Can ETFs Increase Market Fragility? Effect of Information Linkages in ETF Markets*

Ayan Bhattacharya† and Maureen O'Hara‡

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Abstract

We show how inter-market information linkages in ETFs can lead to market instability and herding. When underlying assets are hard-to-trade, informed trading may take place in the ETF. Underlying market makers, then, have an incentive to learn from ETF price when setting prices in their respective markets. We demonstrate that this learning is imperfect: market makers pick up information unrelated to asset value along with pertinent information. This leads to propagation of shocks unrelated to fundamentals and causes market instability. Further, if market makers cannot instantaneously synchronize their prices, inter-market learning can lead to herding, where speculators across markets trade in the same direction using similar signals, unhinged from fundamentals.

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†Samuel Curtis Johnson Graduate School of Management, Cornell University, Ithaca, NY 14853. Email: ab2278@cornell.edu.

‡Samuel Curtis Johnson Graduate School of Management, Cornell University, Ithaca, NY 14853. Email: mo19@cornell.edu.

1 Introduction

> *Exchange-traded funds account for as much as a third of all publicly traded stocks. Or think of it this way: An ETF that tracks a basket of hard-to-trade emerging-market stocks or high-yield bonds will, on any day, attract more buy and sell orders than a bellwether like Microsoft or General Electric. Among bright ideas on Wall Street, this notion of promising investors instant liquidity in some of the most opaque corners of the global marketplace ranks with earlier innovations like securitization in consequence if not risk.*
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> New York Times. February 22, 2016.

What is the risk of introducing Exchange-Traded Funds (ETFs) on hard-to-trade assets? Do such ETFs amplify market volatility or do they act as shock absorbers, introducing an extra layer of liquidity? Shock absorbers or not, what is the mechanism by which they affect market trading? Can these ETFs lead to trading frenzies in which many speculators rush to trade on the same market signal causing large price dislocations? What can regulators do to promote dampening effects of ETFs, if any, over amplifying effects? Questions like these have become increasingly important in recent years as interest in this instrument has exploded.[1] ETFs today are often the preferred vehicle for access to assets that have limited participation or liquidity otherwise, making them a systemically important cog that may move risks across markets.[2] Despite widespread interest among practitioners and regulators, academic research — especially theoretical work — on these market impacts of ETFs is scant.

In this paper we address this gap in the literature by investigating the risks and

[1] Flood, C. (2016, January 03). Record number of companies launch exchange traded funds. Retrieved February 5, 2016, from http://www.ft.com/ (Financial Times). See also Jenkins, C. and Perrotta, A. J. (2015, March 3). Institutional Investors Embrace Corporate Bond ETFs as Cash Bond Liquidity Struggles. Retrieved March 15, 2015, from http://tabbforum.com/opinions/ (Tabb Forum).

[2] Evans, J. (2014, September 7). Bond ETFs' lure of backdoor to liquidity. Retrieved March 15, 2015, from http://www.ft.com/ (Financial Times).

benefits of ETFs that track hard-to-trade assets. We are particularly interested in understanding when trading in ETFs may lead to greater market fragility. The term *fragility* has been used in the literature for many different kinds of market vulnerabilities. In this paper, we use fragility to refer to two specific phenomena: (i) *market instability*, driven by propagation of shocks unrelated to fundamental value of an asset, and, (ii) *rational herding*, wherein all market speculators trade in the same direction, on the same market signals, unhinged from asset fundamentals. Our analysis identifies a feedback channel in which herding and instability can arise due to the presence of ETFs.

Specifically, we develop a tractable model of ETF trading that features learning and feedback effects from the ETF to the underlying asset markets and vice versa. In our model the ETF tracks the weighted average of a basket of underlying assets, and markets are organized as conventional Kyle-style auctions. Informed trading can occur in both the ETF and the underlying assets, but in many settings, particularly those in which the underlying assets are hard-to-trade, informed traders may not have synchronous or symmetric access to both the ETF and the underlying assets. For instance, trade in some foreign sovereign stocks requires licenses that ETF traders often do not have, rendering the underlying assets out of bounds for speculators except as part of country ETFs. Similarly, in certain commodity ETFs, trade in the underlying requires the capacity to carry the physical asset, precluding ETF speculators from also participating in the underlying. For many bond ETFs on the other hand, underlying markets are still over-the-counter and illiquid, and trade may be difficult (and expensive). At the same time, speculators trading in such underlying asset markets may be specialized, with little incentive to trade the full ETF basket.[3] Lack of symmetric access may also arise due to asynchronous trading. Trading hours for ETFs and underlying assets do not overlap in many ETF markets,

[3]An alternative explanation as to why some speculators may stick with ETFs, while others trade in only certain specific underlying stocks, comes from the behavioral literature that explores cognitive biases like familiarity and home bias.

and in turbulent conditions the underlying market may even close while the ETF remains open (a case in point being Greek ETFs in summer 2015). In such scenarios, ETF prices can serve as a source of information for market makers in underlying assets, and vice versa, setting the stage for important feedback effects between prices.

Market instability arises in such settings because an underlying market maker, when learning from ETF prices, cannot perfectly distinguish between price changes caused by factors pertinent to his asset, and other factors, irrelevant to him. This leads to a situation where idiosyncratic shocks pertinent to one asset begin to affect the price of another independent asset — through the ETF price channel — thus causing market instability. We show that ETF markets bring both benefits and costs for underlying asset markets. At the level of the aggregate basket, ETF trading helps move underlying prices closer to fundamental value. Yet at the level of individual assets, it may lead to persistent distortions from fundamentals. Assets with high beta and high weightage in the ETF are especially vulnerable to such distortions.

We also show how herding can arise in the ETF ecosystem. When ETF market makers cannot instantaneously synchronize their price with underlying prices through the arbitrage mechanism, market makers in the ETF and underlying markets set initial clearing prices based on order flow in their own markets, and then revise them as they see prices in other markets evolve. This staggered information flow offers speculators an opportunity to use short-horizon strategies, closing out positions without waiting for the liquidation value to realize, if they can correctly guess the information flow from other markets. When speculators across different markets use common signals for trade, the staggered flow of information into price is predictable, and we show that short-horizon speculator strategies constitute an equilibrium. The common signal is most likely to be about the systematic factor — since the systematic factor affects all assets and all speculators tend to track it. Thus, speculators across the ETF ecosystem end up herding on the systematic factor signal. This herding phenomenon is reminiscent of a Keynesian beauty contest: speculators raise the weight on the systematic factor, not because it affects the liquidation value, but because other speculators do the same.

That ETFs could exert independent effects on market behavior reflects the underlying enigma posed by these securities. As derivatives, ETF prices should be determined by the values of their underlying assets. Yet, in many cases, the trading volume (and liquidity) of the ETF far exceeds that of the underlying asset markets, making these the preferred vehicle of trading interest. And with that trading interest comes the possibility that informed trading (and price discovery) now takes place in the ETF. When this occurs, it is akin to the "tail wagging the dog", in that the ETF price changes the underlying prices rather than the underlying prices changing the ETF. As events like the market dislocation on August 24, 2015 made clear, ETFs are no longer simple appendages to the market, but rather are now capable of affecting markets in their own right.

The paper is organized as follows. Section 2 provides a brief overview to the literature. Section 3 then introduces our basic model. In Section 4 we study ETF markets where the underlying is hard to trade, resulting in informed speculation occurring primarily in the ETF. In such markets, we show that in equilibrium, shocks unrelated to fundamentals may propagate from asset to asset through the ETF channel. In Section 5 we allow informed speculation in both underlying markets and in the ETF. We show that such markets admit a herding equilibrium where all speculators use only the systematic factor signal to determine their order size and asset prices become unhinged from fundamentals. In Section 6 we use data from Greek ETFs trading during the financial crisis in summer 2015 to illustrate many of the phenomena we describe in the paper. Section 7 discusses policy implications and concludes. All proofs are in the Appendix.

2 Literature Review

Our paper is related to a growing and diverse body of literature. One such area is models with feedback effects and strategic complementarities in financial markets (see Hirshleifer, Subrahmanyam and Titman (1994), Barlevy and Veronesi (2003), Veldcamp (2006), Ganguli and Yang (2009), Amador and Weill (2010), Garcia and Strobl (2011), Goldstein, Ozdenoren and Yuan (2011), Goldstein, Ozdenoren and Yuan (2013), Hassan and Mertens (2014)). Most of these papers focus on strategic complementarities that arise in information acquisition and interpretation, but none to our knowledge have focused on the particular complications introduced by ETFs. Though the driving mechanism is very different, the herding portion of our model is related to the model in Froot, Sharfstein and Stein (1992). In their model herding arises because of execution uncertainty: half the orders sent by an informed speculator execute in the first period, while the other half execute in the second period. Thus if a speculator wants to close out his position after the second period, he imitates other speculators. In our model, on the other hand, the reason for herding is the information linkage between the markets that arises due to the way market makers learn.

Our work is also related to market microstructure research looking at the impact of an index on trading activity. Subrahmanyam (1991) examines the optimal strategy for discretionary liquidity traders when they can trade in both the index and the underlying securities, and shows that adverse selection costs are typically lower in indexes. Introduction of an index, therefore, reduces liquidity in underlying securities because liquidity traders find the index more attractive. Gorton and Pennacchi (1993) show that when prices are not fully revealing, the return on composite securities cannot be replicated by holding the underlying individual assets when investors have immediate needs to trade. Like Subrahmanyam (1991), they show that an index can improve the welfare of uninformed traders. The focus of our paper is quite different from these papers, however, as we look at how inter-market information linkages in ETF markets can cause instability and lead to herding. There is a small but growing empirical literature that looks at the impact of equity ETFs on the stock maket. Ben-David, Franzoni and Moussawi (2014) and Krause, Ehsani and Lien (2014) find that ETFs increase the volatility of underlying assets, and Ben-David, Franzoni and Moussawi (2014) further show that this is not accompanied by increased price discovery at the stock level. Da and Shive (2015) find that ETFs contribute to equity return co-movement. Israeli, Lee and Sridharan (2015) find that increased ETF ownership is accompanied by increased bid-ask spreads, decreased pricing efficiency, and increased co-movement of the underlying stocks. The findings in these papers are broadly consistent with our model.

Finally, another related literature involves the limits to arbitrage (see for example Shleifer and Vishny (1997); Gromb and Vayanos (2010)). This literature has largely focused on understanding how the capital and risk required in real world arbitrage can result in asset prices diverging from true values. Because an ETF is a composite security, in principle its price at any time is simply the summation of the underlying component prices. Deviations from these prices are "corrected" by an arbitrage process that involves the creation and redemption of ETF shares. In practice, such perfect synchronization is not automatic, particularly when limited (or asynchronous) access (or simply greater liquidity) makes the ETF a preferred venue for informed trading. Our focus here on ETFs based on hard to trade assets underscores how information flows and market rigidities can create natural impediments to arbitrage — and the potential for market fragility.

3 A Model of ETF Trading

To study market instability that arises through propagation of shocks unrelated to asset fundamentals, we develop a model of ETF trading based on the classic Kyle (1985) setup. In the real world, an ETF originates when an issuer of ETF (ETF sponsor) designates chosen market participants as ETF market makers (authorized participants). ETF market makers have a special agreement with the ETF issuer: they can create/redeem ETF shares at net asset value by delivering the constituents of the ETF. When ETF and underlying asset markets are liquid — and trade synchronously — ETF market makers can arbitrage away price differences between the underlying basket and ETF, ensuring that they move in-step. With risk neutral market participants (like in our model), the same price adjustment results if we assume that market makers learn from price changes in other markets. On the other hand, when some underlying asset markets are hard-to-trade — or trading is asynchronous — the learning mechanism that we describe in the paper is the key driver of the price adjustment process, because arbitrage may no longer be feasible.

Asset Value

We begin by setting up a model in which there is one exchange traded fund (denoted by e) tracking the weighted average of N underlying assets. The initial value of security i ($i = 1, ..., N$), $P_{i,0}$, is public knowledge. The liquidation value of the asset, v_i, is given by

$$P_{i,3} = P_{i,0} + b_i\gamma + \epsilon_i, \quad i = 1, ..., N, \tag{1}$$

where $\gamma, \epsilon_1, ..., \epsilon_N$ are all mutually independent normally distributed random variables, each with mean zero. Consistent with "factor model" representation of security prices, shocks to the asset value may be decomposed into a systematic (or common) factor component, γ, and an idiosyncratic component, ϵ_i, with b_i denoting the factor loading. When it simplifies calculations without loss of generality, we assume $var\,(\epsilon_i) = var\,(\epsilon_j) = var\,(\epsilon)\ \forall i, j \in \{1, ..., N\}$, i.e., the variances of the idiosyncratic components are equal.

The value of the ETF is simply the weighted average of the underlying asset prices.

Thus the liquidation value of the ETF is

$$P_{e,3} = \sum_{i=1}^{N} w_i P_{i,0} + \sum_{i=1}^{N} w_i b_i \gamma + \sum_{i=1}^{N} w_i \epsilon_i, \tag{2}$$

where w_i is the weight of asset i in the basket.

Market Participants and their Information

The ETF and assets are traded simultaneously in separate markets. Each market is organized as a Kyle (1985) type auction with a designated market maker. All traders in the model are risk-neutral. There are N *informed speculators* who trade only in the ETF market. Each speculator receives two signals: one about the systematic factor, and another about an idiosyncratic factor. No two speculators receive information about the same idiosyncratic factor; in other words, each speculator in the ETF market is "associated" with an asset market. This assumption is meant to mimic the fact that in the real world, a speculator tracks only a few selected markets. Having a single ETF speculator for each asset also allows us to sidestep the complications of within-market coordination, and focus on inter-market information linkages.

We also have one informed speculator in each of the N underlying asset markets. This speculator receives a signal about the idiosyncratic factor affecting his specific market, and a signal about the common factor. We analyze a model with speculators in both ETFs and underlying markets in Section 5, but as a useful preliminary, in Section 4, we consider a world in which informed speculators are only active in the ETFs. For simplicity, we assume that signals received by speculators in all the markets — ETF and underlying — have no noise: a speculator in the ETF market tracking market i observes ϵ_i and γ, and so does the speculator trading in market i.

There are $N + 1$ *market makers* in the model: one for the ETF market, and one for each underlying asset market. Like in Kyle (1985), competition is assumed to drive their profits to zero, so they clear markets at expected value. As is standard in such models, there are *liquidity traders* in the ETF and underlying markets who are assumed to have exogenous reasons for trade. Liquidity traders in market i place an order of $z_i \sim \mathcal{N}\left(0, var\left(z_i\right)\right)$; in the ETF market, the liquidity order is $z_e \sim \mathcal{N}\left(0, var\left(z_e\right)\right)$. We assume that $var\left(z_e\right) = \sum_{i=1}^{N} var\left(z_i\right)$, and that the variance of liquidity orders in underlying markets are identical, i.e., $var\left(z_i\right) = var\left(z_j\right) = var\left(z\right)\ \forall i, j \in \{1, ..., N\}$.

Timing of Trade

There are three dates, $t = 1, 2, 3$. On date 1, informed speculators in the ETF and underlying markets trade in their respective markets according to their information. On date 2, market makers in the underlying markets update their prices after observing the date 1 ETF price, and the ETF market maker updates the ETF price after observing the date 1 underlying market prices. On date 3, the liquidation value of the assets realize. (Figures 1 and 3, in Sections 4 and 5 respectively, illustrate the timeline graphically.)

The objective of the ETF speculator tracking market i is to choose an order size, x_{ei}, that satisfies

$$x_{ei} = arg\max_{x'_{ei}} E\left(x'_{ei}\left(\sum_{j=1}^{N} w_j\left(\epsilon_j + b_j\gamma\right) - P_{e,1}\right)\middle|\,\epsilon_i, \gamma\right), \tag{3}$$

where $P_{e,1}$ denotes the ETF price on date 1. Similarly, the objective of a speculator in underlying market i is to choose

$$x_i = arg\max_{x'_i} E\left(x'_i\left(\epsilon_i + b_i\gamma - P_{i,1}\right)\middle|\,\epsilon_i, \gamma\right). \tag{4}$$

The total order flow in the ETF market is denoted by $q_e = x_e + z_e$. Similarly, in underlying market i, the total order flow is $q_i = x_i + z_i$.

Learning and Equilibrium

The model developed here involves $N+1$ securities, $2N$ informed speculators, and $N+1$ market makers. In principle, a complete solution to this model could involve a complex equilibrium in which underlying market makers learn not only from their own order flow, the price movements of every other underlying asset, and the price movements of the ETF, but also from the lack of order flows and price movements in their own and other securities. Such a complicated learning problem is intractable. In the following analysis, we focus on more tractable learning scenarios. In the next section, we characterize how a market maker would learn from the ETF and impound that information in the underlying security. The subsequent section allows learning from both the ETF and own security and focusses on equilibria that result in herding.

4 Informed Speculators in ETF

In this section, we analyze equilibrium when the underlying asset markets are not easily accessible to informed speculators. As noted in the introduction, a number of high yield bond, commodity and country ETF markets possess the inaccessibility characteristic that we model here, and so too would any setting in which the ETF trades when the underlying market is not open. The practical import of this assumption is to restrict our attention to informed speculation happening only in the ETF. For analytical simplicity, we assume complete inaccessibility, but our results go through, qualitatively, when the inaccessibility is partial — as long as there is substantial price-discovery in the ETF that forces underlying market makers to learn from ETF price changes.[4] In the next section we analyze a setting with informed speculation in both the underlying and the ETF.

[4]Technically, our results hold in any equilibrium where ETF speculators are 'long-horizon', i.e. maximize the objective function in equation (3). If there are informed speculators trading *exclusively* in underlying markets, this may not be the case. We discuss this scenario in the next section.

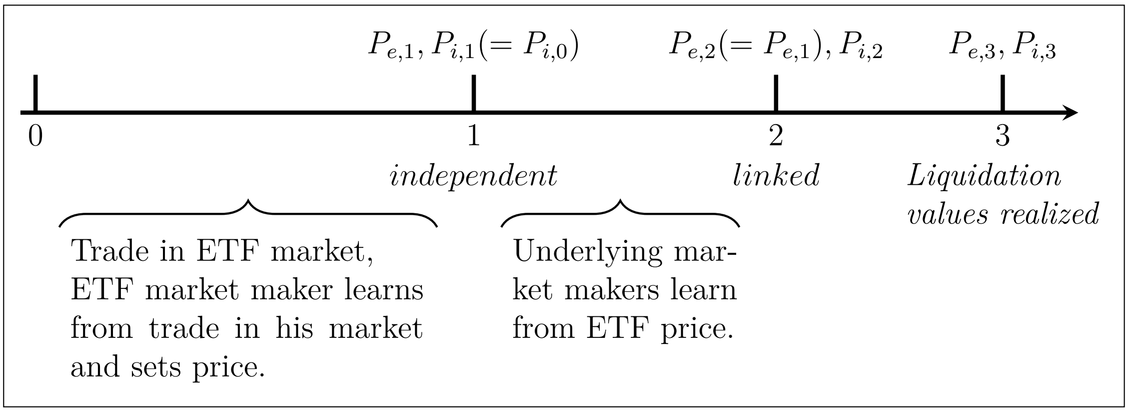


Figure 1: Timeline for model when informed speculation occurs in ETF

Figure 1 below presents the timeline for this setup. Underlying market makers track ETF price changes for information about their own assets. With no informed trading in asset markets, $P_{i,1} = P_{i,0}$, and since ETF market makers have nothing to learn from underlying markets, $P_{e,2} = P_{e,1}$. As is standard in the literature, we look at symmetric linear equilibrium strategies for informed speculators. Each market participant conjectures the strategies of all other participants; in equilibrium, the conjectures are consistent.

An underlying market maker, on seeing a change in ETF price, can infer the order flow as $q_e = (P_{e,1} - P_{e,0}) / \lambda_e$, where λ_e denotes the price impact factor in the ETF market determined in equilibrium. From this order flow, the underlying market maker tries to discern information pertinent to his asset. As a Bayesian, he therefore revises the price in his own market to

$$P_{i,2} = P_{i,1} + \lambda_{ei} w_i q_e = P_{i,1} + \frac{cov\left(\epsilon_i + b_i \gamma, w_i q_e\right)}{var\left(w_i q_e\right)} w_i q_e, \tag{5}$$

where λ_{ei} denotes the impact of the ETF price change on underlying asset i.

An ETF speculator takes into account the impact of his and other speculators' trade on the ETF price before placing an order; hence the clearing price offered by the market maker in the ETF aggregates the information of all ETF speculators. But this, in turn, implies that the order flow inferred by an underlying asset market maker has information pertinent to the asset mixed not just with random noise, but also with systematic information related to other underlying assets. In other words, if $\nu_e w_j$ and θ_e denote the optimal weights that an ETF speculator tracking asset j places on his idiosyncratic and systematic factor signals in equilibrium, the order flow inferred by underlying market maker i is

$$q_e = z_e + \sum_{j=1}^{N} \left(\nu_e w_j \epsilon_j + \theta_e \gamma\right). \tag{6}$$

In equation (6), $\nu_e \epsilon_i + N\theta_e \gamma$ is the only component of the order flow that is pertinent for underlying market maker i, the rest of it obfuscates the information. Substituting the value for q_e in equation (5) above, we obtain the impact of the ETF price adjustment on underlying asset market i:

$$P_{i,2} = P_{i,1} + \frac{\nu_e w_i^2 var\left(\epsilon_i\right) + N\theta_e b_i w_i var\left(\gamma\right)}{w_i^2 \nu_e^2 \sum_{j=1}^{N} w_j^2 var\left(\epsilon_j\right) + w_i^2 N^2 \theta_e^2 var\left(\gamma\right) + w_i^2 var\left(z_e\right)} \left[w_i z_e + w_i \sum_{j=1}^{N} \left(\nu_e w_j \epsilon_j + \theta_e \gamma\right) \right]. \tag{7}$$

Solving for parameters ν_e, θ_e, λ_e and λ_{ei} gives us the proposition below.

Proposition 1. *The equilibrium price set by the ETF market maker is*

$$P_{e,1} = P_{e,0} + \lambda_e q_e, \tag{8}$$

the equilibrium price set by the market maker in underlying market i, $i = 1, ..., N$, is

$$P_{i,2} = P_{i,1} + \lambda_{ei} w_i \left(z_e + \sum_{j=1}^{N} x_{ej} \right), \tag{9}$$

and the optimal order size of an informed ETF speculator tracking market i is

$$x_{ei} = w_i \nu_e . \epsilon_i + \theta_e . \gamma,$$

where

$$\nu_e = \frac{1}{2\lambda_e},\ \theta_e = \frac{\sum_{j=1}^{N} w_j b_j}{(N+1)\,\lambda_e}, \tag{10}$$

$$\lambda_e = \sqrt{\frac{\left(\sum_{j=1}^{N} var\left(\epsilon_j\right) w_j^2\right)/4 + var\left(\gamma\right)\left(\sum_{j=1}^{N} b_j w_j\right)^2 N/\left(N+1\right)^2}{var\left(z_e\right)}}, \tag{11}$$

$$\textit{and}\ \ \lambda_{ei} = \frac{\lambda_e\left(N+1\right) var\left(\epsilon_i\right) + 2\lambda_e N b_i \left(\sum_{j=1}^{N} w_j b_j\right) var\left(\gamma\right)/w_i}{\left(N+1\right)\left(\sum_{j=1}^{N} w_j^2 var\left(\epsilon_j\right)\right) + 2N\left(\sum_{j=1}^{N} w_j b_j\right)^2 var\left(\gamma\right)}. \tag{12}$$

Note that while ν_e and θ_e are increasing in N, λ_e (and hence λ_{ei}) is decreasing in N. This happens because an increase in the number of speculators increases competition among them, leading to a decrease in the adverse selection faced by the market maker.

Proposition 1 illustrates how ETFs tracking hard-to-trade assets may lead to *market instability*. Recall that market instability, in our context, refers to the propagation of unrelated shocks across assets. By unrelated, we mean shocks that are independent of factors that determine the fundamental value of the asset. As Proposition 1 shows, the underlying market maker in asset i is now influenced by information related to the collection of assets.

A novel feature of information transmission through inferred ETF order flow is that it leads to underlying markets getting 'coupled'. Observe that the only source of information for market makers in the underlying asset markets is informed trading in the ETF, and equation (5) above describes how they learn from the ETF price. Coupling, in this case, happens through two channels. The first channel is the price impact factor, λ_{ei}, in equation (5). Equation (12) shows that the price impact factor in market i is affected by the weights, betas, and variance of idiosyncratic factor of other assets in the ETF, as well as the number of assets in the ETF — even though these variables are not related to asset i's liquidation value. The second channel is the order flow variable in equation (6): from the aggregate order flow that he infers, an underlying market maker has no way of distinguishing shocks pertinent to his asset, from irrelevant shocks to idiosyncratic factors of other assets. It is this inability to discriminate that allows unrelated shocks to affect underlying asset prices. Our model allows a precise characterization of the transmission.

Proposition 2. (Market instability) *A shock of* η_j *to the idiosyncratic component of asset* j *leads to a shock of*

$$\frac{w_i^2\left(N+1\right)var\left(\epsilon_i\right)/2+w_iNb_i\left(\sum_{j=1}^{N}w_jb_j\right)var\left(\gamma\right)}{\left(N+1\right)\left(\sum_{j=1}^{N}w_j^2var\left(\epsilon_j\right)\right)+2N\left(\sum_{j=1}^{N}w_jb_j\right)^2var\left(\gamma\right)}\eta_j \tag{13}$$

to $P_{i,2}$*, the price of asset* i*.*

Proposition 2 demonstrates an important way in which ETFs affect overall market stability. Equation (13) shows that, unlike other common financial instruments, ETFs can act as conduits for transmission of risks across the market ecosystem — in this sense, therefore, ETFs make the ecosystem more coupled. Given that there are over 1400 ETFs trading in the US alone as of December 2014, the influence of these instruments on market fragility can be substantial.

Equation (13) also allows us to delineate important determinants of shock propagation.

Proposition 3. *Ceterus paribus, the impact of an unrelated shock is higher for an asset with higher beta.*

Proposition 3 follows directly from equation (13). To understand it intuitively, note that since all speculators use identical systematic factor signals to decide their order size, underlying market makers expect to find good information about the systematic factor in the inferred ETF order flow. Higher beta implies that the systematic factor has a higher relative weightage for the value of the asset, thus a market maker gives higher importance to information in ETF order flow. This, in turn, implies a greater vulnerability to unrelated shocks.

Proposition 4. *Ceterus paribus, the impact of an unrelated shock is higher for an asset with higher weight in the ETF.*

Proposition 4 reflects the fact that speculators with information about assets that have a higher weight in the ETF trade more, because they have a greater relative informational advantage. Consequently, underlying market makers in those markets learn more from the ETF price adjustment and are thus more susceptible to unrelated shocks.

ETF markets therefore bring both benefits and costs for underlying asset markets. The cost is that irrelevant information, blended with pertinent information, now affects prices; the benefit is the access to more information. In the classic setup of Kyle (1985), informativeness of prices is measured by the change in variance of the market maker's value distribution for the asset after a round of trading. Kyle shows that the posterior variance is one-half of the prior variance, and interprets this as revelation of half the information of a speculator in each round. In the following proposition, we show that in our model too, information in ETF order flow brings down the variance for underlying market makers.

Proposition 5. *The posterior variance of underlying market maker i's distribution for the asset value is*

$$\frac{\sum_{j=1,j\neq i}^{N} w_j^2 var^2\left(\epsilon_j\right) + var\left(\epsilon_i\right) var\left(z_e\right) + \left(N^2\theta_e^2 + b_i^2\nu_e^2\sum_{j=1}^{N} w_j^2 - 2b_i w_i \nu_e \theta_e N\right) var\left(\epsilon_i\right) var\left(\gamma\right) + b_i^2 var\left(\gamma\right) var\left(z_e\right)}{w_i^2\nu_e^2\sum_{j=1}^{N} w_j^2 var\left(\epsilon_j\right) + w_i^2 N^2\theta_e^2 var\left(\gamma\right) + w_i^2 var\left(z_e\right)} \tag{14}$$

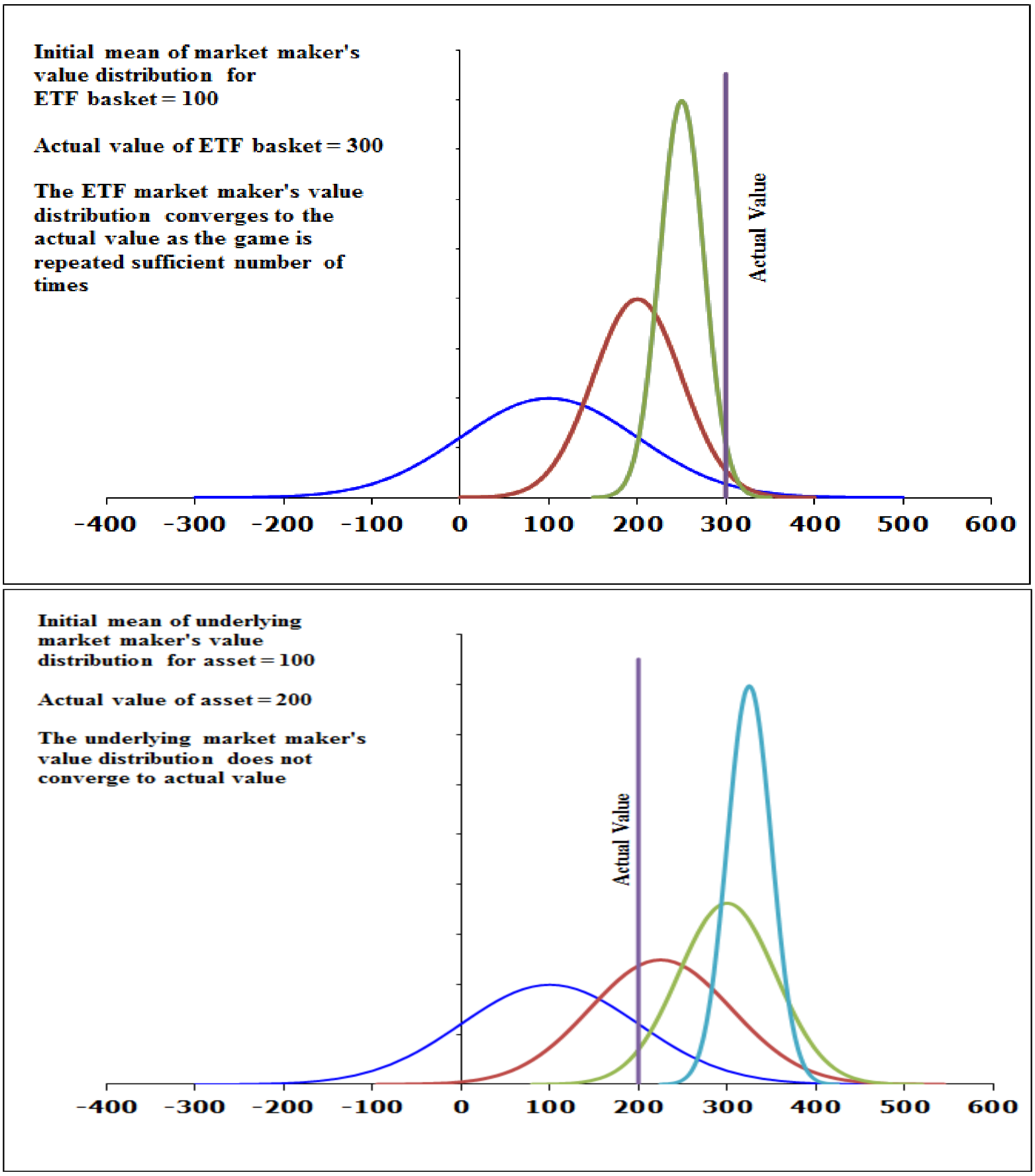
after one round or trading, where ν_e and θ_e are as defined in Proposition 1.

Corollary 1. *The posterior variance of underlying market maker i's distribution for the asset value is lower than his prior variance.*

It is important to place Proposition 5 in the right perspective. Corollary 1 shows that market makers are less uncertain about the value of the asset after they learn from the ETF order flow. This indicates that speculators have conveyed information through the trading process. In the classic Kyle (1985) model, this implies that, on average, prices have moved closer to the true value. In other words, if the trading game were repeated a sufficiently large number of times in Kyle (1985), prices will have converged to the true asset value. As Figure 2 below illustrates, in our model, the implication is more subtle.

At the level of the aggregate underlying basket, the Kyle implication holds true. Like in Kyle (1985), this follows directly from the random nature of liquidity trading. By the weak law of large numbers, $Lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N}\left(z_e\right)_n = 0$, and thus after sufficiently large number of rounds, speculator order flow gets separated perfectly from liquidy order flow; hence aggregate price of the basket converges to the true value. Unlike Kyle (1985), however, the speculator order flow is not homogeneously informative. Different speculators have information about different underlying assets, and the aggregate order flow represents the totality of information held by all speculators. Unlike the random nature of liquidity trading, order flow from each speculator has a systematic bias due to the information driving the trade. Consequently, it is impossible for an underlying market maker to distinguish perfectly information pertinent only to his specific asset — from the ETF price — however many times the game be repeated. This, in essence, represents one of the central dichotomies of the effect of ETFs: at the level of the aggregate basket, prices are better informed, but at level of individual prices, there can be persistent distortions from fundamentals.

Figure 2: Illustrative plots showing that while the value distribution for the ETF basket converges to the true value as the game is repeated, an underlying market maker's value distribution may not converge



The simulations were run using 5 assets, i.e. N=5, with each asset having equal weight in the ETF. The actual value of the assets were taken to be 100, 200, 300, 400 and 500. The plots above represent one of the many possible paths. In the case of the ETF, all paths converge to 300.

5 Underlying Markets with Speculators

Having analyzed the inference problem from the ETF, we now turn to the model in which informed speculation takes place in both the ETF and the underlying asset. Specifically, as described in the model section, each underlying market has one informed speculator, as well as there being N speculators trading only the ETF. Figure 3 below gives the timeline for the setup. Thus $P_{i,1}$ and $P_{e,2}$ no longer need be equal to $P_{i,0}$ and $P_{e,1}$ respectively. This sort of setting is seen in commodity and country ETFs, which have committed participants that trade only the underlying asset.

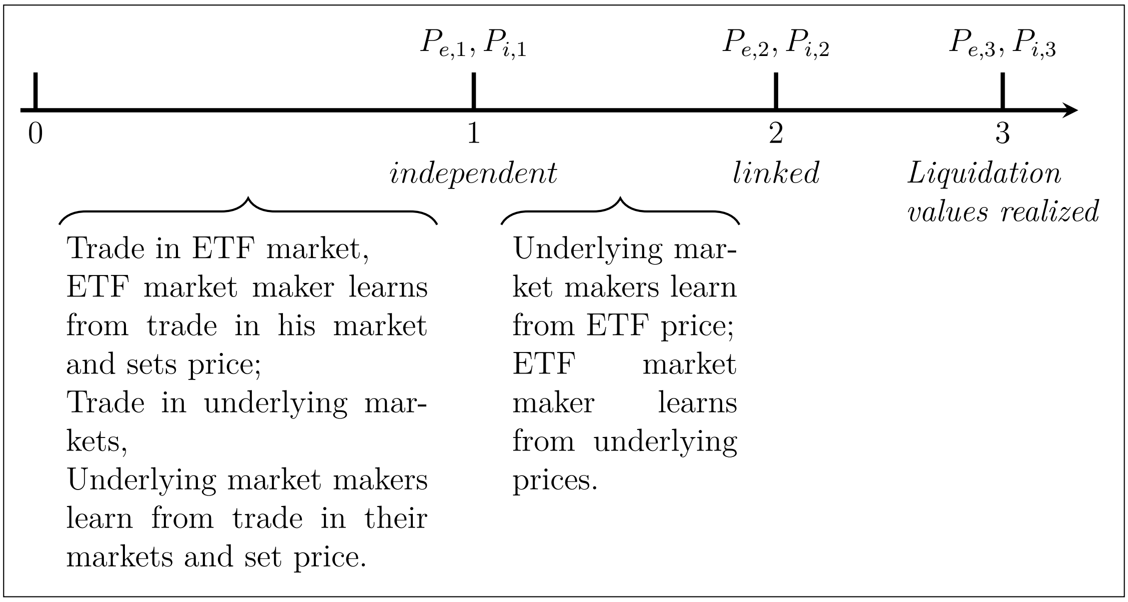


Figure 3: Timeline for model when informed speculation occurs in both ETF and underlying

Though the institutional context is very different, some features of the equilibrium that we obtain in this section are similar to the *herding equilibrium* discussed in Froot, Sharfstein and Stein (1992). When there is informed speculation in both ETF and underlying markets, speculators may have profitable *"short-horizon"* trading strategies. In our context, short-horizon trading strategies for speculators are those where they exit the market on intermediate date 2, without waiting for the liquidation value of the assets to realize. This is profitable for speculators because the market makers learn in stages: initially they set prices to reflect information in just their own market; at a later stage, when they see prices in other markets, market makers revise their prices to reflect new information. The intermediate revision of prices offers speculators an opportunity to close out positions profitably, without waiting for realization of liquidation value, if they can foresee correctly the information in other markets. Following Froot, Sharfstein and Stein (1992), we ignore a speculator's cost of reversing a position when he liquidates and exits the market. This assumption simplifies the exposition greatly, but all our results continue to hold qualitatively even if we work with a more complicated model for liquidation.

When ETF speculators hold assets for the *"long-horizon"*, i.e. till liquidation value is realized on date 3, they maximize the objective functions in equations (3) and (4). We have already solved the ETF speculator's problem, in this scenario, in Proposition 1 of the previous section. The optimal long-horizon strategy of speculators in underlying markets can be obtained similarly (or alternatively, by putting $N = 1$ in equation (10)) giving

$$\nu_i = \frac{1}{2\lambda_i},\ \theta_i = \frac{b_i}{2\lambda_i}\ and\ \lambda_i = \frac{1}{2}\sqrt{\frac{var\left(\epsilon_i\right) + b_i^2 var\left(\gamma\right)}{var\left(z_i\right)}}. \tag{15}$$

.

Obtaining the optimal short-horizon strategy of speculators is more involved. In this case, speculators maximize the objective function

$$x_k = arg\max\ E\left(x_k\left(P_{k,2} - P_{k,1}\right)|\mathcal{F}_k\right), \tag{16}$$

$k = \{e, 1, ..., N\}$, where $\mathcal{F}_k$ represents the information set of the speculator, i.e. the relevant idiosyncratic factor signal and the systematic factor signal. For short-horizon equilibria, we focus on the subset of symmetric linear strategies where speculators with information about a particular asset place the same weight on their signals, irrespective of whether they trade the ETF or underlying asset. Recall that q_k denotes the total order flow in market k on date 0, and is the sum of x_k, the speculator order flow, and z_k, the liquidity order flow. If the conjectured demand of a speculator in underlying market i is $\bar{\nu_i}\left(\epsilon_i\right)+\bar{\theta}\left(\gamma\right)$, the price set by the underlying market maker on date 1 is

$$P_{i,1}=\frac{cov\left(\epsilon_i+b_i\gamma,\bar{\nu_i}\epsilon_i+\bar{\theta}\gamma+z_i\right)}{var\left(\bar{\nu_i}\epsilon_i+\bar{\theta}\gamma+z_i\right)}q_i=\frac{\bar{\nu_i}var\left(\epsilon_i\right)+b_i\bar{\theta}var\left(\gamma\right)}{\bar{\nu_i}^2var\left(\epsilon_i\right)+\bar{\theta}^2var\left(\gamma\right)+var\left(z\right)}q_i. \tag{17}$$

Then, the conjectured demand for a speculator in the ETF tracking underlying market i is also $\bar{\nu_i}\left(\epsilon_i\right)+\bar{\theta}\left(\gamma\right)$, and we have that the price set by the ETF market maker on date 1 is

$$P_{e,1}=\frac{cov\left(\sum_1^N w_i\left(\epsilon_i+b_i\gamma\right),\sum_1^N\left(\bar{\nu_i}\epsilon_i+\bar{\theta}\gamma\right)+z_e\right)}{var\left(\sum_1^N\left(\bar{\nu_i}\epsilon_i+\bar{\theta}\gamma\right)+z_e\right)}q_e=\frac{\sum_{i=1}^N w_i\left(\bar{\nu_i}var\left(\epsilon_i\right)+b_i\bar{\theta}var\left(\gamma\right)\right)}{\sum_{i=1}^N\bar{\nu}_i^2var\left(\epsilon_i\right)+\left(\sum_1^N\bar{\theta}\right)^2var\left(\gamma\right)+var\left(z_e\right)}q_e. \tag{18}$$

On observing the order flow in his own market, a market maker can form an expectation of the order flow in other markets, because speculators tracking an asset get the same signal regardless of whether they trade the ETF or the underlying. Hence, in the case of underlying market maker i,

$$E_i\left[q_e-\sum_{j=1,j\neq i}^N q_j\middle|q_i\right]=\frac{cov\left(q_e-\sum_{j\neq i,j=1}^N q_j,q_i\right)}{var\left(q_i\right)}q_i=\frac{\bar{\nu}_i^2var\left(\epsilon_i\right)+\bar{\theta}^2var\left(\gamma\right)}{var\left(\bar{\nu_i}\epsilon_i+\bar{\theta}\gamma\right)+var\left(z\right)}q_i. \tag{19}$$

Similarly, the ETF market maker expects the order flow in underlying markets to be

$$E_e\left[\sum_{j=1}^N q_j\middle|q_e\right]=\frac{cov\left(\sum_{j=1}^N q_j,q_e\right)}{var\left(q_e\right)}q_e=\frac{\sum_{i=1}^N\bar{\nu}_i^2var\left(\epsilon_i\right)+\bar{\theta}^2var\left(\gamma\right)}{var\left(\sum_{i=1}^N\bar{\nu}_i\epsilon_i+N\bar{\theta}\gamma\right)+var\left(z_e\right)}q_e. \tag{20}$$

After the conclusion of trading on date 1, market makers get to know the price changes in other markets. This enables them to infer the actual order flow in those markets.

Comparing the actual order flow to expected order flow gives market makers additional information about asset value, and they revise their prices. After incorporating the new order flow information, the market maker in underlying market i revises his price to

$$P_{i,2} = P_{i,1} + \frac{cov\left(\epsilon_i + b_i\gamma, q_e - \sum_{j\neq i} q_j\right)}{var\left(q_e - \sum_{j\neq i} q_j \middle| q_i\right)} \left(q_e - \sum_{j=1,j\neq i}^{N} q_j - E\left[q_e - \sum_{j=1,j\neq i}^{N} q_j \middle| q_i\right]\right) \quad (21)$$

which simplifies to

$$P_{i,2} = \frac{\bar{\nu}_i var\left(\epsilon_i\right) + b_i\bar{\theta} var\left(\gamma\right)}{2\bar{\nu}_i^2 var\left(\epsilon_i\right) + 2\bar{\theta}^2 var\left(\gamma\right) + var\left(z\right)} \left(q_i + q_e - \sum_{j=1,j\neq i}^{N} q_j\right). \quad (22)$$

Similarly, the ETF market maker revises his price to

$$P_{e,2} = P_{e,1} + \frac{cov\left(\sum_1^N w_j\left(\epsilon_j + b_j\gamma\right), \sum_1^N z_j\right)}{var\left(\sum_1^N q_j \middle| q_e\right)} \left(\sum_{j=1}^{N} q_j - E\left[\sum_{j=1}^{N} q_j \middle| q_e\right]\right). \quad (23)$$

The equation above simplifies to

$$P_{e,2} = \frac{\sum_{j=1}^N w_j\bar{\nu}_j var\left(\epsilon_j\right) + N\bar{\theta}\left(\sum_{j=1}^N w_j b_j\right) var\left(\gamma\right)}{\sum_{j=1}^N \bar{\nu}_j^2 var\left(\epsilon_j\right) + N^2\bar{\theta}^2 var\left(\gamma\right) + var\left(z_e\right)/2} \left(\frac{q_e + \sum_{j=1}^N q_j}{2}\right). \quad (24)$$

Having obtained prices at dates 1 and 2, we can substitute them in the speculator objective function in (16). Any value for $\bar{\theta}$ and $\bar{\nu}_i$, $i = 1, ..., N$, that solves the resulting $2\left(N+1\right)$ equations, simultaneously, gives speculator signal weights in a short-horizon equilibrium.

Speculators compare expected profits from short and long-horizon strategy, and if they expect equilibrium profits from short-horizon strategy to be higher, they liquidate their position on the intermediate date itself, exiting the market before final asset values are realized. As described above, s*hort-horizon strategies involve all speculators trading on the same signal (systematic factor signal).* This is a classic setting for *rational herding.* As

Froot, Sharfstein and Stein (1992) describe, though rational, in such herding equilibrium, asset prices do not reflect the fundamentals, and they are usually welfare inefficient. In our context, when speculators hold assets till liquidation date, the weights that they place on the idiosyncratic and systematic factor signals, when choosing order size, reflects the weights of the components in the liquidation value of the asset. But when they close their positions at the intermediate stage, we show (in the proposition below) that the weights chosen may be decoupled from fundamental value.

Proposition 6. (Herding equilibrium) *If all speculators use short-horizon strategies, there exists an equilibrium where speculators use only the systematic factor signal to determine their order size. The equilibrium order size for all speculators is* $\bar{\theta}.\gamma$*, with* $\bar{\theta} = \sqrt{var(z)/var(\gamma)}$*, and the equilibrium market maker prices are given by equations (17), (18), (22) and (24).*

For speculators, the expected profit from this short-horizon strategy is higher than the long-horizon strategy of holding the asset till liquidation value when the idiosyncratic and common factor signals, ϵ_i *and* γ*, satisfy the condition*

$$\frac{(\epsilon_i + b_i\gamma)^2}{\gamma^2} \leq \frac{b_i}{3}\sqrt{\frac{var(\epsilon_i) + b_i^2 var(\gamma)}{var(\gamma)}}, \tag{25}$$

for each i=1,...,N.

Condition (25) in the proposition above checks that the value derived by a speculator from knowledge of the idiosyncratic factor is not too high. If that happens, the long-horizon equilibrium strategy that places non-zero weight on the idiosyncratic factor signal dominates the proposed short-horizon strategy that ignores it. Observe that in the herding equilibrium, asset prices do not contain any idiosyncratic fundamental information, so in a strict sense, this equilibrium does not possess the propagation-of-unrelated-idiosyncratic-shock feature. Yet, in a certain sense, the overall outcome of herding is the same as propagation of unrelated shocks: asset markets are more coupled and asset price movements are not related to change in fundamentals.[5]

The dynamics of price formation in the herding equilibrium is quite different from the standard Kyle (1985) type equilibrium where speculators hold the asset till liquidation value is realized. In a certain sense, speculators in a herding equilibrium behave like participants in a Keynesian beauty contest. In our setup, short-horizon strategies are profitable because of the inter-market learning among ETF and underlying markets. When underlying market i's speculator puts higher relative weight on the systematic factor, it gets to have a stronger effect on intermediate ETF price, $P_{e,2}$ (because the ETF market maker learns from underlying markets before setting $P_{e,2}$). This leads to ETF speculators putting more weight on the systematic factor. Since underlying market makers learn from the ETF price, in turn this pushes a speculator in market j to put higher weight on the systematic factor. A full blown feedback cycle can ensue, leading to an equilibrium with no weight on the idiosyncratic factor, for all speculators.

6 A Simple Case Study

As a simple case study of the linkages discussed above, we relate our analysis to the intriguing behavior of ETFs during the recent Greek debt crisis in summer 2015. We caution that this example is best viewed not as a rigorous empirical validation, but more along the lines of a heuristic exposition to illustrate features of our model.

The Greek ETF on NYSE (Ticker Symbol: GREK) tracks the FTSE/ATHEX Custom Capped Index, which is a market capitalization weighted index of the 20 largest companies

[5]As is true for a majority of models in the market microstructure literature, our results come with an important caveat. It is important to keep in mind that we have confined our discussion to a small subset of equilibria that are tractable and interesting. The game we described may have multiple equilibria, and it may be possible that certain real world situations corresponds to other equilibria.

on the Athens stock exchange.[6] It is one of only two major global ETFs that provide international investors with exposure to Greece, the other being the Lyxor ETF FTSE Athex 20, traded in Europe. At the height of the Europe-Greece bailout crisis in summer 2015, the Greek stock markets and the European Lyxor ETF shut down from June 29 to August 02, 2015. During this period, however, the Greek ETF on NYSE, as well as foreign listings of some prominent Greek companies continued to trade.[7] In terms of our model, during this phase, the foreign venues of the Greek companies (foreign listings on London Stock Exchange (LSE) and American Depository Receipts (ADRs)) constituted the underlying markets, while the NYSE platform trading GREK was the ETF market. Since underlying stock markets were closed, the Greek ETF on NYSE was an important venue for price discovery of Greek stocks. Thus the conditions seemed propitious for the kind of phenomena described in this paper.

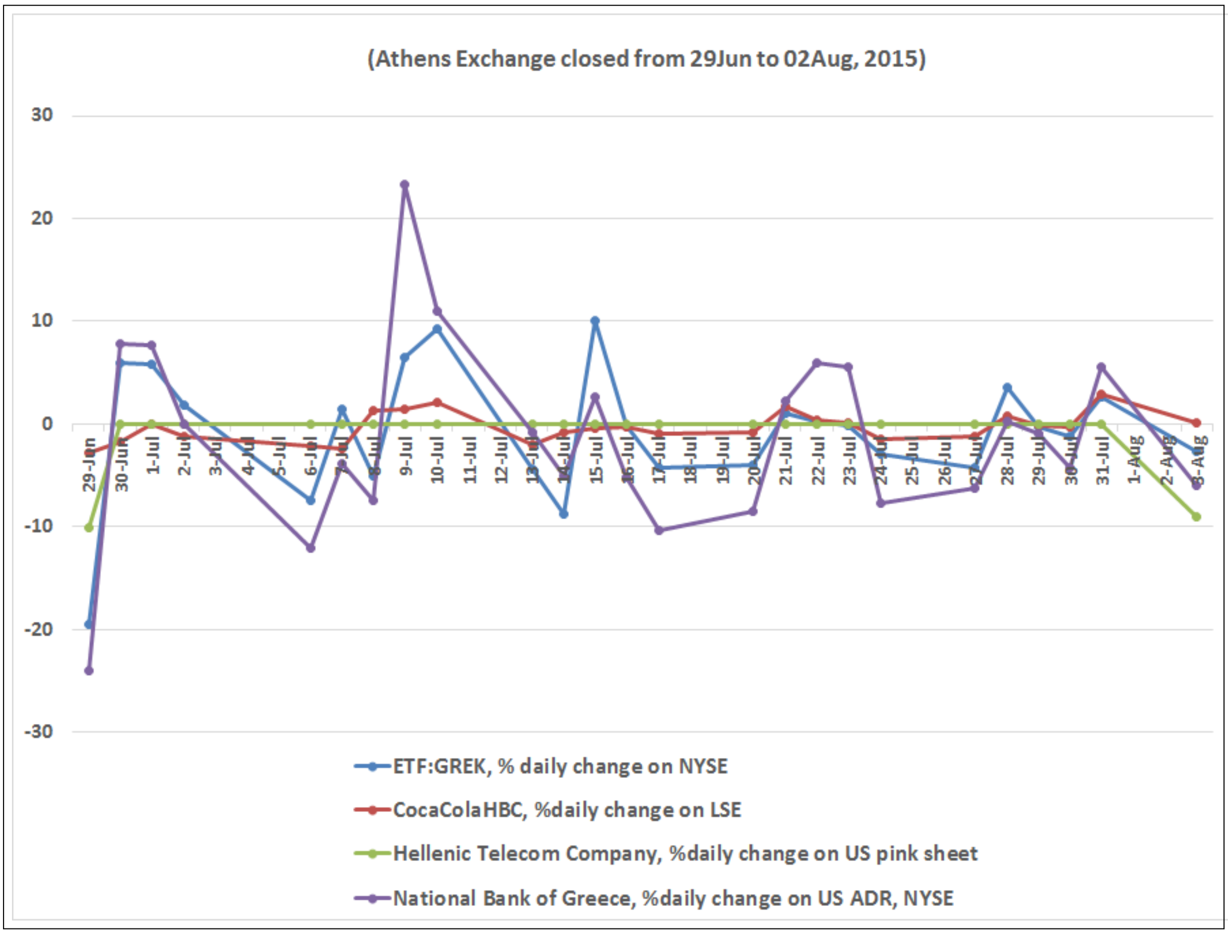
Though GREK tracks the value weighted sum of the top 20 companies on the Athens exchange, the top three constituents — Coca Cola Hellenic Bottling Company (HBC), Hellenic Telecom, and National Bank of Greece — comprised close to 45% of the ETF's holding on 29 June, 2015, when the exchange closed down.[8] Coca Cola HBC and National Bank of Greece continued to trade throughout the period on their alterntate foreign venues — LSE and NYSE ADR respectively — notwithstanding the Athens exchange shutdown. However, trade in Hellenic Telecom's alternative venue, the US pink sheets, was suspended from 29 June till 31 July. These three companies belong to different sectors, and their exposure to Greece varies widely: 95% of Coca Cola HBC's sale was outside Greece, 56% of National Bank of Greece's revenue was earned outside Greece,

[6] Ground Rules, FTSE/ATHEX Custom Capped Index, v1.7, October 2015, available at http://www.ftse.com/products/downloads/FTSE_ATHEX_Custom_Capped_Index.pdf?32.

[7] eCosta, S. H., Detrixhe, J., and Ciolli, J. (2015, June 30). Greek ETF Halted in Europe Amid Volume Frenzy in U.S. Fund. Retrieved October 10, 2015, from http://www.bloomberg.com/news/articles/2015-06-30/greek-etf-stays-halted-in-europe-amid-volume-frenzy-in-u-s-fund (Bloomberg news).

[8] The precise holdings were: Coca Cola HBC 25.05%, Hellenic Telekom 9.94%, and National Bank of Greece 9.36%. Source: http://www.globalxfunds.com/GREK

Figure 4: Greek ETF on NYSE (symbol: GREK) and its major constituents



and 36% of Hellenic Telecom Company's revenue was from sale outside Greece, in 2014.[9] Thus one would not expect the effect of the Greek bailout crisis to be similar on all firms. Yet as Figure 4 shows, from June 29 to August 02, all prices seemed to move largely in-sync, suggesting propagation-of-shocks and herding.

7 Conclusion and Policy Implications

When the S&P Depository Receipt (SPDR) — the first ETF — was launched in 1993, it was a sideshow to underlying markets. Money in ETFs came mostly from passive long term investors seeking an alternative to mutual funds. There was little independent information in those markets and underlying market players took little notice of them. There was little danger of phenomenon of the kind that we describe in this paper affecting the markets. A lot has changed over the years. While ETFs still remain small relative to the underlying in highly liquid markets, in other, less-liquid markets, ETFs dominate trading. Today we have ETFs on many assets which are inaccessible to traders otherwise.

In this paper, we showed that when the information content in ETF prices does not overlap perfectly with the information in underlying markets, markets can become more fragile. A worrisome problem for regulators is the propagation of market instability arising from the feedback effects of ETFs. In the absence of ETFs, market maker learning is more focused on own market order flow. With an ETF, however, the market maker also extracts information from the ETF price, meaning that both own market and ETF market information affects prices. This can result in greater volatility as disturbances in the ETF can affect underlying market prices, even when such information is irrelevant for a particular underlying asset. When this occurs, it is akin to the "tail wagging the dog" in that the ETF price changes the underlying prices rather than the underlying prices

[9]Source: Company Financial Reports

changing the ETF price. Moreover, we demonstrated that ETFs can introduce persistent distortions from fundamentals at the individual asset level, even while enhancing price efficiency at the aggregate ETF level. Assets with high beta and high weighting in the ETF are especially vulnerable to such distortions.

That ETFs can lead to greater market fragility is surely an issue of regulatory concern. Our model suggests some approaches to alleviate such incipient instability. One regulatory solution, for example, could be to restrict ETFs to baskets where the underlying assets are easily tradeable. Another could be to reduce the size of the basket and permit only low beta assets in them, since that reduces the likelihood of noise transmission. Our analysis also suggests that enhancing the quality of information on underlying assets would help, since it reduces the information difference among markets. Thus, actions such as enhanced transparency of individual bond prices, perhaps through more real time trade reporting to TRACE, may be helpful. Regulators may also wish to encourage the nascent electronic trading of bonds, as greater transparency on order information as well as greater accessibility will serve to dampen ETF-induced volatility. Since information difference is a pre-condition for herding across markets, these measures are also likely to reduce the propensity of herding.

Appendix: Proof of Results

Proof of Proposition 1. Let an ETF speculator, tracking market i, make the consistent conjecture that the demand of an informed speculator tracking market j is $x_{ej} = w_j \nu \epsilon_j + \theta_e \gamma \ \forall j \neq i$. Let λ_e denote the conjectured price impact factor in the ETF market. Denoting this trader's order by x', we have then that,

$$E\left[P_{e,1}\right] = \lambda_e \left(x' + \gamma\theta\left(N-1\right)\right),$$

Replacing this expected price in the objective function of the speculator, equation (3), we get his optimal order size as

$$x = argmax_{x'} \ x' \left(\epsilon_i w_i + \gamma \sum_1^N w_j b_j \right) - x' \left(\lambda_e \left(x' + \gamma\theta \left(N-1 \right) \right) \right)$$

This can be solved to obtain

$$x = \frac{\epsilon_i w_i + \gamma \sum_1^N w_j b_j}{2\lambda_e} - \frac{\theta \left(N-1 \right) \gamma}{2}.$$

Setting this equal to $w_i \nu_e \epsilon_i + \theta_e \gamma$ yields the equilibrium ν_e and θ_e in equation (10).

To obtain the equilibrium price impact factor, note that

$$\lambda_e = \frac{cov \left(\sum_1^N \epsilon_i w_i + \gamma \sum_1^N w_j b_j, q_e \right)}{var \left(q_e \right)} = \frac{\nu_e \sum_1^N w_i^2 var \left(\epsilon_i \right) + var \left(\gamma \right) N\theta_e \sum_1^N w_j b_j}{\nu_e^2 \sum_1^N w_i^2 var \left(\epsilon_i \right) + N^2 \theta_e^2 var \left(\gamma \right) + var \left(z_e \right)}$$

Replacing the values for ν_e and θ_e from equation (10) gives a quadratic equation in λ_e, which can be solved to obtain the equilibrium value of λ_e in (11).

The coefficient of the weighted order flow $w_i \left(z_e + \sum_{j=1}^N \left(\nu_e w_j \epsilon_j + \theta_e \gamma \right) \right)$ in equation (7) is λ_{ei}, so the equilibrium λ_{ei} can be obtained by substituting values for ν_e and θ_e. □

Proof of Proposition 2. A shock of of η_j to the idiosyncratic component of asset j, ϵ_j, causes an ETF speculator tracking market j to increase his demand by $w_i \nu_i \eta_j$. This implies a price jump in the ETF of $\lambda_e w_i \nu_i \eta_j$. In turn, this leads to a price jump of $\lambda_{ei} w_i^2 \nu_e \eta_j$ in underlying asset i. Replacing the equilibrium values for ν_e and λ_{ei} from Proposition 1 gives the magnitude of the shock propagated to asset i. □

Proof of Propositions 3 and 4. Follow directly from the expression for shock propagation, in equation (13). □

Proof of Proposition 5. Since all variables are normal, the posterior variance of a market maker in underlying market i, after seeing the ETF price change is,

$$var\left(\epsilon_i + b_i\gamma | P_{e,1} - P_{e,0}\right) = var\left(\epsilon_i + b_i\gamma\right) - \frac{cov^2\left(\epsilon_i + b_i\gamma, w_i z_e + w_i \sum_{j=1}^N \left(\nu_e w_j \epsilon_j + \theta_e \gamma\right)\right)}{var\left(w_i z_e + w_i \sum_{j=1}^N \left(\nu_e w_j \epsilon_j + \theta_e \gamma\right)\right)}. \tag{26}$$

Since,

$$\begin{aligned} cov\left(\epsilon_i + b_i\gamma, w_i z_e + w_i \sum_{j=1}^N \left(\nu_e w_j \epsilon_j + \theta_e \gamma\right)\right) &= E_i\left[\left(\epsilon_i + b_i\gamma\right)\left(w_i z_e + w_i \sum_{j=1}^N \left(\nu_e w_j \epsilon_j + \theta_e \gamma\right)\right)\right] \\ &= E_i\left[w_i^2 \nu_e \epsilon_i^2 + N\theta_e b_i w_i \gamma^2\right] \\ &= w_i^2 \nu_e var\left(\epsilon_i\right) + N\theta_e b_i w_i var\left(\gamma\right), \end{aligned}$$

and

$$var\left(w_i z_e + w_i \sum_{j=1}^N \left(\nu_e w_j \epsilon_j + \theta_e \gamma\right)\right) = w_i^2\left(var\left(z_e\right) + \nu_e^2 \sum_{j=1}^N w_j^2 var\left(\epsilon_j\right) + N^2\theta_e^2 var\left(\gamma\right)\right),$$

we have the expression in equation (14) after substitution. □

Proof of Corollary 1. Follows directly from equation (26) in the Proof of Proposition 5 above. □

Proof of Proposition 6. Denote the date 1 price impact in the ETF by $\lambda_{e,1}$, that is (from equation (18)),

$$\lambda_{e,1} = \frac{\sum_{i=1}^N w_i\left(\bar{\nu}_i var\left(\epsilon_i\right) + b_i\bar{\theta} var\left(\gamma\right)\right)}{\sum_{i=1}^N \bar{\nu}_i^2 var\left(\epsilon_i\right) + \left(\sum_1^N \bar{\theta}\right)^2 var\left(\gamma\right) + var\left(z_e\right)},$$

and similarly, from equation (24),

$$\lambda_{e,2} = \frac{\sum_1^N w_i\bar{\nu}_i var\left(\epsilon_i\right) + N\bar{\theta}\left(\sum_{i=1}^N w_i b_i\right) var\left(\gamma\right)}{\sum_{i=1}^N \bar{\nu}_i^2 var\left(\epsilon_i\right) + N^2\bar{\theta}^2 var\left(\gamma\right) + var\left(z_e\right)/2}.$$

Let an ETF speculator, tracking market i, make the consistent conjecture that the demand of an informed speculator tracking market j is $x_j = \bar{\nu}_j \epsilon_j + \bar{\theta}\gamma \; \forall j \neq i$. Then, to maximize his objective function, he selects order size x such that

$$x = argmax_{x'} \, x' \left(\frac{\lambda_{e,2}}{2} \left(x' + \gamma (N-1) \bar{\theta} + \bar{\nu}_j \epsilon_i + \gamma N \bar{\theta} \right) \right) - x' \left(\lambda_{e,1} \left(x' + (N-1) \bar{\theta} \right) \right).$$

If there exists an equilibrium where all speculators behave identically and herd on the systematic factor, then $\nu_i = 0 \; \forall i \in \{1, ..., N\}$. From the optimal order size equation above, this implies that

$$\bar{\theta}\gamma = \frac{\bar{\theta}\gamma \left((N-1) \lambda_{e,2} + \lambda_{e,2}/2 - (N-1) \lambda_{e,1} \right)}{2\lambda_{e,1} - \lambda_{e,2}}.$$

which simplifies to $\bar{\theta} = \sqrt{var(z)/var(\gamma)}$. The same expression for $\bar{\theta}$ is obtained if one solves the utility maximization problem of a speculator in an underlying market. Thus $\bar{\theta} = \sqrt{var(z)/var(\gamma)}$ is the weight on the common factor in an equilibrium where speculators use short-horizon strategies.

An ETF speculator, tracking market i, expects profit $E_{ei}[\pi_s]$ if he uses the short-horizon strategy, where

$$E_{ei}[\pi_s] = E_{ei}[P_{e,2} - P_{e,1}] \frac{sd(z)}{sd(\gamma)} \gamma$$

Using equation (24), this can be re-written as

$$\begin{aligned} E_{ei}[\pi_s] &= \frac{cov\left(\sum_1^N w_i (\epsilon_i + b_i \gamma), \sum z_j\right)}{var\left(\sum q_j | q_e\right)} \left(\sum q_j - E\left[\sum q_j \middle| q_e \right] \right) \frac{sd(z)}{sd(\gamma)} \gamma \\ &= \frac{\bar{\theta} N \left(\sum_1^N w_i b_i \right) var(\gamma)}{2N^2 \bar{\theta}^2 var(\gamma) + var(z_e)} N \bar{\theta} \gamma \left(\frac{var(\gamma)}{N^2 \bar{\theta}^2 var(\gamma) + var(z_e)} \right) \frac{sd(z)}{sd(\gamma)} \gamma \\ &= \frac{sd(z)}{sd(\gamma)} \frac{N \left(\sum_1^N w_i b_i \right)}{(2N+1)(N+1)} \gamma^2. \end{aligned} \quad (27)$$

Similarly, a speculator in underlying market i, expects the following profit if he follows the short-horizon strategy:

$$\begin{aligned} E_i\left[\pi_s\right] &= E_i\left[P_{i,2}-P_{i,1}\right]\frac{sd\left(z\right)}{sd\left(\gamma\right)}\gamma \\ &= \frac{cov\left(\epsilon_i+b_i\gamma, q_e-\sum_{j\neq i}q_j\right)}{var\left(q_e-\sum_{j\neq i}q_j\middle|q_i\right)}\left(q_e-\sum_{j=1,j\neq i}^{N}q_j-E\left[q_e-\sum_{j\neq i}q_j\middle|q_i\right]\right)\frac{sd\left(z\right)}{sd\left(\gamma\right)}\gamma \\ &= \frac{\bar{\theta}b_i var\left(\gamma\right)}{2\bar{\theta}^2 var\left(\gamma\right)+var\left(z\right)}\bar{\theta}\gamma\left(\frac{var\left(\gamma\right)}{N^2\bar{\theta}^2 var\left(\gamma\right)+var\left(z_e\right)}\right)\frac{sd\left(z\right)}{sd\left(\gamma\right)}\gamma \\ &= \frac{sd\left(z\right)}{sd\left(\gamma\right)}\frac{b_i}{6}\gamma^2. \end{aligned} \tag{28}$$

If, on the other hand, an ETF speculator, tracking market i, uses the long-horizon strategy, he expects his profit to be

$$\begin{aligned} E_{ei}\left[\pi_l\right] &= E_{ei}\left[P_{e,3}-P_{e,1}\right]\left(\bar{\nu}_i\epsilon_i+\bar{\theta}\gamma\right) \\ &= \left(\epsilon_i+\sum_{i=1}^{N}b_i\gamma-\lambda_{e,1}\left(\bar{\nu}_i\epsilon_i+N\bar{\theta}\gamma\right)\right)\left(\bar{\nu}_i\epsilon_i+\bar{\theta}\gamma\right) \\ &= \left(\frac{\epsilon_i}{2}+\gamma\left(\sum_{1}^{N}b_i-\frac{N\sum_1^N w_i b_i}{N+1}\right)\right)\left(\frac{\epsilon_i}{2}+\frac{\sum_1^N w_i b_i}{N+1}\gamma\right)\frac{1}{\lambda_{e,1}} \end{aligned} \tag{29}$$

Similarly, a speculator in underlying market i, using the long-horizon strategy, expects his profit to be

$$\begin{aligned} E_i\left[\pi_l\right] &= E_i\left[P_{i,3}-P_{i,1}\right]\left(\bar{\nu}_i\epsilon_i+\bar{\theta}\gamma\right) \\ &= \left(\epsilon_i+b_i\gamma-\lambda_{i,1}\left(\bar{\nu}_i\epsilon_i+\bar{\theta}\gamma\right)\right)\left(\bar{\nu}_i\epsilon_i+\bar{\theta}\gamma\right) \\ &= \left(\epsilon_i+b_i\gamma\right)^2\frac{1}{4\lambda_{i,1}} \end{aligned} \tag{30}$$

A speculator chooses the short-horizon strategy over the long-horizon strategy when $E\left[\pi_s\right]\geq E\left[\pi_l\right]$. Condition (25) can be obtained by comparing equations (27) and (29), and (28) and (30) above. Since ϵ_i affects only the long-horizon expected profit (and not the short-horizon profit), when the profit from knowledge of idiosycratic factor is not too high, the condition is fulfilled. □

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