (>)(1 - \frac{\sqrt{3}}{2})R$.

To see how the above inequality can explain the difference in organizational forms across businesses, consider some service that consists mainly of routine tasks, e.g., manufacturing standard products. Since those tasks are often common in the industry, there can be well-established conceptual predicates that describe those tasks concisely. For example, suppose a contract requests a worker to assemble car engines. While assembling an engine may consist of many elementary actions, a competent worker may interpret the established conceptual predicate "assembling an engine" appropriately given a specific situation. Thus, this type of service has a rich set of conceptual predicates that saves the writing cost, i.e., a high β. The inequality condition then suggests that the principal may prefer the employment relationship for this type of business.

Now, consider some service that constantly deals with new projects or cases, e.g., consulting. Then, it is hard to have a conceptual predicate that can be interpreted properly for each case. For example, if an employment contract states a worker's duty is "proposing the best marketing strategy," it seems unrealistic to expect a worker to find out the actions to be performed since people often have different ideas about "the best strategy" for new cases. Then, since this type of service lacks effective conceptual predicates to describe a duty, the contract has to be written with an enormous number of elementary predicates, i.e., a low β. Then, the inequality condition suggests the principal may prefer the partnership for this type of service.

Needless to say, the above analysis is highly stylized and ignores other potentially important elements.[32] Nevertheless, the simple analysis illustrates how the current framework can provide fresh insight into other important economic questions.

[32]For example, Levin and Tadelis [2002] provides an alternative approach based on the observability of service quality.

6 Concluding remarks

This paper provides a formal writing cost framework that helps us to apprehend why actual contracts are often imprecise and how imprecise contracts can work effectively. The main innovation of the current paper is the use of predicate logic as a model of contractual language. The explicit model of contractual language allows us to analyze how a contract is written by the principal in addition to how it is interpreted by the agent and the court. It is shown that even if any service can be describable and contractible, any equilibrium contract exhibits contractual impreciseness if the service space, the set of conceptual predicates, and its semantic structure are rich enough. Moreover, both kinds of impreciseness persist even under a vanishingly small writing cost. The result is contrary to typical writing cost models in which a non-standard feature disappears as the writing cost approaches zero. The current paper also captures the fact that some actual contracts are fairly precise; for example, the equilibrium contract for trade in a simple commodity can be precise in the current model.

While the standard principal-agent model usually has a descriptively and semantically precise contract, contractual impreciseness of this paper can be compatible with the standard model under a certain situation. While contractual impreciseness persists under an arbitrarily small writing cost, the equilibrium service of the current model coincides with that of the standard contract model under a sufficiently small writing cost. Thus, when the writing cost is small enough given the size of a contract, analyzing the principal-agent problem with the standard model is, in fact, without loss of generality.

Writing a contract for a fairly complex service should be as costly as performing any office work. When the writing cost is not negligible, it distorts the equilibrium contract. The current framework can capture not only the distortion studied in the existing literature but also the new kind that depends on the available contractual language for the service. As illustrated in Section 5-4, the framework can also provide new insight into other economic phenomena such as the formation of organizational forms.

The current writing cost framework can also be applied to study the simplicity of actual contracts. For instance, the wage schedule in actual contracts is often simpler than the optimal wage schedule in contract theory. The traditional writing cost approach claims that the actual wage schedule is simpler since the benefit of writing the wage for every state does not justify the writing cost; for example, the cost of describing a nonlinear wage function can be prohibitively large if every wage-state pair needs to be written explicitly. However, some economists might not find this argument convincing since the writing cost of describing the nonlinear wage schedule can be quite small if it is described with the help of mathematics, the language developed to describe a highly complex structure economically with great precision.[33] The current writing cost framework offers a defense against such criticism; since a contract needs to be mutually comprehensible for all parties, the relevant writing cost is not the cost of writing a contract with some language but that with the available contractual language. The growing gap between contract theory and actual contracts might be attributed partially to the fact that the former gets increasingly sophisticated by utilizing the power of mathematical languages, while the latter is largely restricted by the use of natural language.

7 Appendix

This section provides the omitted proofs.

7.1 Proof of Lemma 1

Note that the agent's optimal service provision is

$$f^*(\psi, t, 1) = \begin{cases} \zeta(\psi) & \text{if } c(\zeta(\psi)) < c(S_\psi(\theta_\psi) \cup \xi(\psi)) + \gamma \\ \xi(\psi) & \text{if } c(\zeta(\psi)) > c(S_\psi(\theta_\psi) \cup \xi(\psi)) + \gamma \end{cases}$$

Since the principal's optimal acceptance strategy is

$$\sigma^*(\psi, t, 1, S) = \begin{cases} 1 & \text{if } v(S_\psi(\theta_\psi) \cup S) \leq v(S) \\ 0 & \text{if } v(S_\psi(\theta_\psi) \cup S) > v(S) \end{cases},$$

$\zeta(\psi)$ solves $\max_{S \in \{S' : v(S_\psi(\theta_\psi) \cup S) \leq v(S)\}} \{t - c(S)\}$.

If $S_\psi(\theta_\psi) \not\subset \zeta(\psi)$, then $c(S_\psi(\theta_\psi) \cup \zeta(\psi)) > c(\zeta(\psi))$ from Assumption 1. Then, from Assumption 2 and 3, $v(S_\psi(\theta_\psi) \cup \zeta(\psi)) > v(\zeta(\psi))$, a contradiction. Thus, $S_\psi(\theta_\psi) \subset \zeta(\psi)$. If $S_\psi(\theta_\psi) \subsetneq S$, then $c(S_\psi(\theta_\psi)) < c(S)$ from Assumption 1. Since $S_\psi(\theta_\psi) \in \{S : v(S_\psi(\theta_\psi) \cup S) \leq v(S)\}$, it must be $\zeta(\psi) = S_\psi(\theta_\psi)$.

Now, I claim that we never have $f^*(\psi, t, 1) = \xi(\psi)$. To see this, note that since $\zeta(\psi) = S_\psi(\theta_\psi)$

[33] For example, any nonlinear wage function can be described by a polynomial as accurate as one wishes.

and $\gamma > 0$, $c(S_\psi(\theta_\psi)) < c(S_\psi(\theta_\psi) \cup S) + \gamma$ for any S by Assumption 1. Hence, $f^*(\psi, t, 1) \neq \xi(\psi)$. It follows that $f^*(\psi, t, 1) = S_\psi(\theta_\psi)$.

7.2 Proof of Lemma 2

Consider the agent's problem at period 2. The agent's expected payoff from accepting an offer (ψ, t) given (σ^*, f^*) is $t - c(f^*(\psi))$. The agent's optimal acceptance decision is then

$$\rho^*(\psi, t) = \begin{cases} 1 & \text{if } t \geq c(f^*(\psi)) \\ 0 & \text{if } t < c(f^*(\psi)) \end{cases}.$$

From Lemma 1, we know that $f^*(\psi) = S_\psi(\theta_\psi)$. Then, given ψ, the optimal transfer solves

$$\max_{t \in T} \{v(S_\psi(\theta_\psi)) - t\} \mathbf{1}_{\{t' : t' \geq c(S_\psi(\theta_\psi))\}}(t) - \kappa(\psi)$$

where $\mathbf{1}_{\{t' : t' \geq c(S_\psi(\theta_\psi))\}}(t)$ is the indicator function. Thus, if the principal wishes to induce the agent's acceptance, i.e., $x = 1$, she chooses $t = c(S_\psi(\theta_\psi))$. Her payoff is then $v(S_\psi(\theta_\psi)) - c(S_\psi(\theta_\psi)) - \kappa(\psi)$. If the principal induces the agent's rejection, i.e., $x = 0$, by setting some $t \in [0, c(S_\psi(\theta_\psi)))$, her payoff is $-\kappa(\psi)$. Thus, if $\psi \in \Psi \backslash \Psi^-$, i.e., ψ is such that $v(S_\psi(\theta_\psi)) - c(S_\psi(\theta_\psi)) \geq 0$, then $t = c(S_\psi(\theta_\psi))$; if $\psi \in \Psi^-$, i.e., ψ is such that $v(S_\psi(\theta_\psi)) - c(S_\psi(\theta_\psi)) < 0$, then $t \in [0, c(S_\psi(\theta_\psi)))$.

7.3 Proof of Proposition 1

From Lemma 1, $f^*(\psi) = S_\psi(\theta_\psi)$. Moreover, from Lemma 2, $t = c(S_\psi(\theta_\psi))$ given any $\psi \in \Psi \backslash \Psi^-$. Note that if the principal uses the null description, then $S_\psi(\theta_\psi) = \emptyset$ and $\kappa(\psi) = 0$, which guarantee the principal to have the payoff of zero. Thus, the principal never chooses $\psi \in \Psi^-$, which yields a negative payoff, i.e., $-\kappa(\psi)$. Then, the principal's equilibrium description solves $\max_{\psi \in \Psi} \{v(S_\psi(\theta_\psi)) - c(S_\psi(\theta_\psi)) - \kappa(\psi)\}$.

7.4 Proof of Lemma 3

To prove Lemma 3, I establish the following claim.

Claim: *If ψ^* contains a disjunction $\vee_{\phi \in L'} l$ as a subformula, the equilibrium service does not satisfy $\vee_{\phi \in L'} l$ under θ_{ψ^*}.*

The proof is by contradiction; it will be shown that if the equilibrium service satisfies a subformula $\vee_{\phi\in L'} l$ under θ_{ψ^*}, then ψ^* is not optimal, contradicting the premise.

Consider literals $l', l'' \in L'$. Let ψ' be the description that is the same as ψ^* except that $\vee_{\phi\in L'} l$ is replaced by $\vee_{\phi\in L'\setminus\{l''\}} l$. Similarly, let ψ'' be the description that is the same as ψ^* except that $\vee_{\phi\in L'} l$ is replaced by $\vee_{\phi\in L'\setminus\{l'\}} l$. Without loss of generality, suppose

$$\min_{S\in\{S':\psi'(S';\theta_{\psi^*})=1\}} c(S) \leq \min_{S\in\{S':\psi''(S';\theta_{\psi^*})=1\}} c(S) \tag{1}$$

The proof of Claim consists of two steps.

Step-1: $S_{\psi'}(\theta_{\psi^*}) = S_{\psi^*}(\theta_{\psi^*})$.

Recall that $S_{\psi^*}(\theta_{\psi^*})$ solves $\min_{S\in\{S':\psi^*(S';\theta_{\psi^*})=1\}} c(S)$. Moreover, by construction,

$$\{S : \psi'(S;\theta_{\psi^*}) = 1\} \cup \{S : \psi''(S;\theta_{\psi^*}) = 1\} = \{S : \psi^*(S;\theta_{\psi^*}) = 1\}.$$

Then, from inequality (1), we have $S_{\psi^*}(\theta_{\psi^*}) \in \arg\min_{S\in\{S':\psi'(S';\theta_{\psi^*})=1\}} c(S)$. Then, from Assumption 3, $S_{\psi'}(\theta_{\psi^*}) = S_{\psi^*}(\theta_{\psi^*})$.

Step 2. The principal strictly prefers ψ' to ψ^*.

First, I claim that $\theta_{\psi'} = \theta_{\psi^*}$. To see the claim, suppose $\theta_{\psi'} \neq \theta_{\psi^*}$. Then,

$v(S_{\psi'}(\theta_{\psi'}))-c(S_{\psi'}(\theta_{\psi'})) \geq v(S_{\psi'}(\theta_{\psi^*}))-c(S_{\psi'}(\theta_{\psi^*}))$. Since $S_{\psi'}(\theta_{\psi^*}) = S_{\psi^*}(\theta_{\psi^*})$, we have$v(S_{\psi'}(\theta_{\psi'}))-c(S_{\psi'}(\theta_{\psi'})) \geq v(S_{\psi^*}(\theta_{\psi^*}))-c(S_{\psi^*}(\theta_{\psi^*}))$. Note that, from Lemma 1 and 2, the principal's payoff from ψ' is $v(S_{\psi'}(\theta_{\psi'}))-c(S_{\psi'}(\theta_{\psi'}))-\kappa(\psi')$, whereas her payoff from ψ^* is $v(S_{\psi^*}(\theta_{\psi^*}))-c(S_{\psi^*}(\theta_{\psi^*}))-\kappa(\psi^*)$. Then, since $\kappa(\psi') < \kappa(\psi^*)$, the principal strictly prefers ψ' to ψ^*, contradicting the optimality of ψ^*. It follows that $\theta_{\psi'} = \theta_{\psi^*}$.

From Step 1 and $\theta_{\psi'} = \theta_{\psi^*}$, $S_{\psi'}(\theta_{\psi'}) = S_{\psi^*}(\theta_{\psi^*})$. Then, from Lemma 1, $v(f^*(\psi^*))-c(f^*(\psi^*)) = v(f^*(\psi'))-c(f^*(\psi'))$. Since $\kappa(\psi') < \kappa(\psi^*)$, the principal strictly prefers ψ' to ψ^*, contradicting the optimality of ψ^*.

Note that since each conjunction of literals, i.e., $\wedge_{l\in L'} l$, can be considered as one predicate, we can also show that if ψ^* contains a subformula $\bigwedge_{l\in L'} l \vee \bigwedge_{l\in L''} l$, the equilibrium service does not satisfy $\bigwedge_{l\in L'} l \vee \bigwedge_{l\in L''} l$ under θ_{ψ^*}. It follows that ψ^* must consist of conjunctions, i.e., $\wedge_{\phi\in L'} l$, and/or $\vee_{\phi\in L'} l$ with negation (or some odd number of negations). I claim that if ψ^* has $\vee_{\phi\in L'} l$ with (an odd number of) negation, it can be written as a logically equivalent conjunction of literals that preserves the number of predicates. To see this, suppose ψ^* contains $\neg(\vee_{\phi\in L'} l)$. Then, by De Morgan's law, $\neg(\bigvee_{\phi\in L'} l) = \bigwedge_{\phi\in L'} \neg l$. Hence, any negated disjunction can be written as a logically equivalent conjunction of literals, while preserving the number of predicates.

7.5 Proof of Lemma 4

To prove Lemma 4, suppose $\neg\phi' \in L^*$. Then, consider the following alternative description:

$$\psi' = \bigwedge_{l \in L^* \setminus \{\neg\phi'\}} l.$$

By construction, $\{S : \psi^*(S;\theta) = 1\} = \{S : (\psi' \wedge \neg\phi')(S;\theta) = 1\}$. Since ψ^* has an additional property $\neg\phi'$ to be satisfied, $\{S : \psi'(S;\theta) = 1\} \supset \{S : \psi^*(S;\theta) = 1\}$. Note that if $\psi'(S;\theta) = 1$ and $\psi^*(S;\theta) = 0$, then $\neg\phi'(S;\theta) = 0$ or equivalently $\phi'(S;\theta) = 1$. Thus, if $S' \in \{S : \psi'(S;\theta) = 1\} \setminus \{S : \psi^*(S;\theta) = 1\}$, then $S' \supset S_{\psi^*}(\theta) \cup S_{\phi'}(\theta)$.

From Assumption 1, $c(S_{\psi^*}(\theta)) < c(S_{\psi^*}(\theta) \cup S_{\phi'}(\theta))$. Since $S_{\psi^*}(\theta) \in \{S : \psi^*(S;\theta) = 1\}$ and $S_{\psi'}(\theta) \in \arg\min_{S \in \{S' : \psi'(S';\theta)=1\}} c(S)$, we have $S_{\psi'}(\theta) \in \{S : \psi^*(S;\theta) = 1\}$. Then, from Assumption 3, $S_{\psi'}(\theta) = S_{\psi^*}(\theta)$. Moreover, since $\theta_\psi \in \arg\max_{\theta \in \Theta}\{v(S_\psi(\theta)) - c(S_\psi(\theta))\}$ and $S_{\psi'}(\theta) = S_{\psi^*}(\theta)$, we have $\theta_{\psi'} = \theta_{\psi^*}$.

Note that if $S_{\psi'}(\theta_{\psi'}) = S_{\psi^*}(\theta_{\psi^*})$, $v(f^*(\psi^*)) - c(f^*(\psi^*)) = v(f^*(\psi')) - c(f^*(\psi'))$ from Lemma 1. Then, since $\kappa(\psi') < \kappa(\psi^*)$, we have

$$v(f^*(\psi^*)) - c(f^*(\psi^*)) - \kappa(\psi^*) < v(f^*(\psi')) - c(f^*(\psi')) - \kappa(\psi').$$

Then, from Lemma 1, the principal strictly prefers ψ' to ψ^*, contradicting the optimality of ψ^*.

7.6 Proof of Lemma 5

To prove Lemma 5, suppose $\phi_{a'}, \phi_{m'} \in \Phi^*$, but $a' \in S_{\phi_{m'}}(\theta_{\psi^*})$. Then, consider the alternative description

$$\psi' = \bigwedge_{l \in \Phi^* \setminus \{\phi_{a'}\}} .$$

Since $a' \in S_{\phi_{m'}}(\theta_{\psi^*})$, $\psi'(S;\theta_{\psi^*}) = \psi^*(S;\theta_{\psi^*})$ for all S. Then, $S_{\psi'}(\theta_{\psi^*}) = S_{\psi^*}(\theta_{\psi^*})$. I claim that $\theta_{\psi'} = \theta_{\psi^*}$. To establish the claim, first, suppose $\theta_{\psi'} \neq \theta_{\psi^*}$. Then, $v(S_{\psi'}(\theta_{\psi'})) - c(S_{\psi'}(\theta_{\psi'})) \geq v(S_{\psi'}(\theta_{\psi^*})) - c(S_{\psi'}(\theta_{\psi^*}))$. From $S_{\psi'}(\theta_{\psi^*}) = S_{\psi^*}(\theta_{\psi^*})$, $v(S_{\psi'}(\theta_{\psi'})) - c(S_{\psi'}(\theta_{\psi'})) \geq v(S_{\psi^*}(\theta_{\psi^*})) - c(S_{\psi^*}(\theta_{\psi^*}))$. Note that, from Lemma 1 and 2, the principal's payoff from ψ is $v(S_\psi(\theta_\psi)) - c(S_\psi(\theta_\psi)) - \kappa(\psi)$. Then, since $\kappa(\psi') < \kappa(\psi^*)$, the principal strictly prefers ψ' to ψ^*, contradicting the optimality of ψ^*. Hence, if ψ^* is an equilibrium description, we need to have $\theta_{\psi'} = \theta_{\psi^*}$.

Since $\theta_{\psi'} = \theta_{\psi^*}$ and $S_{\psi'}(\theta_{\psi^*}) = S_{\psi^*}(\theta_{\psi^*})$, we have $S_{\psi'}(\theta_{\psi'}) = S_{\psi^*}(\theta_{\psi^*})$. Moreover, from Lemma 1, $f^*(\psi') = f^*(\psi^*)$. Then, since $\kappa(\psi') < \kappa(\psi^*)$, the principal strictly prefers ψ' to ψ^*, a contradiction.

7.7 Proof of Proposition 2

To prove the necessity of the condition, note that since $S_{\psi^*}(\theta_{\psi^*})$ is the most economical service that satisfies ψ^* under θ_{ψ^*}, whenever $S \not\supset S_{\psi^*}(\theta_{\psi^*})$, we must have $\psi^*(S; \theta_{\psi^*}) = 0$. Thus, if there is no $S' \in \mathcal{S}$ such that $S' \supsetneq S_{\psi^*}(\theta_{\psi^*})$, $S_{\psi^*}(\theta_{\psi^*})$ is the only service that satisfies ψ^*. That is, ψ^* is descriptively precise.

To prove the sufficiency of the condition, suppose there exists $S' \in \mathcal{S}$ such that $S' \supsetneq S_{\psi^*}(\theta_{\psi^*})$. From Lemma 4, ψ^* can be written as the conjunction of Φ^*, which is cost and logical-equivalent to ψ^*. Hence, $\psi^*(S_{\psi^*}(\theta_{\psi^*}); \theta_{\psi^*}) = \psi^*(S'; \theta_{\psi^*}) = 1$. That is, ψ^* is descriptively imprecise.

7.8 Proof of Proposition 3

Necessity: Note that

$$S_{\psi^*}(\theta) = \{a : \phi_a \in \Phi^*\} \cup \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta).$$

Thus, if

$$\bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta) = \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta_{\psi^*})$$

for all θ, then $S_{\psi^*}(\theta) = S_{\psi^*}(\theta_{\psi^*})$. Since $\psi(S; \theta) = 1$ iff $S \supset S_\psi(\theta)$, we have $\psi^*(S; \theta) = \psi^*(S; \theta_{\psi^*})$ for all S. That is, ψ^* is semantically precise.

Sufficiency: Suppose, for some θ,

$$\bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta) \neq \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta_{\psi^*})$$

Case 1. There exists θ' such that

$$\bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta') \cap \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta_{\psi^*}) \neq \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta'), \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta_{\psi^*}).$$

In this case, there exists a' such that $a' \in \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta_{\psi^*})$ and $a' \notin \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta')$. From Lemma 5, $\phi_{a'} \notin \Phi^*$. Hence, $\psi^*(S_{\psi^*}(\theta'); \theta_{\psi^*}) = 0$, whereas $\psi^*(S_{\psi^*}(\theta'); \theta') = 1$. That is, ψ^* is semantically imprecise.

Case 2. There is no θ' such that

$$\bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta') \cap \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta_{\psi^*}) \neq \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta'), \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta_{\psi^*}).$$

In this case, for any θ, it is either

$$\bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta) \supsetneq \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta_{\psi^*})$$

or

$$\bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta) \subsetneq \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta_{\psi^*})$$

Step 1. If there exists θ' such that $\bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta') \supsetneq \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta_{\psi^*})$, then

$$\bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta') \not\subset S_{\psi^*}(\theta_{\psi^*}).$$

To prove the claim, suppose $\bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta') \subset S_{\psi^*}(\theta_{\psi^*})$. Then, let

$$S' = S_{\psi^*}(\theta_{\psi^*}) \backslash \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta').$$

Then, consider the following description:

$$\psi' = \bigwedge_{\phi \in \{\phi_m : \phi_m \in \Phi^*\} \cup \{\phi_a : a \in S'\}} \phi.$$

Note that, by construction, $S_{\psi'}(\theta') = S_{\psi^*}(\theta_{\psi^*})$. Moreover, since $\bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta') \supsetneq \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta_{\psi^*})$,

$$|\bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta') \backslash \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta_{\psi^*})| > 0.$$

Thus, $\kappa(\psi') < \kappa(\psi^*)$.

Now, I claim that ψ^* cannot be the equilibrium description. First, if $\theta_{\psi'} \neq \theta'$, then, by definition, $v(S_{\psi'}(\theta_{\psi'})) - c(S_{\psi'}(\theta_{\psi'})) \geq v(S_{\psi'}(\theta')) - c(S_{\psi'}(\theta'))$. Since $S_{\psi'}(\theta') = S_{\psi^*}(\theta_{\psi^*})$, $v(S_{\psi'}(\theta_{\psi'})) - c(S_{\psi'}(\theta_{\psi'})) \geq v(S_{\psi'}(\theta_{\psi^*})) - c(S_{\psi'}(\theta_{\psi^*}))$. But then, since $\kappa(\psi') < \kappa(\psi^*)$, the principal strictly prefers ψ' to ψ^*, contradicting the optimality of ψ^*. Second, if $\theta_{\psi'} = \theta'$, then $S_{\psi'}(\theta_{\psi'}) = S_{\psi^*}(\theta_{\psi^*})$. But then, since $\kappa(\psi') < \kappa(\psi^*)$, the principal strictly prefers ψ' to ψ^*, a contradiction.

Step 2. If there exists θ' such that $\bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta') \supsetneq \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta_{\psi^*})$, then ψ^* is semantically imprecise.

From Step 1, we can focus on the case in which

$$\bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta') \not\subset S_{\psi^*}(\theta_{\psi^*}).$$

In this case, there exists $a' \in S_{\psi^*}(\theta')$ such that $a' \in \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta')$ and $a' \notin S_{\psi^*}(\theta_{\psi^*})$. Thus, $\psi^*(S_{\psi^*}(\theta_{\psi^*}); \theta_{\psi^*}) = 1$, but $\psi^*(S_{\psi^*}(\theta_{\psi^*}); \theta') = 0$. Hence, ψ^* is semantically imprecise.

Step 3. If $\bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta) \subsetneq \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta_{\psi^*})$ for all θ, then ψ^* is semantically imprecise.

In this case, there exists $a' \in \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta_{\psi^*})$ but $a' \notin \bigcup_{\phi_m \in \Phi^*} S_{\phi_m}(\theta')$ for some θ'. Then, clearly, $a' \notin S_{\psi^*}(\theta')$. Thus, $\psi^*(S_{\psi^*}(\theta'); \theta') = 1$, but $\psi^*(S_{\psi^*}(\theta'); \theta_{\psi^*}) = 0$. Hence, ψ^* is semantically imprecise.

7.9 Proof of Lemma 6

Suppose not. Then, the equilibrium description ψ^* consists entirely of elementary predicates. Moreover, from Lemma 3 and 4, the equilibrium description ψ^* takes the form of $\bigwedge_{a \in S} \phi_a$. Note that $\kappa(\bigwedge_{a \in S} \phi_a) = \kappa(|S|)$. Then, from Proposition 1 and the definition of $\hat{S}$,

$$\psi^* = \bigwedge_{a \in \hat{S}} \phi_a.$$

Consider the alternative description:

$$\psi' = \bigwedge_{a \in \hat{S} \setminus S_{\phi'_m}(\theta')} \phi_a \wedge \phi'_m$$

where θ' is such that $\hat{S} \supset S_{\phi'_m}(\theta')$ and $|S_{\phi'_m}(\theta')| > 1$. Since $\hat{S} \supset S_{\phi'_m}(\theta')$ and $|S_{\phi'_m}(\theta')| > 1$, $\kappa(\psi') < \kappa(\psi^*)$.

If $\theta_{\psi'} = \theta'$, then $S_{\psi'}(\theta_{\psi'}) = S_{\psi^*}(\theta_{\psi^*})$ by construction. The principal then prefers ψ' to ψ^*, contradicting the optimality of ψ^*. Suppose $\theta_{\psi'} \neq \theta'$. Then, since $S_{\psi'}(\theta') = S_{\psi^*}(\theta_{\psi^*})$, we have $v(S_{\psi'}(\theta_{\psi'})) - c(S_{\psi'}(\theta_{\psi'})) > v(S_{\psi^*}(\theta_{\psi^*})) - c(S_{\psi^*}(\theta_{\psi^*}))$. Then, the principal uses ψ' rather than ψ^*, a contradiction.

7.10 Proof of Lemma 7

Consider the following service description

$$\psi' = \bigwedge_{a \in S^{1st}} \phi_a.$$

From Lemma 1, $f^*(\psi') = S^{1st}$. Thus, the principal's payoff from ψ' is $v(S^{1st}) - c(S^{1st}) - \alpha k(|S^{1st}|)$.

Now, define

$$\alpha^* = \frac{1}{k(|S^{1st}|)} \left[v(S^{1st}) - c(S^{1st}) - \max_{S \in \{S' : S' \neq S^{1st}\}} \{v(S) - c(S)\} \right].$$

Clearly, $\alpha^* > 0$. If $\alpha \in (0, \alpha^*)$, by construction, $v(S^{1st}) - c(S^{1st}) - \alpha k(|S^{1st}|) > v(S_\psi(\theta_\psi)) - c(S_\psi(\theta_\psi))$ for any ψ such that $S_\psi(\theta_\psi) \neq S^{1st}$. Then, the principal's payoff from ψ' is higher than her payoff from any ψ such that $S_\psi(\theta_\psi) \neq S^{1st}$. Hence, the principal never chooses ψ such that $S_\psi(\theta_\psi) \neq S^{1st}$ in any equilibrium.

7.11 Proof of Proposition 5

Claim 1: *A higher α decreases efficiency.*

Let ψ^*_α be the equilibrium description under α. Suppose $\alpha'' > \alpha'$. From Proposition 1,

$$v(S_{\psi^*_{\alpha'}}(\theta_{\psi^*_{\alpha'}})) - c(S_{\psi^*_{\alpha'}}(\theta_{\psi^*_{\alpha'}})) - \alpha' k(n(\psi^*_{\alpha'})) \geq v(S_{\psi^*_{\alpha''}}(\theta_{\psi^*_{\alpha''}})) - c(S_{\psi^*_{\alpha''}}(\theta_{\psi^*_{\alpha''}})) - \alpha' k(n(\psi^*_{\alpha''})).$$

By rewriting the above inequality,

$$v(S_{\psi^*_{\alpha''}}(\theta_{\psi^*_{\alpha''}})) - c(S_{\psi^*_{\alpha''}}(\theta_{\psi^*_{\alpha''}})) - [v(S_{\psi^*_{\alpha'}}(\theta_{\psi^*_{\alpha'}})) - c(S_{\psi^*_{\alpha'}}(\theta_{\psi^*_{\alpha'}}))] \leq \alpha'[k(n(\psi^*_{\alpha''})) - k(n(\psi^*_{\alpha'}))]$$

Now, suppose α'' weakly improves efficiency. Then,

$$v(S_{\psi^*_{\alpha''}}(\theta_{\psi^*_{\alpha''}})) - c(S_{\psi^*_{\alpha''}}(\theta_{\psi^*_{\alpha''}})) - \alpha'' k(n(\psi^*_{\alpha''})) \geq v(S_{\psi^*_{\alpha'}}(\theta_{\psi^*_{\alpha'}})) - c(S_{\psi^*_{\alpha'}}(\theta_{\psi^*_{\alpha'}})) - \alpha' k(n(\psi^*_{\alpha'})).$$

By rewriting the above inequality,

$$v(S_{\psi^*_{\alpha''}}(\theta_{\psi^*_{\alpha''}})) - c(S_{\psi^*_{\alpha''}}(\theta_{\psi^*_{\alpha''}})) - [v(S_{\psi^*_{\alpha'}}(\theta_{\psi^*_{\alpha'}})) - c(S_{\psi^*_{\alpha'}}(\theta_{\psi^*_{\alpha'}}))] \geq \alpha'' k(n(\psi^*_{\alpha''})) - \alpha' k(n(\psi^*_{\alpha'}))$$

By combining the above inequalities,

$$\alpha'[k(n(\psi^*_{\alpha''})) - k(n(\psi^*_{\alpha'}))] \geq \alpha'' k(n(\psi^*_{\alpha''})) - \alpha' k(n(\psi^*_{\alpha'})).$$

But then $\alpha' \geq \alpha''$, a contradiction.

Claim 2. *There exists an environment in which a higher α decreases the value of an equilibrium service.*

Suppose $\mathcal{S} = P(A)$ and $\Phi = \{\phi_a\}_{a \in A}$, i.e., $M = \emptyset$. That is, all predicates are elementary. Let

$$\psi_S = \bigwedge_{a \in S} \phi_a.$$

Then, the principal's payoff from ψ_S is $v(S) - c(S) - \alpha k(|S|))$.

Now, consider $S' \subsetneq S^{1st}$. From Assumption 1, 2, and 3, $v(S^{1st}) > v(S')$. Let $\alpha_{S'}$ be the solution of $v(S^{1st}) - c(S^{1st}) - [v(S') - c(S')] = \alpha k(|S^{1st} \backslash S'|)$. Let $\hat{\mathcal{S}} = \{S : v(S) \geq v(S^{1st})\}$. Since S^{1st} is the first best, $v(S^{1st}) - c(S^{1st}) \geq v(S) - c(S)$ for all $S \in \hat{\mathcal{S}}$. Thus, if $\alpha > \alpha_{S'}$, the principal strictly prefers $\psi_{S'}$ to ψ_S for any $S \in \hat{\mathcal{S}}$. It follows that $\psi^* \neq \psi_S$ for any $S \in \hat{\mathcal{S}}$ if $\alpha > \alpha_{S'}$. That is, $v(f^*(\psi^*)) < v(S^{1st})$ if $\alpha > \alpha_{S'}$.

Turning to the case of a small α, from Lemma 6, $\psi^* = \psi_{S^{1st}}$ and $v(f^*(\psi^*)) = v(S^{1st})$ if $\alpha < \alpha^*$. Hence, a larger α reduces the equilibrium service value.

Claim 3. *There exists an environment in which a higher α increases the value of an equilibrium service.*

Suppose $M = \{m\}$ and $S_{\phi_m}(\theta) \supsetneq S^{1st}$ for all θ. Moreover, assume $|S^{1st}| > 1$. Let $\psi' = \phi_m$. Then, consider v and c such that

$$v(S_{\phi_m}(\theta_{\psi'})) - c(S_{\phi_m}(\theta_{\psi'})) > \max_{a \in A}\{v(\{a\}) - c(\{a\})\} \tag{2}$$

Then, since $\kappa(\phi_m) = \kappa(\phi_a)$, the principal strictly prefers ψ' to any elementary predicate.

Now, consider $\alpha' > 0$ such that

$$\alpha' > \frac{v(S^{1st}) - c(S^{1st}) - [v(S_{\phi_m}(\theta_{\psi'})) - c(S_{\phi_m}(\theta_{\psi'}))]}{k(2) - k(1)}.$$

Then, $v(S^{1st}) - c(S^{1st}) - \alpha' k(2) < v(S_{\phi_m}(\theta_{\psi'})) - c(S_{\phi_m}(\theta_{\psi'})) - \alpha' k(1)$. Note that the left hand side of the above inequality is the principal's highest possible equilibrium payoff that can be induced by a description with more than one predicate. Then, from Inequality (4), the equilibrium description under α' is ψ'. Since $S_{\phi_m}(\theta) \supsetneq S^{1st}$ for all θ, $v(S_{\phi_m}(\theta_{\psi'})) > v(S^{1st})$ from Assumption 1, 2, and 3.

Turning to the case of a small α, from Lemma 6, if $\alpha \in (0, \alpha^*)$, $f^*(\psi^*) = S^{1st}$. Since $\alpha' > \alpha^*$, a higher α increases the value of the equilibrium service.

7.12 Proof of Proposition 6

Clam 1: *Any enrichment weakly improves efficiency.*

Let $\Psi(\Phi)$ be the set of predicate formulas generated from $\Phi = \{\phi_a\}_{a \in A} \cup \{\phi_m\}_{m \in M}$. If M' is an enrichment of M, then the new set of predicates is $\Phi' = \{\phi_a\}_{a \in A} \cup \{\phi_m\}_{m \in M'}$. Clearly, $\Phi' \supsetneq \Phi$. Thus, $\Psi(\Phi') \supsetneq \Psi(\Phi)$. From Proposition 1, ψ^* solves $\max_{\psi \in \Psi} v(S_\psi(\theta_\psi)) - c(S_\psi(\theta_\psi)) - \kappa(\psi)$. Thus, if $\Psi(\Phi') \supsetneq \Psi(\Phi)$, then

$$\max_{\psi \in \Psi(\Phi')} v(S_\psi(\theta_\psi)) - c(S_\psi(\theta_\psi)) - \kappa(\psi) \geq \max_{\psi \in \Psi(\Phi)} v(S_\psi(\theta_\psi)) - c(S_\psi(\theta_\psi)) - \kappa(\psi).$$

Claim 2: *There exists an environment in which a refinement increases efficiency.*

Suppose $M = \{m\}$ and $S_{\phi_m}(\theta) \supsetneq S^{1st}$ for all θ. Moreover, assume $|S^{1st}| > 1$. From Lemma 1, $f^*(\psi) = S_\psi(\theta_\psi)$. Since $S_{\phi_m}(\theta) \supsetneq S^{1st}$ for all θ, whenever ψ contains ϕ_m, $S_\psi(\theta_\psi) \supsetneq S^{1st}$. Form Lemma 6, if $\alpha \in (0, \alpha^*)$, $f(\psi^*) = S^{1st}$. Thus, the equilibrium description is

$$\psi^* = \bigwedge_{a \in S^{1st}} \phi_a$$

if $\alpha \in (0, \alpha^*)$.

Now, consider a refinement $M' = \{m', m''\}$ such that $S_{\phi_{m'}}(\theta) \subset S^{1st}$ for all θ and $|S_{\phi_{m'}}(\theta)| > 1$

for some θ, whereas $S_{\phi_{m''}}(\theta) \not\subset S^{1st}$ for all θ. Then, consider the following alternative description.

$$\psi_\theta = \phi_{m'} \wedge \bigwedge_{a \in S^{1st} \setminus S_{\phi_{m'}}(\theta)} \phi_a.$$

Note that since $S_{\phi_{m'}}(\theta) \subset S^{1st}$ for all θ, $S_{\psi_\theta}(\theta) = S^{1st}$. Thus, $\theta_{\psi_\theta} = \theta$.

Let $\tilde{\theta}$ be the solution of $\min_\theta |S^{1st} \setminus S_{\phi_{m'}}(\theta)|$. Then, by construction, $\kappa(\psi_{\tilde{\theta}}) \leq \kappa(\psi_\theta)$ for all θ. Moreover, since $|S_{\phi_{m'}}(\tilde{\theta})| > 1$, $\kappa(\psi_{\tilde{\theta}}) < \kappa(\bigwedge_{a \in S^{1st}} \phi_a)$. From Lemma 6, if $\alpha \in (0, \alpha^*)$, $f(\psi^*) = S^{1st}$. Then, since $\psi_{\tilde{\theta}}$ is the most economical description that induces S^{1st}, $\psi^* = \psi_{\tilde{\theta}}$ under M'. That is, the principal's equilibrium payoff under M' is higher than that under M.

Claim 3: *There exists an environment in which a refinement reduces efficiency.*

Suppose $M = \{m\}$ and $S_{\phi_m}(\theta) \subset S^{1st}$ for all θ. Let

$$\psi_{\tilde{\theta}} = \phi_{m'} \wedge \bigwedge_{a \in S^{1st} \setminus S_{\phi_{m'}}(\tilde{\theta})} \phi_a.$$

where $\tilde{\theta}$ solves $\min_\theta |S^{1st} \setminus S_{\phi_{m'}}(\theta)|$. By construction, $S_{\psi_{\tilde{\theta}}}(\theta) = S^{1st}$ and $\psi_{\tilde{\theta}}$ is the most economical description that induces S^{1st}. From Lemma 6, if $\alpha \in (0, \alpha^*)$, $f(\psi^*) = S^{1st}$. Thus, $\psi^* = \psi_{\tilde{\theta}}$ under M.

Now, consider $M' = \{m', m''\}$, which is a refinement of M. Let

$$\psi'_{\tilde{\theta}} = \phi_{m'} \wedge \phi_{m''} \wedge \bigwedge_{a \in S^{1st} \setminus S_{\phi_{m'}}(\tilde{\theta}) \cup S_{\phi_{m''}}(\tilde{\theta})} \phi_a.$$

Since $S_{\phi_m}(\theta) = S_{\phi_{m'}}(\theta) \cup S_{\phi_{m''}}(\theta)$ for all θ, $S_{\psi_{\tilde{\theta}}}(\theta) = S_{\psi'_{\tilde{\theta}}}(\theta) = S^{1st}$. Since $\tilde{\theta}$ solves $\min_\theta |S^{1st} \setminus S_{\phi_{m'}}(\theta)|$, it also solves $\min_\theta |S^{1st} \setminus S_{\phi_{m'}}(\theta) \cup S_{\phi_{m''}}(\theta)|$. Thus, $\psi_{\tilde{\theta}}$ is the most economical description that induces S^{1st} under M'. Then, from Lemma 6, $\psi^* = \psi'_{\tilde{\theta}}$ under M'. Note that since $\kappa(\psi'_{\tilde{\theta}}) > \kappa(\psi_{\tilde{\theta}})$, the principal's equilibrium payoff under M is higher than that under M'.

Claim 4: *No formalization improves efficiency.*

From the definition of θ_{ψ^*}, $v(S_{\psi^*}(\theta_{\psi^*})) - c(S_{\psi^*}(\theta_{\psi^*})) - \kappa(\psi^*) \geq v(S_{\psi^*}(\theta)) - c(S_{\psi^*}(\theta)) - \kappa(\psi^*)$ for any θ. Thus, if formalization θ' strictly improves efficiency, it must make the principal choose some ψ' such that $v(S_{\psi'}(\theta')) - c(S_{\psi'}(\theta')) - \kappa(\psi') > v(S_{\psi^*}(\theta_{\psi^*})) - c(S_{\psi^*}(\theta_{\psi^*})) - \kappa(\psi^*)$. From the definition of $\theta_{\psi'}$, $v(S_{\psi'}(\theta_{\psi'})) - c(S_{\psi'}(\theta_{\psi'})) \geq v(S_{\psi'}(\theta')) - c(S_{\psi'}(\theta'))$. But then, $v(S_{\psi'}(\theta_{\psi'})) - c(S_{\psi'}(\theta_{\psi'})) - \kappa(\psi') > v(S_{\psi^*}(\theta_{\psi^*})) - c(S_{\psi^*}(\theta_{\psi^*})) - \kappa(\psi^*)$, contradicting the optimality of ψ^*.

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