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Abstract

This paper presents an empirical study on hedging long-dated crude oil futures options with forward price models incorporating stochastic interest rates and stochastic volatility. Several hedging schemes are considered including delta, gamma, vega and interest rate hedge. Factor hedging is applied to the proposed multi-dimensional models and the corresponding hedge ratios are estimated by using historical crude oil futures prices, crude oil option prices and Treasury yields. Hedge ratios from stochastic interest rate models consistently improve hedging performance over hedge ratios from deterministic interest rate models, an improvement that becomes more pronounced over periods with high interest rate volatility, such as during the GFC. An interest rate hedge consistently improves hedging beyond delta, gamma and vega hedging, especially when shorter maturity contracts are used to roll the hedge forward. Furthermore, when the market experiences high interest rate volatility and the hedge is subject to high basis risk, adding interest rate hedge to delta hedge provides an improvement, while adding gamma and/or vega to the delta hedge worsens performance.

Keywords: Stochastic interest rates; Delta hedge; Interest rate hedge; Long-dated crude oil options; JEL: C13, C60, G13, Q40

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1. Introduction

Motivated by the debacle of the German company Metallgesellschaft (MG) at the end of 1993, several research papers have investigated the methods and risks in hedging long-dated over-the-counter forward commodity contracts by using short-dated contracts, typically short-dated futures. The investigation conducted in Edwards and Canter (1995) and Brennan and Crew (1997) conclude that MG’s stack-and-roll hedging strategy was flawed and exposed the company to significant basis risk.

Other papers in the literature that consider using multiple futures contracts to hedge long-dated commodity commitments include Veld-Merkoulova and De Roon (2003), Bühler, Korn, and Schöbel (2004) and Shiraya and Takahashi (2012). Veld-Merkoulova and De Roon (2003) by proposing a term structure model of futures convenience yields, develop a strategy that minimises both spot price risk and basis risk by using two futures contracts with different maturities. Empirical results show that this two-futures strategy outperforms the simple stack-and-roll hedge substantially in hedging performance. Bühler et al. (2004) empirically demonstrate that the oil market is characterised by two pricing regimes - when the spot oil price is high (low), the sensitivity of the futures price is low (high). They then propose a continuous-time, partial equilibrium, two-regime model and show that their model implies a relatively high (low) hedge ratios when oil prices are low (high). Shiraya and Takahashi (2012) propose a mean reversion Gaussian model of commodity spot prices and futures prices are derived endogenously. In their hedging analysis, by using 3 futures contracts with different maturities to hedge the long-dated forward contract, they calculate the hedging positions by matching their sensitivities to the different sources of uncertainty. They empirically demonstrate that their Gaussian model outperforms the stack and roll model used by Metallgesellschaft.

Trolle and Schwartz (2009) propose a multi-dimensional stochastic volatility model for commodity derivatives featuring unspanned stochastic volatility. They show that adding options to the set of hedging instruments significantly improves hedging of volatility trades, such as straddles, compared to using futures only as hedging instruments. Dempster, Medova, and Tang (2008) propose a four-factor model for two spot prices and their convenience yields and develop closed-form pricing and hedging formulae for options on spot and futures spreads of commodity. Chiarella, Kang, Nikitopoulos, and Tô (2013) empirically demonstrate that hump-shaped volatility specifications reduce the hedging error of crude oil volatility trades (straddles) compared to the exponential volatility specification counterpart. However, all these papers assume deterministic interest rates and it is unclear to what extent these models can provide adequate hedges for long-dated options positions. The research literature on hedging long-dated commodity option positions is rather limited and this paper aims to make a contribution by empirically investigating the hedging of crude oil futures options with maturities up to six years.

We use the Light Sweet Crude Oil (WTI) futures and option dataset from the NYMEX\(^1\) spanning a 6-year period from January 2006 to December 2011. A call futures option maturing on December 2011 is hedged from 3rd July, 2006 to 31st October, 2011. During this 6-year period, a number of major events happened, such as the Global Financial Crisis (GFC) and

\(^1\)The database has been provided by CME.
the Arab Spring and Libyan revolution that had significant impact on spot crude oil prices, crude oil futures and its options, as well as interest rates. The gross domestic product growth rates of China and India have increased exponentially during the decade of the 2000s. To support the growth of the economy, the consumption of energy also increased. In particular, China’s demand of crude oil grew at a 7.2% annual logarithmic rate between 1991 and 2006. This phenomenal growth rate, among other factors, increased the demand in crude oil. On the supply side, Saudi Arabia, the biggest oil exporter in the world, reduced its crude oil production in 2007. Consequently, crude oil prices increased sharply, sending the price to a high of US$145 per barrel on 3rd July, 2008, which was immediately followed by a spectacular collapse in prices and by the end of 2008, the spot crude oil price was below US$40 per barrel (see Hamilton (2009) and Hamilton (2008)). The US Treasury yields were above 4.5% before the July 2007 and had decreased steadily to nearly zero by the end of 2008, after the GFC.

The empirical analysis in this paper considers a position in a long-dated call futures option with a maturity in December 2011. This option is hedged over five years using several hedging schemes such as delta hedge, delta-vega hedge and delta-gamma hedge. For the purpose of comparison, two models are used to compute the suitable hedge ratios; one with stochastic interest rates as modelled in Cheng, Nikitopoulos, and Schlögl (2015) and one with deterministic interest rates fitted to a Nelson and Siegel (1987) curve. These models are estimated from historical crude oil futures and option prices and Treasury yields by using the extended Kalman filter, see Cheng, Nikitopoulos, and Schlögl (2016a) for a similar estimation application. The hedge ratios for delta-interest rate (delta-IR), delta-vega-interest-rate (delta-vega-IR), and delta-gamma and delta-gamma-interest-rate (delta-gamma-IR) hedges are also computed but they are limited to the stochastic interest rate model. The hedge ratios are derived using the factor-hedging methodology, the effectiveness of which has been analysed in Cheng, Nikitopoulos, and Schlögl (2016b).

From this analysis, several results have emerged. Firstly, when an interest rate hedge is added to the delta, gamma and vega hedge, there is a consistent improvement to the hedging performance, especially when shorter maturity contracts are used to roll the hedge forward (thus more basis risk is present). Secondly, because of the high interest rate volatility, interest rate hedging was more important during the GFC than in recent years. Over periods of high interest rate volatility (for instance, pre-GFC and during GFC) and when shorter maturity hedging contracts are used, the delta-IR hedge consistently improves hedging performance compared to delta hedge, while adding a gamma or vega hedge to the delta hedge worsens hedging performance. Thirdly, due to lower basis risks, using hedging instruments with maturities closer to the maturity of the option to be hedged reduces the hedging error. Fourthly, the hedging performance from the stochastic interest rate model is consistently better than the hedging performance from the deterministic interest rate model, with the effect being more pronounced during the GFC. However, there is only marginal improvement over the deterministic interest rate model during pre-crisis period and no noticeable improvement after 2010.

The remainder of the paper is structured as follows. Section 2 presents the application of the factor hedging methodology on the three-dimensional stochastic volatility–stochastic interest rate forward price model developed in Cheng et al. (2015). It also derives the hedge ratios for a variety of hedging schemes including delta, delta-IR, delta-vega, delta-vega-IR, delta-gamma and delta-gamma-IR. Section 3 describes the methodology to assess hedging
performance on long-dated crude oil futures options, including the details of the dataset used. Section 4 presents the empirical results and discusses their implications. Section 5 concludes.

2. Factor hedging for a stochastic volatility–stochastic interest rate model

Factor hedging is a broad hedging method that allows one to hedge simultaneously multiple factors and multiple dimensions impacting the forward curve of commodities, the instantaneous volatility component and the interest rate variation and subsequently the value of commodity derivatives portfolios. By considering the $n$-dimensional stochastic volatility and $N$-dimensional stochastic interest rate model developed in Cheng et al. (2015), to hedge the $n$-dimensional forward rate risks (that is $W^r_i(t)$ for $i = 1, 2, \ldots, n$) it is necessary to use $n$ number of hedging instruments such as futures contracts. To further hedge the $n$ number of volatility risks (that is $W^\sigma_i(t)$ for $i = 1, 2, \ldots, n$), it is required to use an additional $n$ number of volatility-sensitive hedging instruments such as futures options. Since the proposed model considers stochastic interest rates, the $N$-dimensional interest rate risks (that is $W^\rho_i(t)$ for $i = 1, 2, \ldots, N$) should be hedged by using $N$ number of interest-rate-sensitive contracts such as bonds. For completeness, we present next the model used to compute the required hedge ratios.

2.1. Model

The model assumes that the time $t$-futures price $F(t; T, \sigma_t)$ of a commodity, for delivery at time $T$, evolves as follows:

$$\frac{dF(t; T, \sigma_t)}{F(t; T, \sigma_t)} = \sum_{i=1}^{n} \sigma^F_i(t; T, \sigma_t)dW^r_i(t),$$

(1)

where $\sigma_t = \{\sigma_1(t), \ldots, \sigma_n(t)\}$ and for $i = 1, 2, \ldots, n$,

$$d\sigma_i(t) = \kappa_i(\bar{\sigma}_i - \sigma_i(t))dt + \gamma_i dW^\sigma_i(t).$$

Also,

$$r(t) = r(t) + \sum_{j=1}^{N} r_j(t)$$

where, for $j = 1, 2, \ldots, N$,

$$dr_j(t) = -\lambda_j(t)r_j(t)dt + \theta_j dW^\rho_j(t).$$

The functional form of the volatility term structure $\sigma^F_i(t; T, \sigma_t)$ is specified as follows:

$$\sigma^F_i(t; T, \sigma_t) = (\xi_0i + \xi_i(T - t))e^{-\eta_i(T-t)}\sigma_i(t)$$
with $\xi_0$, $\xi_i$, and $\eta_i \in \mathbb{R}$ for all $i \in \{1, 2, \ldots, n\}$. The correlation structure of the associated Wiener processes is given by

$$dW^x_i(t)dW^\sigma_j(t) = \begin{cases} \rho^x_{i,j} dt, & \text{if } i = j, \\ 0, & \text{otherwise} \end{cases}$$

$$dW^\tau_i(t)dW^\tau_j(t) = \begin{cases} \rho^\tau dt, & \text{if } j = 1, \\ 0, & \text{otherwise} \end{cases}$$

$$dW^\sigma_i(t)dW^\tau_j(t) = \begin{cases} \rho^\sigma dt, & \text{if } j = 1, \\ 0, & \text{otherwise} \end{cases}$$

for $i \in \{1, \ldots, n\}$, $j \in \{1, \ldots, n\}$, $i \in \{1, \ldots, N\}$ and $j \in \{1, \ldots, N\}$.

### 2.2. Delta hedging

For an $n$-dimensional model, factor delta hedging requires $n$ hedging instruments such as Futures contracts with different maturities, denoted by $F(t, T_j, \sigma_t)$, $j = 1, \ldots, n$. Let $\check{Y}(t, T_M, \sigma_t)$ be the value of the futures option to be hedged and $T_M$ be its maturity. Let $\Delta Y_H^\delta$, and $\Delta Y_H^\delta_i$ denote the change in price of the portfolio and the change in the price of the option on futures (which is the target to be hedged) by the $i^{th}$ shock of the uncertainty in the futures curve (that is $dW^\tau_i(t)$) respectively and $\delta_j$ be the position corresponding to the $j^{th}$ hedging instrument, we have:

$$\Delta Y_H^\delta = \Delta Y_H^\delta + \delta_1 \Delta F_1(t, T_1, \sigma_t) + \delta_2 \Delta F_2(t, T_2, \sigma_t) + \ldots + \delta_n \Delta F_n(t, T_n, \sigma_t).$$

(3)

For an $n$-dimensional model with $n$ amount of hedging instruments the set of equations in equation (3) forms a system of $n$ linear equations which can be solved exactly by matrix inversion. However, from numerical results, some of the values of $\delta_i$ produced by using this method may be unnecessarily large and consequently lead to very large profit and loss of the hedging portfolio. We use an alternative and more general method by simply minimising the sum of the squared hedging errors, $\Delta Y_H^\delta_i$:

$$\min_{\delta_1, \ldots, \delta_n} \left\{ (\Delta Y_H^\delta_{i,1})^2 + (\Delta Y_H^\delta_{i,2})^2 + \ldots + (\Delta Y_H^\delta_{i,n})^2 \right\}$$

(4)

with a constraint that $\delta_i \leq \ell$. \(^2\) The changes in the value of the hedging instruments $\Delta F_i(t, T_j, \sigma_t)$ can be approximated by the discretisation of the stochastic differential equation, as follows:\(^3\)

$$\Delta F_i(t, T_j, \sigma_t) = (F_i(t, T_j, \sigma_t)\sigma_i(t, T_j)\Delta W_i(t)) - \left(F(t, T_j, \sigma_t)\sigma_i(t, T_j)( - \Delta W_i(t))\right)$$

$$= 2F(t, T_j, \sigma_t)\sigma_i(t, T_j)\Delta W_i(t).$$

\(^2\)\(\ell = 1\) seems to be a good choice to give low hedging errors and the hedging results are stable by varying $\ell$ slightly.

\(^3\)This first order approximation is sufficient for the study at hand, because any additional accuracy on the calculation of the hedge would be drowned out by the fact that the model is only an approximation of the empirical reality.
Lastly we can calculate the change in the option on futures $\Delta \Upsilon_i^\sigma$ by the $i^{th}$ shock as:

$$\Delta \Upsilon_i^\sigma(t, T_j) = \Upsilon \left( F_{i,U}(t, T_j, \sigma_i) \right) - \Upsilon \left( F_{i,D}(t, T_j, \sigma_i) \right),$$

(5)

where $F_{i,U}$ and $F_{i,D}$ denote the ‘up’ and ‘down’ moves of the price of the underlying hedging instrument (e.g. futures price):

$$F_{i,U}(t, T_j, \sigma_i) = F(t, T_j, \sigma_i) + F(t, T_j, \sigma_i)\sigma_i(t, T_j)\Delta W_i(t)$$

(6)

$$F_{i,D}(t, T_j, \sigma_i) = F(t, T_j, \sigma_i) - F(t, T_j, \sigma_i)\sigma_i(t, T_j)\Delta W_i(t).$$

(7)

We note that the change in $W_i(t)$ is normally distributed, so an ‘up’ or ‘down’ move is just a notation. An ‘up’ move does not necessarily mean $\Delta W_i(t) > 0$.

2.3. Delta-Vega hedging

The condition in equation (4) only immunises small risks generated from the uncertainty that directly impacts the underlying asset, for instance, the futures curve. It cannot mitigate risks originating from an instantaneous volatility which may be stochastic. In order to account for the risks of a stochastic volatility process, an additional number of $n$ futures options may be used as hedging instruments to simultaneously immunise both volatility and futures price risks. Let $v_i$ be the number of futures options that have values of $\Psi(t, T_j, \sigma_i)$ for $j = 1, \ldots, n$. The number of futures options $v_1, v_2, \ldots, v_n$ are determined similarly to equation (4) such that the overall changes in the value of the hedged portfolio, due to volatility risk, are minimised. Setting up the hedged portfolio, we have:

$$\Delta \Upsilon_i^\sigma(t, T_j) \triangleq \Delta \Upsilon_i^\sigma + v_1 \Delta \Psi_i^\sigma \left( F(t, T_1), t, T_1 \right) + v_2 \Delta \Psi_i^\sigma \left( F(t, T_2), t, T_2 \right) + \ldots + v_n \Delta \Psi_i^\sigma \left( F(t, T_n), t, T_n \right),$$

(8)

where $\Delta \Psi_i^\sigma \left( F(t, T_j), t, T_j \right) = \Psi \left( F(t, T_j), t, T_j, \sigma_i^{U\sigma} \right) - \Psi \left( F(t, T_j), t, T_j, \sigma_i^{D\sigma} \right)$. $\sigma_i^{U\sigma}$ represents the vector of stochastic volatility processes where the $i^{th}$ element $\sigma_i(t)$ has been shocked by an ‘up’ movement in $W_i^\sigma(t)$, denoted by $\sigma_i^{U\sigma}$ and it can be obtained as follows:

$$\sigma_i^{U\sigma} = \sigma_i(t) + \kappa_i(\bar{\sigma}_i - \sigma_i(t))\Delta t + \gamma_i \Delta W_i^\sigma(t).$$

$\sigma_i^{D\sigma}$ is interpreted in a similar way:

$$\sigma_i^{D\sigma} = \sigma_i(t) + \kappa_i(\bar{\sigma}_i - \sigma_i(t))\Delta t - \gamma_i \Delta W_i^\sigma(t).$$

The hedging portfolio consisting of one target option to be hedged and $n$ additional options, with number of contracts $v_i$, which are determined by a similar minimisation procedure to the one outlined in equation (4). This portfolio, however, is not delta-neutral. To make this hedging portfolio also delta-neutral, an additional $n$ number of futures contracts with number of positions $\delta_i$ are included in the vega-neutral portfolio and then the $\delta_i$ such that the portfolio

\[\text{Note that we are not using a futures option as a hedging instrument that matches exactly both the maturity and strike of the target futures option that we are hedging because if we can use an exactly matching option to hedge then the hedged portfolio would have a value of zero at all times.}\]
is both delta- and vega-neutral is determined. Thus a portfolio that is simultaneously delta- and vega-neutral should satisfy:

$$\Delta \Psi^H \overset{\Delta}{=} \Psi + \delta_1 F(t, T_1) + \delta_2 F(t, T_2) + \ldots + \delta_n F(t, T_n) + v_1 \Psi(F(t, T_1), t, T_1) + v_2 \Psi(F(t, T_2), t, T_2) + \ldots + v_n \Psi(F(t, T_n), t, T_n).$$

Since the ‘up’ and ‘down’ shocks of the stochastic volatility process $\sigma_t$ have no impact on futures prices, we can determine $v_1, \ldots, v_n$ by applying $n$ volatility shocks to equation (8) to calculate $\Delta \Psi^H_{t,i}$ and minimise its squared hedging errors as shown in equation (4). The next step is to determine $\delta_i, \ldots, \delta_n$ by applying $n$ shocks to equation (9) (with $v_i$ determined previously and are held as constants) and then minimising its squared errors.

2.4. Delta-Gamma hedging

Delta hedging can provide sufficient protection against small price changes, but not against larger price changes. To hedge larger price changes, a second order hedging is required to take into account the curvature of option prices, in other words, gamma hedging. Gamma measures the rate of change of the option’s delta with respect to the underlying, which is equivalent to the second derivative of the option price with respect to the underlying. If gamma is low, re-balancing of the portfolio infrequently may be sufficient because the delta of the option does not vary much when the underlying moves. However, if gamma is high, it is necessary to re-balance the portfolio frequently because when the underlying moves, the delta of the option is not accurate anymore. Hence, it cannot be used as an efficient hedge against market risk. The hedging portfolio is the same as the portfolio in equation (9) but we use a different method to calculate the positions $v_1, v_2, \ldots, v_n$. To determine the amount of positions, we construct the change of the hedging portfolio given by:

$$\Delta \Psi^H_{t,i} \overset{\Delta}{=} \Delta \Psi^H_{t} + v_1 \Delta \Psi^H_{t}(F(t, T_1), t, T_1) + v_2 \Delta \Psi^H_{t}(F(t, T_2), t, T_2) + \ldots + v_n \Delta \Psi^H_{t}(F(t, T_n), t, T_n),$$

where

$$\Delta \Psi^H_{t}(F(t, T_j), t, T_j) = \Psi(F_{i,U}(t, T_j), t, T_j, \sigma_t) - 2\Psi(F(t, T_j), t, T_j, \sigma_t) + \Psi(F_{i,D}(t, T_j), t, T_j, \sigma_t).$$

$F_{i,U}$ and $F_{i,D}$ are the ‘up’ and ‘down’ moves of the futures price as defined in equation (6) and equation (7). The change of the target futures option $\Delta \Psi^H_{t}$ is calculated similarly to $\Delta \Psi^H_{t}(F(t, T_1), t, T_1)$. The rest of the steps to determine $v_1, \ldots, v_n$ are exactly the same as in subsection 2.2.

2.5. Delta-IR, Delta-Vega-IR and Delta-Gamma-IR hedging

To immunise $N$ risks from the interest rate shocks, $N$ additional bond contracts with different maturities are required. All together, to apply a delta-vega-IR factor hedge, we need $n$ number of futures with different maturities, $n$ number of options on futures with different maturities and $N$ number of bond contracts with different maturities. Since the shocks from the stochastic volatility process have no impact on bonds and futures, the first
step is to determine $v_1, \ldots, v_n$, which are exactly the same as in equation (8). The shocks from the interest rate process have an impact on futures options as well as on bonds, so the next step is to determine the number of bond contracts by minimising the sum of the squared errors of the hedging portfolio consisting of options and bonds. The construction of a delta-gamma-IR factor hedge and the method to determine the number of the bond contracts follow the same procedure as the delta-vega-IR factor hedge and the detail is therefore omitted. To construct a delta-IR hedge we need $n$ number of futures with different maturities and $N$ number of bond contracts with different maturities. Options are not used as hedging instruments to construct this delta-IR hedge.

3. Hedging futures options

Crude oil futures options with maturities beyond five years are hedged by using the above mentioned hedging schemes. The three-dimensional version of the stochastic volatility–stochastic interest rate forward price model (1), (i.e., $n = N = 3$) is employed to compute the required hedging ratios. For a comparison of the hedging performance between the stochastic and the deterministic interest rate model, we also consider the deterministic interest rate counterpart of this three-dimensional model, as fitted to a Nelson and Siegel (1987) curve, see Cheng et al. (2016a) for details. To capture the impact of stochastic interest rates in the hedging performance, hedging under the deterministic interest rate specifications is compared to the stochastic interest rate specifications. Furthermore, to assess the hedging performance of these schemes in terms of the choice of the hedging instruments, several scenarios are considered for varying maturities of the hedging instruments.

3.1. Methodology

A 6-year crude oil dataset of futures and options is used in the investigations starting from January 2006 till December 2011. Crude oil futures options that mature in December are considered as the target option to hedge because December contracts are the only contracts with maturities over five years in our dataset. June contracts are also liquid but they have a maximum maturity of only four years in the dataset. There are only a few futures options with a maturity of December 2011 and with non-zero open interest that persist throughout the whole 5-year period. Thus the futures option with a strike of $62$ and a maturity of December 2011 is used as the target option to be hedged.

The 6-year crude oil dataset of futures and options is sub-divided into 12 half-year periods. The first sub-period is between January 2006 and June 2006, the second is between July 2006 and December 2006, the third sub-period is between January 2007 and June 2007 and so on. The last sub-period, which is the 12th, is between July 2011 and October 2011. Since a crude oil option on futures contract ceases trading around the 17th of a month prior to the maturity month of the futures option, a December contract would cease trading around the 17th of November, and in the dataset we exclude options with maturities under 14 calendar days, the hedging performance analysis is terminated at the end of October 2011. For each of the 12 sub-periods, a set of model parameters is estimated for the stochastic interest rate

\[^5\text{See CME's Crude Oil Futures Contract Specs and Crude Oil Options Contract Specs.}\]
model and for the deterministic interest rate model using Kalman filter, as outlined in Cheng et al. (2016a).

We use model parameters from previous sub-period together with the current state variables to estimate the hedge ratio for the current sub-period. For example, during the hedging analysis between July 2006 and December 2006, the model parameters estimated in the first sub-period are used together with the up-to-date state variables to derive the hedging ratios. This procedure is followed until October 2011. The idea of this is that out-of-sample model parameters are used when we derive the hedge ratios, i.e., the hedge ratios only use market information which is available at the time for which they are constructed.

To compare differences in the hedging performance of a long-dated option between using longer maturity hedging instruments and shorter maturity hedging instruments, four scenarios are investigated where the hedging instruments have varying maturities. For the three-dimensional model used, as illustrated in Section 2, to construct a delta-hedged portfolio, it is necessary to use three hedging instruments, thus futures contracts with three different maturities. In theory any three different maturities would be fine. But in practice, the liquidity of futures contracts with a maturity of more than a year decreases significantly with an exception of futures maturing in June or December. Only futures contracts maturing in December are available with a maturity of more than 3 years. So in this empirical analysis, June and December futures contracts with maturities of less than 3 years are considered, and only December contracts with a maturity longer than 3 years.

In the first scenario, hedging instruments with the three most adjacent June or December contracts are used. For instance, if the trading date is 15th of July, 2006, then hedging instruments with maturities of December 2006, June 2007 and December 2007 are used. By the end of October 2006, the December 2006 contract gets rolled over to June 2008 (and keeping the other two contracts), and so on. The second scenario is similar to the first scenario except that in the beginning of the hedging period the first hedging contract matures in December 2007, followed by June 2008 and December 2008. As the trading date gets to the end of October 2007, the December 2007 contract gets rolled over to June 2009, and so on. The third and fourth scenarios follow this idea. Table 1 shows the maturities of contracts used in these four scenarios. To construct a delta-vega or delta-gamma hedged portfolio, three additional options on futures are required. The maturities of these options are selected according to Table 1. Their strikes are selected based on a combination of liquidity and moneyness. We first filter out near-the-money options with strikes that are ±15% away from the at-the-money strike and then out of these strikes, we select the strike with the highest open interest.

3.2. Monte Carlo simulation

It is unclear what is the best approach to choose the size of the ‘up’ and ‘down’ moves \( \Delta W_i(t), \Delta W'_i(t) \) and \( \Delta W''_i(t) \) required in the factor hedging, as discussed in Section 2. Along the lines of Chiarella et al. (2013), the size of the moves can be determined by using a Monte Carlo simulation approach.

We first consider the delta hedging scheme outlined in Section 2.2. Let \( k \) be the index of the Monte Carlo simulation with 1000 iterations (that is, \( k = 1, 2, \ldots, 1000 \)). Let \( t_d \) be
Table 1: Different maturities of contracts used in the four scenarios. The table displays different maturities of futures / options contracts used in these four scenarios. For instance, in Scenario 4 and assuming delta-hedging, a call option on futures is hedged in 3rd July 2006 by using 3 futures contracts with maturities of Dec 2009, Dec 2010 and Dec 2011. June contracts are not included here because on 3rd July 2006, futures with a maturity of June 2010 are not available. As the call option approaches the end of October 2009, the futures with a maturity of Dec 2009 gets rolled over to June 2012 while the other two futures contracts with maturities of Dec 2010 and Dec 2011 remain.

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As the call option approaches the end of October 2009, the futures with a maturity of Dec 2009 gets rolled over to June 2012 while the other two futures contracts with maturities of Dec 2010 and Dec 2011 remain.

the trading day and \( d = 1, 2, \ldots, 1333 \) be the index of the trading day.\(^6\) At \( t_1 \), we generate \( n \) number of independent \( \Delta W_1 \) by drawing \( n \) random samples from a normally distributed random number generator with mean of 0 and variance of \( \frac{1}{252} \). Using these \( n \) samples and following the steps outlined in Section 2.2, we can construct a hedging portfolio consisting the option to be hedged and \( n \) number of futures with different maturities and with hedge ratios \( \delta_{1}^k, \ldots, \delta_{n}^k \). We add the index \( k \) in these hedge ratios to explicitly show their dependencies on the \( n \) random samples realised in iteration \( k \). The profit and loss (P/L) of the delta-hedged portfolio on trading day \( t_2 \) is defined as:

\[
P/L_{2}^{\Delta} \triangleq \Delta Y_{2}^{\Delta} + \delta_{1}^{k} \Delta F(t_2, T_1) + \delta_{2}^{k} \Delta F(t_2, T_2) + \ldots + \delta_{n}^{k} \Delta F(t_2, T_n),
\]

where \( \Delta F(t_2, T_1) = F(t_2, T_1) - \Delta F(t_1, T_1) \) represents the difference of market quoted futures prices between \( t_1 \) and \( t_2 \) \(^7\) and \( \Delta Y_{2}^{\Delta} \) represents the difference of the market quoted option prices.

\(^6\)Since we start the hedging scheme from July 2006, we have \( t_1 \) representing the trading day of the 3rd of July, 2006, which is a Monday. We note that the 4th of July, 2006 is a public holiday (Independence Day) in the United States, so \( t_2 \) represents the trading day of the 5th of July, 2006. \( t_{1333} \) represents the last trading day under consideration which is on the 31st of October, 2011.

\(^7\)We drop the dependency of \( \sigma \) in these futures prices because these are quoted prices rather than model prices.
prices from $t_1$ to $t_2$. In each of the 1000 simulations, we have a total of 1332 P/Ls ($P/L^휇,1$ to $P/L^휇,1333$) and we define the root-mean-square errors (RMSEs) as follow:

\[
\text{RMSE}^휇,k_{\text{total}} = \sqrt{\frac{1}{1332} \sum_{d=2}^{1333} (P/L^휇,k_d)^2}
\]

\[
\text{RMSE}^휇,k_1 = \sqrt{\frac{1}{50} \sum_{d=2}^{51} (P/L^휇,k_d)^2}
\]

\[
\text{RMSE}^휇,k_2 = \sqrt{\frac{1}{50} \sum_{d=52}^{101} (P/L^휇,k_d)^2}
\]

\[
\vdots
\]

\[
\text{RMSE}^휇,k_{27} = \sqrt{\frac{1}{32} \sum_{d=1302}^{1333} (P/L^휇,k_d)^2}.
\]

To calculate the total average of the RMSEs, we simply take the average over 1000 simulations. That is:

\[
\text{RMSE}^휇_{\text{total}} = \frac{1}{1000} \sum_{k=1}^{1000} \text{RMSE}^휇,k_{\text{total}}
\]

\[
\text{RMSE}^휇_1 = \frac{1}{1000} \sum_{k=1}^{1000} \text{RMSE}^휇,k_1
\]

\[
\text{RMSE}^휇_2 = \frac{1}{1000} \sum_{k=1}^{1000} \text{RMSE}^휇,k_2
\]

\[
\vdots
\]

\[
\text{RMSE}^휇_{27} = \frac{1}{1000} \sum_{k=1}^{1000} \text{RMSE}^휇,k_{27}.
\]

In each path, seven hedging schemes are considered, namely: unhedged, delta, delta-IR, delta-vega, delta-vega-IR, delta-gamma and delta-gamma-IR. The results are the RMSEs of the profits and losses of the hedging portfolios.

So in each of the 1000 simulations, seven RMSEs represent the variability of the hedging portfolios under different hedging schemes for the whole six-year period. To illustrate the

---

8Strictly speaking, one unhedged portfolio with just the option itself and six hedging portfolios are considered.

9Note that the hedging schemes are all run on the same underlying empirical data set covering 1333 days of daily price history, and as explained above, the Monte Carlo simulations only serve to provide the $dW$ “bumps” used to calculate the positions in each hedge instrument in the factor hedge.
variability of the hedging portfolios at various times over the six-year period, the whole period from July 2006 to the end of 2011 is sub-divided into 27 sub-periods. Each of the sub-periods consists 50 trading days, except the last one. So now, given a scenario, the RMSE represents the RMSE of the profits and losses of the hedging portfolio per path per sub-period per hedging scheme. Then the average of the RMSE over 1000 simulations for each hedging scheme is computed, for each of the scenarios. Thus, we have 27 by 7 RMSEs, as shown in Figure 3.

4. Empirical results

The results of our empirical hedging application are depicted in Figure 3 and Figure 4, where the RMSE of the unhedged portfolio are compared with the RMSE of the hedged portfolios for the six different hedging schemes, under the proposed three-dimensional stochastic volatility–stochastic interest rate model. Figure 3 considers the hedging instruments of Scenario 1 and 2, where short-dated futures contracts are considered. Figure 4 considers the hedging instruments of Scenario 3 and 4, where longer-dated contracts are included (see Table 1 for exact contract maturities). Thus, the results are separated into 27 groups of 7 bars. The first group, labelled ‘Aug06’, represents the RMSE over a 50-trading day period from 5th of July, 2006 to 13th of September, 2006 inclusive. The second group, without label, represents the RMSE over a 50-trading day period from 14th of September, 2006 to 24th of November, 2006 inclusive. We continue similarly, so the last group, labelled ‘Oct11’, represents the RMSE over a 32-trading day period from 15th of September, 2011 to 31st of October, 2011. Firstly, we compare the overall hedging error between hedged positions and unhedged positions. Then we evaluate the contribution of stochastic interest rate models on hedging performance compared to deterministic interest rate models. Finally, we discuss the performance of each hedging scheme over the course of these six years.

4.1. Unhedged vs. hedged positions

Table 2 reports the improvement in the overall hedging error for a range of hedging schemes compared to the unhedged futures option position. Several conclusions can be drawn. Firstly, an interest rate hedge incrementally but consistently improves hedging performance when it is added to delta, delta-vega and delta-gamma hedging. When longer maturity futures contracts are used as hedging instruments (thus lower basis risk is present), such as in Scenario 4, then the improvement is about 1%, while the improvement nearly doubles when shorter maturity futures contracts are used as hedging instruments. When short-dated contracts are used as hedging instruments, it is necessary to more frequently rollover the hedge and as a result, a higher basis risk is present. Consequently, part of the basis risk is essentially managed by hedging the interest rate risk. This is in line with what would be expected theoretically, since the difference between futures prices for different maturities is in part due to interest rates (though convenience yields would also play a role).

Secondly, longer maturity hedging instruments (Scenario 4) consistently provide a better hedging performance across all hedging schemes compared to the shorter maturity hedging

10The six hedging schemes are: delta, delta-IR, delta-vega, delta-vega-IR, delta-gamma and delta-gamma-IR.
Table 2: **Improvement of hedging schemes over unhedged positions.** This table shows the percentage improvement of the RMSE of the hedged position over the unhedged portfolio for a range of hedging schemes.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>delta (s)</th>
<th>delta-IR (s)</th>
<th>delta-vega (s)</th>
<th>delta-vega-IR (s)</th>
<th>delta-gamma (s)</th>
<th>delta-gamma-IR (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>74.16%</td>
<td>75.08%</td>
<td>73.95%</td>
<td>74.40%</td>
<td>74.68%</td>
<td>75.56%</td>
</tr>
<tr>
<td>2</td>
<td>74.88%</td>
<td>75.66%</td>
<td>74.78%</td>
<td>75.68%</td>
<td>75.42%</td>
<td>76.42%</td>
</tr>
<tr>
<td>3</td>
<td>76.68%</td>
<td>77.33%</td>
<td>76.52%</td>
<td>77.46%</td>
<td>76.98%</td>
<td>77.64%</td>
</tr>
<tr>
<td>4</td>
<td>82.87%</td>
<td>83.02%</td>
<td>84.18%</td>
<td>84.67%</td>
<td>85.15%</td>
<td>85.77%</td>
</tr>
</tbody>
</table>

Table 3: **Comparison between stochastic interest rate model and deterministic interest rate model.** This table shows the percentage improvement of the RMSE over the unhedged portfolio between the stochastic interest rate model and deterministic interest rate model. In delta(s) and delta(d) columns, only futures are used as hedging instruments with hedge ratios derived from the stochastic interest rate model and deterministic interest rate model, respectively. Interest-rate hedging is not included (i.e., no bonds are used to hedge). Similarly apply for delta-vega and delta-gamma hedges.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>delta (s)</th>
<th>delta-vega (s)</th>
<th>delta-gamma (s)</th>
<th>delta (d)</th>
<th>delta-vega (d)</th>
<th>delta-gamma (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>74.16%</td>
<td>73.95%</td>
<td>74.68%</td>
<td>72.95%</td>
<td>72.87%</td>
<td>73.85%</td>
</tr>
<tr>
<td>2</td>
<td>74.88%</td>
<td>74.78%</td>
<td>75.42%</td>
<td>73.55%</td>
<td>73.73%</td>
<td>74.68%</td>
</tr>
<tr>
<td>3</td>
<td>76.68%</td>
<td>76.52%</td>
<td>76.98%</td>
<td>75.33%</td>
<td>75.43%</td>
<td>75.58%</td>
</tr>
<tr>
<td>4</td>
<td>82.87%</td>
<td>84.18%</td>
<td>85.15%</td>
<td>81.23%</td>
<td>82.83%</td>
<td>83.80%</td>
</tr>
</tbody>
</table>

With the shorter maturity futures contracts, the improvement across the different schemes is ranging between 73.95% and 75.56%. With the longer maturity futures contracts, the improvement across the different schemes is more sound, ranging between 82.87% and 85.77%, with the delta-gamma-IR hedge outperforming all other schemes. Thirdly, delta-gamma hedge tends to provide a better hedge compared to delta-vega hedge. Thus when hedging long-dated options positions, gamma hedge seems to be more efficient compared to vega hedge, with delta hedging being the least efficient.

4.2. **Deterministic vs. stochastic interest rates**

Table 3 compares the hedging error improvement over the unhedged positions for a range of hedging schemes by using models with deterministic and stochastic interest rates. When interest rate hedge is not considered additional to the delta, vega and gamma hedges, then the option position is hedged by using hedge ratios from the deterministic interest rate and the stochastic interest rate model specifications. Overall, the stochastic interest rate model improves incrementally the hedging performance compared to the deterministic interest rate model. The effect is more pronounced when the shorter maturity hedging instruments are used, predominantly due to the additional basis risk.
4.3. Comparison of hedging schemes

Figure 5 and Figure 6 presents the results of Figure 3 and Figure 4 by omitting the errors from the unhedged portfolio. A thorough inspection of the hedging performance of the six different hedging schemes reveals different patterns over the course of these six years.

Figure 1 depicts the annualised monthly standard deviations of the US Treasury yields and Figure 2 plots futures prices for four December contracts and the annualised standard deviation of these log-futures returns, between 2005 and 2013. These figures reveal that there is a period of high volatility of interest rates, high volatility of crude oil futures prices till the beginning of 2008, then there is a period of extreme variation in yields and futures prices between 2008 and 2009 associated with the GFC and then after that a period of relatively low variation in interest rates, especially the 1-year and 2-year yields. Thus the pre-GFC period, the GFC period and the post-GFC period, as specified above, have been used to discuss the results.

As we have already discussed in Section 4.1, an interest rate hedge consistently provides an improvement in the hedging performance, when combined with the other hedging schemes including delta, delta-gamma and delta-vega hedging. This improvement is more sound when the futures option is hedged with shorter maturity contracts, and weakens as the maturity of the hedging instruments increases (thus lower basis risk is involved). Additionally, during the GFC period, the improvement of including IR hedge to the other hedging schemes such as delta, delta-vega and delta-gamma is substantial. In the pre-GFC period, yields were relatively more volatile compared to the post-GFC period. Accordingly, the inclusion of IR hedge has a marginal improvement in the pre-GFC period, and no perceptible improvement in the post-GFC period. These observations are consistent with the numerical findings in Cheng et al. (2016b), where it is demonstrated that during periods of low interest rate volatility, stochastic interest rates marginally improve hedging performance. However, during the high interest rate volatility period of the GFC (see Figure 1), adding IR hedge over delta, delta-vega and delta-gamma hedging, considerably reduces the hedging errors.

Furthermore, delta-IR hedge outperforms consistently and substantially all other hedging schemes, when the market experiences high interest rate volatility and the hedge is subject to high basis risk (e.g. in scenario 1, where shorter maturity hedging instruments are considered, see top panel of Figure 5). In particular, during GFC period, adding IR hedge to delta hedging typically improves hedging performance, while adding gamma or vega to delta hedging worsens hedging performance. This effect is more evident, when more basis risk is present. Note also that, over periods of extremely high volatility such as during the GFC, the hedge is subject to model risk, since models would not be able to capture well these market conditions. Another reason for this could be that the delta-vega or the delta-gamma hedging schemes are more sensitive to model misspecification, i.e., to a mismatch between the assumed stochastic dynamics and the “true” process generating empirical data, since three short-dated options and three short-dated futures are used to hedge the long-dated option. Consequently, for hedging applications that are subject to model risk or basis risk, IR hedge tends to improve further delta hedging, something that other hedging schemes, such as gamma and vega, do not attain.11

11Even though a stochastic volatility model is used, delta-vega hedge does not provide any noticeable
Figure 1: **Annualised monthly standard deviations of the US Treasury yields.**
This plot presents annualised monthly standard deviations are calculated based on computing the standard deviations of the US Treasury yields over 20 trading days and then multiplied by $\sqrt{12}$.

On the other hand, under Scenario 4, that provides a hedge with the least basis risk compared to the other scenarios, the delta-vega-IR hedge tends to perform better during the pre-GFC, and the GFC period, while the delta-gamma-IR hedge typically performs the best at the post-GFC period, see bottom panel of Figure 6. During the post-GFC period, the volatility of the futures contracts is low which makes volatility hedge less necessary (or even counterproductive). Finally, during post-GFC period, where the interest rate volatilities and levels have been unusually low, the IR hedges do not typically provide further improvement compared to the other hedges. These latter results do not generally depend on the basis risk present in the hedging applications.

5. **Conclusion**

This paper employs the stochastic volatility–stochastic interest rate forward price model developed in Cheng et al. (2015) to hedge long-dated crude oil options over six years. The Light Sweet Crude Oil (WTI) futures and option dataset from the NYMEX spanning a 6-year period is used in the empirical analysis. Several hedging schemes, including delta, gamma, vega and interest rate risk hedges, are applied to hedge a call option on futures, from the beginning of 2006 to the end of 2011. Factor hedging is used to simultaneously hedge multi-dimensional risks impacting the forward curve, the stochastic volatility and stochastic interest rates.

Several conclusions are drawn from this empirical analysis on hedging long-dated crude oil options. Firstly, using hedging instruments with maturities that are closer to the ma-
turity of the option reduces the hedging error. This is predominantly due to the fact that hedging instruments with longer maturities require less frequently to roll forward the hedge; hence, leading to lower basis risks. Secondly, delta-gamma hedging is overall more effective than delta-vega hedging, especially when the maturities of the hedging instruments are getting shorter. For hedging instruments with longer maturities, delta-vega performs better compared to delta-gamma in the pre-GFC period.

Thirdly, interest rate hedging consistently improves hedging performance when it is added to delta, delta-vega and delta-gamma hedging with the improvement being more evident when the shorter maturity contracts are used as hedging instruments (thus when basis risk is higher). Consequently, part of the basis risk is essentially managed via the hedging of the interest rate risk. Furthermore, interest rate hedging was more effective during the GFC when interest rate volatility was very high, while interest rate hedging does not improve the performance of the hedge in the post-GFC period, where interest rates volatility was extremely low.

Fourthly, comparing the hedging performance between a stochastic interest rate model and a deterministic interest rate model, during the GFC, there is a significant improvement from the stochastic interest rate model over the deterministic interest rate model. However, there is only marginal hedging improvement from the stochastic interest rate model during the pre-crisis period and no noticeable improvement at all after 2010.

Lastly, delta-IR hedge often outperforms delta-vega and delta-gamma hedges. When hedging is carried out with shorter maturity hedging instruments and over periods of high interest rates volatility, adding IR hedge to delta hedge improves hedging performance, while adding gamma or vega hedge to the delta hedge deteriorates the hedge. This is a key conclusion: When we have more exposure to model risk (due to turbulent market conditions) and basis risk (due to a mismatch between the maturity of the option to be hedged and the hedge instruments), IR hedge beyond delta hedge can consistently outperform all other hedging schemes.

References


Figure 2: Futures prices and annualised standard deviations of the log-return of futures prices for four December contracts. These plots present the futures prices of four December contracts (top) and the annualised standard deviations of the log-return of futures prices of four December contracts (bottom) over eight years.
Figure 3: RMSEs for Scenario 1 and 2. This plot compares the average RMSEs over 1,000 simulation paths of various hedging schemes under Scenario 1 (top) and Scenario 2 (bottom) with the unhedged position.
Figure 4: RMSEs for Scenario 3 and 4. These plots compare the average RMSEs over 1000 simulation paths of various hedging schemes under Scenario 3 (top) and Scenario 4 (bottom) with the unhedged position.
Figure 5: RMSEs for Scenario 1 and 2. These plots display the average RMSEs over 1000 simulation paths of various hedging schemes under Scenario 1 (top) and Scenario 2 (bottom).
Figure 6: **RMSEs for Scenario 3 and 4.** These plots display the average RMSEs over 1000 simulation paths of various hedging schemes under Scenario 3 (top) and Scenario 4 (bottom).
Figure 7: **RMSEs for Scenario 1 and 2.** These plots compare the average RMSEs of portfolios assuming stochastic interest rates and deterministic interest rates, under Scenario 1 (top) and Scenario 2 (bottom).
Figure 8: **RMSEs for Scenario 3 and 4.** These plots compare the average RMSEs of portfolios assuming stochastic interest rates and deterministic interest rates, under Scenario 3 (top) and Scenario 4 (bottom).