Treasury Bill Yields: Overlooked Information*

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Abstract

This paper finds that bond risk premium consists of long-term and short-term components. The long-term factor raises the slope of yield curve, has forecastability horizon of longer than one year, is related to value, size and momentum premiums in the stock market, and forecasts macroeconomic growth. In contrast, the short-term factor is completely hidden from Treasury bond yields yet apparently reduces Treasury bill yields, has forecastability horizon of less than one quarter, is related to aggregate stock market returns, and is largely attributed to liquidity premium.

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1 Introduction

Bond yields are the sum of two components: the average of expected future riskfree interest rates and risk premium. Thus, a change in risk premium can be expected to affect bond yields. However, the recent literature has found the opposite; risk premium in the Treasury bond market does not appear to affect the shapes of yield curves. For example, Cochrane and Piazzesi (2005, p.147) conclude that “the return-forecasting factor is clearly not related to any of the first three principal components,” although the three principal components are responsible for more than 99.9% of total variation of yield curves. Ludvigson and Ng (2009) show that macroeconomic variables are informative of bond risk premium but the information is not spanned by bond yields.

The puzzle can be explained by the negative correlation between risk premium and expected future riskfree interest rate. Suppose, for example, a circumstance in which investors become more risk averse and expect riskfree rate to decrease in the future. If the increase in risk premium is offset by the decrease in expected riskfree rate, the circumstance would not affect bond yields. In other words, the activity is hidden from yield curves.

The existence of the hidden factor has been the central theme of the latest term structure literature. For example, Duffee (2011) explains the theoretical background that risk premium factor can be hidden, estimates the risk premium using the five-factor Kalman filtering method and shows that around one-half of the premium is hidden from yield curves. Joslin, Priebsch, and Singleton (2010) incorporate macroeconomic variables—inflation and real economic growth—into an affine term structure model and show that the macro risk premium is not spanned by bond yields. Chernov and Mueller (2012, p.367) show that the hidden factor “affects inflation expectations at all horizons, but has almost no effect on the nominal yields.”

\[ \text{Equation (4) in the next section shows the composition of bond yields.} \]
This paper seeks to add to this literature by focusing on the possibility that the increase in risk premium and the decrease in riskfree rate almost, but not completely, offset. Thus, the hidden factor might appear to be hidden due to its extremely short half-life in risk-neutral probability measure. One can also expect, however, that Treasury bill yields might have unique risk premium information that is not spanned by Treasury bonds, which is the key finding of this paper.

This argument seems conflicted with the conventional wisdom that long-term maturity bond yields are more informative of bond risk premium than bond yields of short-term maturity. For example, a permanent shock to bond risk premium, if any, would raise the slope of yield curve since long-term yields have higher sensitivity to risk premium. However, short-term interest rates are insensitive to an increase in risk premium since they are considered riskfree by definition. This is the rationale behind Fama and Bliss (1987) and Campbell and Shiller (1991) that the excess returns of holding long-term Treasury bonds can be predicted by the slope of yield curve. The conflict can be solved by assuming multiple risk premium factors of different frequencies.

Risk premium consists of long-term and short-term components. The long-term factor raises the slope of yield curve, while the short-term factor is hidden from Treasury bond yields but reduces the yields of Treasury bills. The long-term factor predicts excess returns over longer than one year, while the short-term factor loses its predictability in one quarter. The two factors also have different economic implications. The long-term factor predicts future macroeconomic growth, and the short-term factor is closely related to liquidity premium. In addition, the long-term factor is related to value, size and momentum premiums in the stock market while the short-term factor is related to aggregate stock market returns.

This is not the first paper that examines the differences between Treasury bonds and Treasury bills. For example, Duffee (1996) documents their market segmentation by using
the correlations of their yields. Pearson and Sun (1994), who estimate a two-factor Cox, Ingersoll, and Ross (1985) term structure model, also conclude that “estimates based on only bills imply unreasonably large price errors for longer maturities.” However, there has been no precedent that compares Treasury bonds with Treasury bills in terms of the informativeness of risk premium. Many papers of bond risk premium have not examined the importance of Treasury bills since their methodology is based on the annual excess returns of holding long-term Treasury bonds over one-year interest rates. Treasury bills whose maturities are less than one year are thus not considered by construction.

The rest of this paper is organized as follows. Section 2 explains the estimation of state variables from bond yields and inflation. Section 3 compares the forecastability difference between long- and short-term risk premium factors. Section 4 shows how the two risk premium factors are related to the principal components of bond yields and, in particular, why bond risk premium seems to be unrelated to the first three principal components—level, slope, and curvature. Section 5 compares the risk premium factors to other financial market variables such as liquidity and stock market risk factors. Section 6 discusses how the state variables are related to macroeconomic growth. Section 7 concludes.

2 Estimation of State Variables

Let \( p_t^{(n)} \) denote the log price of an \( n \)-period maturity discount bond at time \( t \). Its continuously compounded bond yield is derived as

\[
y_t^{(n)} = -\frac{1}{n} p_t^{(n)},
\]

where \( y_t^{(1)} = r_t \) is the one-period interest rate.
The excess return of holding \( n \)-period maturity bonds from time \( t \) for \( h \) periods is defined as its holding return less the \( h \)-period interest rate

\[
\text{exr}_{t, t+h}^{(n)} = \{ p_{t+h}^{(n-h)} - p_t^{(n)} \} - \{ 0 - p_t^{(h)} \} = -(n-h) y_{t+h}^{(n-h)} + n y_t^{(n)} - h y_t^{(h)}.
\]  

(2)

For a unit holding period, \( h = 1 \), the above equation can be

\[
n y_t^{(n)} = r_t + (n-1) y_{t+1}^{(n-1)} + \text{exr}_{t+1}^{(n)}
= r_t + \{ r_{t+1} + (n-2) y_{t+2}^{(n-2)} + \text{exr}_{t+2}^{(n-1)} \} + \text{exr}_{t+1}^{(n)}
= \ldots
= \sum_{i=0}^{n-1} r_{t+i} + \sum_{i=0}^{n-2} \text{exr}_{t+i+1}^{(n-i)}.
\]  

(3)

Therefore,

\[
\therefore \quad y_t^{(n)} = \frac{1}{n} \sum_{i=0}^{n-1} E_t [r_{t+i}] + \frac{1}{n} \sum_{i=0}^{n-2} E_t [\text{exr}_{t+i+1}^{(n-i)}].
\]  

(4)

This equation implies that bond yields are essentially the sum of two components: the average of expected future short interest rates \( (r_{t+i}) \) and excess returns \( (\text{exr}_{t+i}) \). Now the question is how to split bond yields into the two components, which is the main theme of the following two subsections. Section 2.1 separates the former component—the average of expected future short rates—into a persistent part of core inflation and a transitory deviation of one-year bond yields. Section 2.2 shows that the latter one—the average of expected future excess returns—consists of two risk premium state variables with different frequencies. The long-term risk premium state variable has half-life of about 7.1 months and the short-term risk premium’s half life is 1.8 months.
2.1 Decomposition of Short Interest Rates \((r_{t+i})\)

Short interest rates do not revert to a constant mean. Instead, their mean-reversion is toward a time-varying expected value. For example, Fama (2006) concludes that the “mean-reversion is toward a non-stationary (permanent) long-term mean.”

It implies that at least two state variables are needed to model the dynamics of short interest rates: one needs to be persistent and the other transitory. In fact, it has long been assumed, either explicitly or implicitly, that short interest rates consist of multiple components with different frequencies. For example, in almost all multifactor term structure models with at least a level and a slope factor, such as Rudebusch and Wu (2008), short interest rates are implicitly assumed to be driven by persistent and transitory components. The level factor usually appears as a persistent unit-root process as opposed to the slope factor being a stationary mean-reverting variable. Other examples include Campbell, Sunderam, and Viceira (2012), who explicitly model that nominal short interest rates are composed of three components: permanent inflation, transitory inflation, and transitory real interest rate.

The literature suggests that the persistent component of short interest rates is primarily determined by the long-run mean of inflation (Kozicki and Tinsley, 2001; Gürkaynak, Sack, and Swanson 2005; Atkeson and Kehoe 2009; Goodfriend and King 2009). Thus, I estimate the persistent component using the history of realized core inflation, and the transitory component is estimated as the residuals from regressing 1-year Treasury bond yields on the persistent inflation component. This estimation strategy is borrowed from Cieslak and Povala (2011).

The persistent component \((\tau_t)\) is estimated as an exponentially-weighted average of
realized core inflation over the past 10 years:

\[
\tau_t \equiv \frac{\sum_{i=0}^{120} v^i CPI_{t-i}}{\sum_{i=0}^{120} v^i},
\]

where \((1-v)\) denotes constant gain. Cieslak and Povala (2011) estimate the gain parameter at \(v = 0.9868\) (standard error 0.0025) by comparing realized inflation with inflation survey forecasts.

The top panel of Figure 1 shows the time series of the estimated persistent component of inflation. Core inflation data are downloaded from the FRED Economic Data.\(^2\) The estimated series start in January 1968 since the database provides core inflation since January 1958 and 10-year histories are used to estimate the variable. According to the figure, the inflation component reached the peak in the early 1980s and has gradually declined since then. The augmented Dickey-Fuller (ADF) test does not reject the null hypothesis for the persistent inflation of being a unit-root process.

Provided that short interest rates are determined by two components, the transitory one can be estimated as the residuals of short interest rates orthogonalized by the persistent inflation component. Following Cieslak and Povala (2011), Fama-Bliss one-year bond yields are used as a proxy of the short interest rates.

\[
\delta_t \equiv y_t^{(1y)} - \hat{\beta}_0 - \hat{\beta}_1 \tau_t,
\]

where \(\delta_t\) denotes the transitory deviation of short interest rates. \(\hat{\beta}_0\) and \(\hat{\beta}_1\) are OLS coefficients of regressing \(y_t^{(1y)}\) on \(\tau_t\).

The bottom panel of Figure 1 shows the time series of the transitory component. It drops rapidly during recession periods, which is consistent with conventional monetary policy that employs federal funds rate to moderate business cycle.

\(^2\)http://research.stlouisfed.org/fred2/series/CPILFESL?cid=32424
Figure 1: Decomposition of Short Interest Rates

This figure decomposes one-year Treasury bond yields into two components: one is persistent and the other is transitory. The persistent component is estimated as an exponentially-weighted average of realized core inflation over the past 10 years, and the transitory component as the residuals from the regression of one-year yields on the persistent inflation. Shaded areas denote NBER recessions.
2.2 Decomposition of Risk Premium \( (E_t[exr_{t+i}]) \)

Risk premium can be defined as expected excess returns of risky assets. The early literature of risk premium in the bond market, such as Campbell and Shiller (1991) and Dai and Singleton (2002), shows that the excess returns are predictable, and this finding has two implications. First, it rejects the expectations hypothesis that forward interest rates are equal to expected future short interest rates. Second, it implies that the market price of risk is time-varying and spanned by observable variables.

This paper assumes two risk premium factors. The first one, \( rpl \), is estimated following Cieslak and Povala (2011)’s approach. The authors estimate the risk premium factor as the residual from regressing T-bond yields on the two components of short interest rates. They first regress each of 2- to 5-year bond yields on the persistent inflation \( \tau_t \)

\[
y_t^{(n)} = \beta_0^{(n)} + \beta_1^{(n)} \tau_t + \epsilon_t^{(n)} \quad \text{for } n = 2, \cdots, 5 \text{ years} \tag{7}
\]

and then regress the average of the residuals \( \bar{\epsilon}_t \equiv \frac{1}{4} \sum_{n=2}^{5} \epsilon_t^{(n)} \) on the transitory interest rate component \( \delta_t \) without an intercept.

\[
\bar{\epsilon}_t = \gamma_1 \delta_t + u_t. \tag{8}
\]

Cieslak and Povala (2011) show that equation (8)’s residuals \( \hat{u}_t \) outperform both Cochrane and Piazzesi (2005) and Ludvigson and Ng (2009)’s risk premium predictors in the predictive regression of excess returns in the bond market. This paper uses the residuals as a proxy of \( rpl \). The CRSP Fama-Bliss Discount Bond Yields are used for the 2- to 5-year bond yields.

The second risk premium factor, \( rps \), is estimated as 3-month Treasury bill yields.

\(^3\)Piazzesi (2003) explains the rejection of the expectations hypothesis in detail.
Figure 2: Time Series of Risk Premium Factors

This figure compares the time series of long- and short-term risk premium factors. The former is estimated from Treasury bond yields meanwhile the latter is from Treasury bill yields. The factors are demeaned and normalized. Shaded areas denote NBER recessions.

(a) Risk Premium Factor from Treasury Bonds ($r_{pl,t}$)

(b) Risk Premium Factor from Treasury Bills ($r_{ps,t}$)
that are orthogonalised by the other three state variables.

\[
\text{rps}_t = - \left\{ y_t^{(3m)} - \hat{\beta}_0 - \hat{\beta}_1 \tau_t - \hat{\beta}_2 \delta_t - \hat{\beta}_3 \text{rpl}_t \right\},
\]

(9)

where the \(\hat{\beta}\)s are OLS coefficients. \(\text{rps}\) turns positive if 3-month T-bill yield is lower than the projected value by the three state variables. Both \(\text{rpl}\) and \(\text{rps}\) are normalised by sample mean and standard deviation. The 3-month T-bill yields are downloaded from the FRED Economic Data.

To explain the rationale of \(\text{rps}\), consider a simple hypothetical example with two state variables whose risk-neutral dynamics are

\[
\begin{pmatrix}
  r_{t+1} \\
  \text{rps}_{t+1}
\end{pmatrix} = \mu^q + \begin{pmatrix}
  k_{11}^q & k_{12}^q \\
  k_{21}^q & k_{22}^q
\end{pmatrix} \begin{pmatrix}
  r_t \\
  \text{rps}_t
\end{pmatrix} + \sum \epsilon^q_{t+1},
\]

(10)

where \(r_t\) denotes short interest rate at time \(t\) and the superscript \(q\) denotes risk-neutral probability measure. The loadings of \(m\)-period zero-coupon bond yields on the state variables can be derived as

\[
B_m = \frac{1}{m} \begin{pmatrix}
  1 & 0
\end{pmatrix} \{I - K^q\}^{-1} \{I - (K^q)^m\}.
\]

(11)

If \(k_{12}^q\) and \(k_{22}^q\) are sufficiently small but not exactly zero, the loading of \(m\)-period bond yields on \(\text{rps}\) converge to zero very quickly with maturity \(m\). Therefore, the information of \(\text{rps}\) would have appeared not in T-bonds but in T-bills. 3-month T-bills are chosen since it has the longest history since 1954. In comparison, the secondary market rate of 4-week Treasury bills is provided only since July 2001.

\[\text{Figure 2}\] shows the time series of the two risk premium factors, \(\text{rpl}\) and \(\text{rps}\). The three state variables (\(\tau_t\), \(\delta_t\), and \(\text{rpl}_t\)) are based on the replication of Cieslak and Povala (2011),

\[\text{http://research.stlouisfed.org/fred2/series/DTB3?cid=116}\]
and this paper adds one more state variable—short-term risk premium factor \( (r_{ts}) \)—to the framework and focuses on the comparison of \( rpl \) and \( rps \). Although not reported in this paper, other benchmark risk premium factors such as Fama and Bliss (1987)’s forward interest rate slope and Cochrane and Piazzesi (2005)’s tent-shaped factor were also used as a proxy of \( rpl \) and the results were not changed.

### 3 Forecast of Excess Returns in the Bond Market

Risk premium is not observable. However, if future excess returns can be predicted by some observable variables at time \( t \), one can consider that the unobservable risk premium can be projected on the space that is spanned by the observable predictors. Therefore, if \( rpl \) and \( rps \) were indeed related to risk premium, they would have been able to predict future excess returns.

Table 1 tests the forecastability of annual excess returns. The dependent variable is the excess returns of holding Treasury bonds over the next one year, \( \text{exr}_{t, t+1}^{(n)} \), in which risk-free short interest rates are estimated as one-year T-bond yields. Since Fama and Bliss (1987), it has been a norm to use annual excess returns for the test of risk premium factors’ forecastability. The excess returns are estimated from the T-bond data provided by the Federal Reserve Board of Governors which smoothes yield curves using the Svensson curve approximation method. Gürkaynak, Sack, and Wright (2006) explain the construction of the dataset in detail.

Panel A regresses the excess returns on \( rpl \) only, replicating Cieslak and Povala (2011)’s predictability test. For comparison, Panel B and C regress the returns on the other two benchmark risk premium factors. Panel B uses Cochrane and Piazzesi (2005)’s tent-shaped factor which is estimated from the predicted excess returns projected on five

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### Table 1: Forecast of Annual Excess Returns

The dependent variable is excess returns of holding Treasury bonds over the next one year, \( e.r_t^{(n)} \), whose maturities are specified by the top row. Predictors in each panel are respectively attributed to [Cieslak and Povala (2011)](http://www.econ2.jhu.edu/people/Duffee/), [Cochrane and Piazzesi (2005)](http://www.econ2.jhu.edu/people/Duffee/), and [Duffee (2011)](http://www.econ2.jhu.edu/people/Duffee/). \( rpl \)'s estimation method is explained in Section 2.2. All risk premium factors are normalized. Numbers in parentheses are Newey-West \( t \) statistics with 12 lags. ***, **, and * denote significances at 1%, 5%, and 10% level respectively.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>2 years</th>
<th>3 years</th>
<th>5 years</th>
<th>7 years</th>
<th>10 years</th>
<th>15 years</th>
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<tr>
<td><strong>Panel A. Forecast by ( rpl ) (Cieslak and Povala, 2011)</strong></td>
<td></td>
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<tr>
<td>( rpl )</td>
<td>0.967***</td>
<td>1.826***</td>
<td>3.296***</td>
<td>4.601***</td>
<td>6.615***</td>
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<td>528</td>
<td>528</td>
<td>485</td>
<td>482</td>
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<td>( R^2 )</td>
<td>0.302</td>
<td>0.327</td>
<td>0.353</td>
<td>0.368</td>
<td>0.485</td>
<td>0.424</td>
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<td><strong>Panel B. Cochrane and Piazzesi (2005)’s Tent-Shaped Factor</strong></td>
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<tr>
<td>( tent )</td>
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<td>3.485***</td>
<td>4.735***</td>
<td>7.098***</td>
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<td>0.223</td>
<td>0.212</td>
<td>0.231</td>
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<td><strong>Panel C. Duffee (2011)’s Hidden Factor</strong></td>
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<td></td>
</tr>
<tr>
<td>( hidden )</td>
<td>-0.703***</td>
<td>-1.271***</td>
<td>-2.221***</td>
<td>-3.062***</td>
<td>-4.624***</td>
<td>-6.833***</td>
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<td></td>
<td>(-4.091)</td>
<td>(-4.049)</td>
<td>(-3.951)</td>
<td>(-3.908)</td>
<td>(-4.100)</td>
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<td>434</td>
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<tr>
<td>( R^2 )</td>
<td>0.158</td>
<td>0.158</td>
<td>0.164</td>
<td>0.170</td>
<td>0.196</td>
<td>0.206</td>
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</table>

forward interest rates. Panel C uses [Duffee (2011)](http://www.econ2.jhu.edu/people/Duffee/)'s hidden factor which is estimated as a higher-order state variable from a Kalman filtering under the restriction that the factor be hidden from the cross-section of bond yields. The hidden factor can be downloaded from Gregory Duffee’s website. Figure 3 shows the time series of the two benchmark risk premium factors.

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This figure shows the time series of Cochrane and Piazzesi (2005)’s tent-shaped risk premium factor and Duffee (2011)’s hidden factor. Cochrane and Piazzesi (2005) estimate the tent-shaped factor as the predicted annual excess returns of holding Treasury bonds by five forward interest rates, and Duffee (2011) estimates the hidden factor as a higher-order state variable of a Kalman filtering method. Both factors are normalised, and Duffee (2011)’s hidden factor is multiplied by $-1$.

Table 1 shows that $rpl$ outperforms other benchmarks by a large margin. For example, $rpl$’s $R^2$ in Panel A range from 30.2% to 42.4%. In comparison, the tent-shaped factor’s $R^2$s in Panel B are from 16.7% to 23.1%, and the hidden factor’s $R^2$s in Panel C are from 15.8% to 20.6%. $rpl$ also outperforms the benchmarks in terms of statistical significance. $rpl$’s Newey-West $t$-statistics are 8.0 to 8.7 as opposed to the statistics of 4.2~4.6 in Panel B and 3.9~4.1 in Panel C.

In particular, the large improvement of forecastability from Panel B to Panel A is notable since both predictors are estimated from the same dataset—the cross-section of five Fama-Bliss bond yields—except that $rpl$ orthogonalised the yields with inflation rates. $R^2$s almost double by simply getting rid of the persistent inflation component from

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$^7$ I intentionally specify the “Fama-Bliss” bond yields since, as shown by Dai, Singleton, and Yang (2004), the tent shape is turned into a wave pattern if bond yields are estimated from different data sources.
predictors. Thus, it implies that the persistent inflation component (i.e., the level factor as will be shown in Section 4) is not related to the time-varying market price of risk. In a similar vein, Dai, Singleton, and Yang (2004) show that the tent shape of Cochrane and Piazzesi (2005)'s risk premium factor is due to a mechanical effect to offset the level factor from forward interest rates. Cochrane and Piazzesi (2008) also show that the level factor is not related to the market price of risk although its uncertainty is priced by the market.

However, strong forecastability of annual excess returns does not necessarily mean strong forecastability for shorter investment horizons. First of all, the predictive regression of annual returns with monthly observations is likely to be contaminated by the long-horizon forecast bias since its dependent variable is mechanically autocorrelated. For example, Boudoukh, Richardson, and Whitelaw (2007) show that long-horizon forecast regressions exaggerate $R^2$ and statistical significance. Moreover, annual excess returns inevitably ignore the risk premium born by one-year Treasury bonds since one-year bonds are considered a riskfree asset for an annual horizon but a risky asset for a monthly period. One stylized fact in the bond market is that, as shown by Duffee (2010), shorter-maturity bonds offer higher Sharpe ratios. Thus, the act of ignoring one-year bonds’ risk premium would have probably undermined the authenticity of excess return forecasts.

Table 2 tests the forecastability of monthly excess returns. The dependent variable is the excess returns from holding Treasury bonds for the next one month over one-month riskfree interest rates, $exr_{t,t+1m}$. The one-month riskfree rates are from the Ibbotson Associates. Panel A uses $rpl$ as the only predictor. As suspected, its $R^2$'s drop to 2.7%~3.4%, which are only one tenth of the $R^2$'s in Table 1. Although not reported here, the $R^2$'s of the other benchmark predictors also drop to 1%~2%.

Panel B, however, shows that the monthly return forecast is significantly improved by

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8The one-month riskfree interest rates are downloaded from Kenneth French’s website.
Table 2: Forecast of Monthly Excess Returns

The dependent variable is excess returns of holding Treasury bonds over the next one month, \( \text{extr}^{(n)}_{t,t+1m} \), whose maturities are specified by the top row. \( rpl \) and \( rps \) denote risk premium factors which are estimated respectively from Treasury bonds and Treasury bills. \( \tilde{y}^{(1m)}_t \) and \( \tilde{y}^{(3m)}_t \) denote 1- and 3-month Treasury bills orthogonalized by the persistent and transitory components of risk-free interest rates and \( rpl \). Numbers in parentheses are Newey-West t statistics with 12 lags. ***, **, and * denote significances at 1%, 5%, and 10% level respectively.

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<th>7 years</th>
<th>10 years</th>
<th>15 years</th>
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<tbody>
<tr>
<td>( rpl )</td>
<td>0.134***</td>
<td>0.202***</td>
<td>0.317***</td>
<td>0.414***</td>
<td>0.553***</td>
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<td>( R^2 )</td>
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<td>0.034</td>
<td>0.033</td>
<td>0.033</td>
<td>0.031</td>
</tr>
</tbody>
</table>

Panel A. Forecast by \( rpl \) Only

| \( rpl \) | 0.134*** | 0.202*** | 0.317*** | 0.414*** | 0.542*** | 0.756*** |
|           | (3.259)  | (3.683)  | (4.183)  | (4.312)  | (4.066)  | (4.036)  |
| \( rps \) | 0.143*** | 0.192*** | 0.290*** | 0.405*** | 0.611*** | 0.992*** |
|           | (2.884)  | (2.860)  | (3.092)  | (3.449)  | (3.832)  | (4.387)  |
| obs       | 540      | 540      | 540      | 540      | 497      | 494      |
| \( R^2 \) | 0.058    | 0.059    | 0.062    | 0.065    | 0.074    | 0.082    |

Adding \( rps \) as a predictor. \( R^2 \)s almost double to 5.8%~8.2% and both \( rpl \) and \( rps \) appear significant. \( rpl \) and \( rps \) do not have multicollinearity issue since \( rps \) is orthogonalised by \( rpl \) and thus they have zero correlation. According to the results, the excess return of holding 10-year bonds for one month is expected to increase by 0.542% if \( rpl \) is one standard deviation high and by 0.611% when \( rps \) is one standard deviation high. This result implies that \( rps \) (or equivalently 3-month Treasury bills) has unique risk premium information that is not spanned by the first three state variables.

As Figure 2 shows, \( rps \) is far more volatile than \( rpl \). In particular, \( rpl \)'s half-life is
Table 3: Forecast of Monthly Excess Returns in Further Periods

The dependent variable is monthly excess returns of holding Treasury bonds in further periods, $exr_{t+\phi-1,t+\phi}^{(n)}$ for $\phi = 2, 3$ and 16 months. $rpl_t$ and $rps_t$ denote risk premium factors which are estimated respectively from Treasury bonds and Treasury bills. Numbers in parentheses are Newey-West $t$ statistics with 12 lags. ***, **, and * denote significances at 1%, 5%, and 10% level respectively.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>2 years</th>
<th>3 years</th>
<th>5 years</th>
<th>7 years</th>
<th>10 years</th>
<th>15 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A. Monthly Excess Returns in 2 Months, $exr_{t+1m,t+2m}^{(n)}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$rpl_t$</td>
<td>0.155***</td>
<td>0.233***</td>
<td>0.371***</td>
<td>0.487***</td>
<td>0.653***</td>
<td>0.896***</td>
</tr>
<tr>
<td></td>
<td>(4.381)</td>
<td>(4.796)</td>
<td>(5.220)</td>
<td>(5.413)</td>
<td>(5.466)</td>
<td>(5.455)</td>
</tr>
<tr>
<td>$rps_t$</td>
<td>0.145***</td>
<td>0.204***</td>
<td>0.305***</td>
<td>0.405***</td>
<td>0.558***</td>
<td>0.771***</td>
</tr>
<tr>
<td></td>
<td>(3.015)</td>
<td>(3.254)</td>
<td>(3.669)</td>
<td>(3.897)</td>
<td>(3.813)</td>
<td>(3.471)</td>
</tr>
<tr>
<td>obs</td>
<td>540</td>
<td>540</td>
<td>540</td>
<td>540</td>
<td>498</td>
<td>495</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.068</td>
<td>0.073</td>
<td>0.077</td>
<td>0.077</td>
<td>0.081</td>
<td>0.074</td>
</tr>
<tr>
<td>Panel B. Monthly Excess Returns in 3 Months, $exr_{t+2m,t+3m}^{(n)}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$rpl_t$</td>
<td>0.145***</td>
<td>0.209***</td>
<td>0.315***</td>
<td>0.405***</td>
<td>0.552***</td>
<td>0.822***</td>
</tr>
<tr>
<td>$rps_t$</td>
<td>0.064</td>
<td>0.088</td>
<td>0.123</td>
<td>0.167*</td>
<td>0.287**</td>
<td>0.407**</td>
</tr>
<tr>
<td></td>
<td>(1.587)</td>
<td>(1.609)</td>
<td>(1.635)</td>
<td>(1.784)</td>
<td>(2.351)</td>
<td>(2.236)</td>
</tr>
<tr>
<td>obs</td>
<td>540</td>
<td>540</td>
<td>540</td>
<td>540</td>
<td>499</td>
<td>496</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.038</td>
<td>0.039</td>
<td>0.038</td>
<td>0.037</td>
<td>0.043</td>
<td>0.044</td>
</tr>
<tr>
<td>Panel C. Monthly Excess Returns in 16 Months, $exr_{t+15m,t+16m}^{(n)}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$rpl_t$</td>
<td>0.069</td>
<td>0.106*</td>
<td>0.168*</td>
<td>0.218*</td>
<td>0.306**</td>
<td>0.409*</td>
</tr>
<tr>
<td></td>
<td>(1.571)</td>
<td>(1.754)</td>
<td>(1.862)</td>
<td>(1.853)</td>
<td>(1.962)</td>
<td>(1.894)</td>
</tr>
<tr>
<td>$rps_t$</td>
<td>-0.042</td>
<td>-0.062</td>
<td>-0.113</td>
<td>-0.168</td>
<td>-0.301</td>
<td>-0.453*</td>
</tr>
<tr>
<td></td>
<td>(-0.695)</td>
<td>(-0.779)</td>
<td>(-1.000)</td>
<td>(-1.168)</td>
<td>(-1.613)</td>
<td>(-1.779)</td>
</tr>
<tr>
<td>obs</td>
<td>534</td>
<td>534</td>
<td>534</td>
<td>534</td>
<td>506</td>
<td>503</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.009</td>
<td>0.011</td>
<td>0.013</td>
<td>0.014</td>
<td>0.019</td>
<td>0.018</td>
</tr>
</tbody>
</table>
estimated to be about 7.1 months meanwhile \( rps \)' is 1.8 months long. Since \( rps \) has a shorter half-life, one can also expect that \( rps \) might have a shorter forecast horizon than \( rpl \), which is tested in Table 3.

The dependent variables in each panel of Table 3 are monthly excess returns in 2, 3 and 16 months \( (exr_{t+1m,t+2m}^{(n)}, exr_{t+2m,t+3m}^{(n)}, \text{ and } exr_{t+15m,t+16m}^{(n)}) \). The table shows that \( rps \) shows even stronger forecastability in two months than in one month of the previous table but loses most of its forecastability in three months. In contrast, \( rpl \)'s forecastability remains significant for the horizon of up to 16 months. Particularly, \( rpl \) appears significant at 1% confidence level even in one year. Thus, the table confirms that \( rps \)'s forecastability is limited to a short horizon whereas \( rpl \)'s forecastability remains strong for more than one year.

Every term structure model is essentially a function to convert state variables into yield curves. In other words, excess returns of bonds can be written as a function of the changes in state variables. Given the previous finding that the excess returns are predicted by \( rpl \) and \( rps \), one can expect the risk premium factors to also predict the changes in other state variables. Moreover, it is of a particular interest to see the forecast of the persistent inflation \( (\tau_t) \) and transitory short interest rate \( (\delta_t) \) since they account for the level and slope factors (will be shown in the next section) and thus explain almost 99% of the total variation of yield curves.

For this aim, Table 4 estimates the VAR(1) system of the state variables to understand their dynamics. Its dependent variables are the changes in state variables, which are specified in the first row. Note that \( \tau_t \)'s coefficient in column (1) is close to zero, implying that the inflation component is as persistent as a random walk process. In comparison, \( \delta_t \)'s coefficient of \(-0.041\) in column (2) implies that its half-life is about 15.4 \( (= -\log(2)/\log(1 - 0.041)) \) months. \( rpl_t \)'s coefficient of \(-0.093\) in column (3) and \( rps_t \)'s coefficient of \(-0.320\) in column (4) imply that their half-lives are 7.1
Table 4: Forecast of State Variables and Bond Yields

The dependent variable is the changes in four state variables: $\tau$, $\delta$, $rpl$, and $rps$. $\tau$ and $\delta$ denote the persistent and transitory components of risk-free interest rates whereas $rpl$ and $rps$ denote long- and short-term risk premium factors. Their estimation is explained in Section 2.1 and 2.2. $\tau$ and $\delta$ are multiplied by 100 to match with the scales of $rpl$ and $rps$. Numbers in parentheses are Newey-West $t$ statistics with 12 lags. ***, **, and * denote significances at 1%, 5%, and 10% level respectively.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>dep. var.</td>
<td>$\Delta \tau_{t+1}$</td>
<td>$\Delta \delta_{t+1}$</td>
<td>$\Delta rpl_{t+1}$</td>
<td>$\Delta rps_{t+1}$</td>
</tr>
<tr>
<td>$\tau_t$</td>
<td>0.000</td>
<td>-0.001</td>
<td>0.005</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
<td>(-0.051)</td>
<td>(0.537)</td>
<td>(0.037)</td>
</tr>
<tr>
<td>$\delta_t$</td>
<td>0.008***</td>
<td>-0.041***</td>
<td>0.004</td>
<td>-0.027</td>
</tr>
<tr>
<td></td>
<td>(4.298)</td>
<td>(-2.644)</td>
<td>(0.439)</td>
<td>(-1.249)</td>
</tr>
<tr>
<td>$rpl_t$</td>
<td>-0.011***</td>
<td>0.003</td>
<td>-0.093***</td>
<td>0.039</td>
</tr>
<tr>
<td></td>
<td>(-3.504)</td>
<td>(0.093)</td>
<td>(-4.51)</td>
<td>(0.936)</td>
</tr>
<tr>
<td>$rps_t$</td>
<td>-0.001</td>
<td>-0.067***</td>
<td>-0.025</td>
<td>-0.320***</td>
</tr>
<tr>
<td></td>
<td>(-0.740)</td>
<td>(-2.076)</td>
<td>(-1.436)</td>
<td>(-7.162)</td>
</tr>
<tr>
<td>obs</td>
<td>539</td>
<td>539</td>
<td>539</td>
<td>539</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.262</td>
<td>0.038</td>
<td>0.049</td>
<td>0.166</td>
</tr>
</tbody>
</table>

($= -\log(2)/\log(1 - 0.093)$) and $1.8 (= -\log(2)/\log(1 - 0.320))$ months respectively. These results are consistent with Figure 1 and 2’s findings that $\tau_t$ is the most persistent state variable, followed by $\delta_t$, $rpl_t$, and $rps_t$. Moreover, $\delta_t$’s coefficient in column (1) is significantly positive since the Federal Reserve Board raises federal funds rate in the anticipation of high inflation in subsequent periods.

The most important implication in the table is based on $rpl_t$ and $rps_t$’s coefficients in column (1) and (2), which imply that both risk premium factors predict a decrease in short interest rates. However, they do so in different ways. $rpl_t$ predicts a decrease in the persistent inflation ($\tau_t$) meanwhile $rps_t$ does a decrease in the transitory component of short interest rates ($\delta_t$). This result confirms that their predictability is based on different dimensions.

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4 Principal Components of Yield Curves and the Literature of Hidden Factor

Bond yields, as shown by equation \(4\), are the sum of expected future riskfree interest rates and risk premium. Thus, one can expect that high risk premium would raise bond yields. The intuition motivates Fama and Bliss (1987) to show that high risk premium is related to steep yield curve slope. Their finding implies that long-maturity bond yields have higher loadings on risk premium than short-maturity yields.

However, the recent literature has discovered some evidence that opposes the intuition. For example, Cochrane and Piazzesi (2005) estimate a tent-shaped risk premium factor by regressing the annual excess returns of holding Treasury bonds on five forward interest rates and conclude that “the return-forecasting factor is clearly not related to any of the first three principal components.” Since the first three principal components—level, slope, and curvature—are able to explain 99.9% of the total variations of yield curves, their finding can be rephrased as that the tent-shaped risk premium factor does not affect the shape of yield curve. It is why bond risk premium has been called as the “hidden factor” by the recent literature.

The latest literature such as Duffee (2011) and Joslin, Priebsch, and Singleton (2010) explains that the risk premium factor is hidden since an increase in risk premium is offset by a decrease in expected short interest rates. The hidden factor affects the relative proportion of riskfree rates and risk premium, but not their sums, i.e., bond yields. Chernov and Mueller (2012) also show that the hidden factor “affects inflation expectations at all horizons, but has almost no effect on the nominal yields.”

But Cieslak and Povala (2011) reach a different conclusion. Their risk premium factor does raise the slope of yield curves. They explain that the factor seems to be hidden because the slope factor—the second principal component—is driven by not only risk
premium but also the transitory component of short interest rates ($\delta_t$). The former raises the long end of yield curve meanwhile the latter raises the short end. The Shapley decomposition shows that 70% and 27% of the slope factor’s variations are explained by transitory short rates and risk premium respectively.

My paper adds a new perspective to the debate. I find positive evidence backing all of the aforementioned papers. For example, my results support Fama and Bliss (1987) and Cieslak and Povala (2011) since the long-term risk premium factor ($rpl_t$) indeed raises the slope of yield curves. This paper also agrees with Duffee (2011) and Joslin, Priebsch, and Singleton (2010) since the short-term risk premium factor ($rps_t$) is almost completely hidden from T-bond yields. As the hidden factor literature explains, $rps$ predicts high risk premium but does not affect the shape of yield curves. Cochrane and Piazzesi (2005) are also supported since higher-order principal components are dominantly determined by the two risk premium factors.

Figure 4 shows how Treasury bond yields are affected when each state variable is deviated by $+/-$ two standard deviations. Each subfigure is labeled according to a given state variable. The horizontal axis denotes maturities in years from 1 to 15 years, and the vertical axis denotes annualized bond yields. The figure is plotted based on the OLS coefficients of regressing T-bond yields on the state variables. Simple OLS regression is used to estimate the factor loadings instead of affine term structure model since, as Duffee (2009) and Joslin, Singleton, and Zhu (2011) point out, the results are invariant to the imposition of no-arbitrage restrictions and thus the affine model adds very little improvement over the OLS regression.

Panel (a) shows that the persistent inflation component ($\tau_t$) shifts the level of yield curves. This effect is intuitive given the near-unit-root persistence of the variable. Panel (b) and (c) show that yield curves’ slope is determined by the transitory short interest rate ($\delta_t$) as well as the long-term risk premium factor ($rpl_t$). High $\delta_t$ flattens the slope
Figure 4: Factor Loadings of Treasury Bond Yields

This figure shows how the shapes of Treasury bond yield curves are affected by state variables. The horizontal axis denotes maturities in years, and the vertical axis denotes annualized bond yields. Each subplot corresponds to the changes of +/- two standard deviations in a given state variable.

(a) Persistent Inflation ($\tau_t$)
(b) Transitory Short Rates ($\delta_t$)
(c) Long-term Risk Premium Factor ($rpl_t$)
(d) Short-term Risk Premium Factor ($rps_t$)
by raising short-maturity bond yields meanwhile high \( rpl_t \) steepens it by raising long-maturity yields. Panel (d) confirms that the short-term risk premium factor \( (rps_t) \) is almost completely hidden from T-bond yields.

In comparison, Figure 5 shows the factor loadings of Treasury bill yields. The horizontal axis denotes maturities in months from 1 to 12 months, and the vertical axis denotes annualized bond yields. The factor loadings are estimated by regressing Fama TBill Structures (available from CRSP) on the state variables.

The comparison of Figure 4 and Figure 5 offers interesting differences. First, the level of T-bill yields is determined by the two components of short interest rates, \( \tau_t \) and \( \delta_t \), meanwhile the level of T-bond yields is determined only by \( \tau_t \). Second, T-bill yields’ slope is jointly determined by the two risk premium factors, \( rpl_t \) and \( rps_t \), but the slope of T-bond yields is driven by \( \delta_t \) and \( rpl_t \).

In particular, the two figures imply that Treasury bills are likely to become expensive when \( rps \) is high although it does not affect the prices of Treasury bonds at all. Therefore, one may hypothesise that \( rps \)’ implication in the financial markets is probably based on the market segmentation between T-bonds and T-bills. For example, Duffee (1996) shows that the market segmentation has increased since the early 1980s. Given that T-bills provide convenience yields to investors, as argued by Grinblatt (2001), we can expect \( rps_t \) to be related to liquidity measures in the financial market since convenience yields are proportional to the demands of liquidity. This hypothesis will be tested in the next section.

Lastly, Table 5 shows the contribution of each state variable to the explained variations of principal components. For example, its Panel A implies that 83.46% and 13.07% of the level factor (PC1)’s variations can be explained by \( \tau_t \) and \( \delta_t \) respectively. The contributions are estimated by the Shapley decomposition method. Numbers in any row are summed to one hundred. Panel A uses 15 T-bond yields to estimate principle com-

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Figure 5: Factor Loadings of Treasury Bill Yields

This figure shows how the shapes of Treasury bill yield curves are affected by state variables. The horizontal axis denotes maturities in months, and the vertical axis denotes annualized bond yields. Each subplot corresponds to the changes of $\pm$ two standard deviations in a given state variable.

(a) Persistent Inflation ($\tau_t$)
(b) Transitory Short Rates ($\delta_t$)
(c) Long-term Risk Premium Factor ($r_{pl_t}$)
(d) Short-term Risk Premium Factor ($r_{ps_t}$)
Table 5: Decomposition of Principal Components by State Variables

This table shows the contributions of the four state variables to the explained variance of the respective principal components. Each state variable’s contribution is computed using the Shapley decomposition. Numbers in one row are summed to one hundred. In Panel A, fifteen Treasury bond yields with the maturities of 1 to 15 years are used for the estimation of principal components. In Panel B, three Treasury bill yields of 1, 3, and 6-month maturities are used along with the fifteen Treasury bond yields.

<table>
<thead>
<tr>
<th>Inflation (τ)</th>
<th>Short Rates (δ)</th>
<th>Risk Premium (rpl)</th>
<th>Risk Premium (rps)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A. 15 T-bond Yields (1 to 15 Years)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PC1 (Level)</td>
<td>83.46</td>
<td>13.07</td>
<td>3.46</td>
</tr>
<tr>
<td>PC2 (Slope)</td>
<td>3.45</td>
<td>69.79</td>
<td>26.74</td>
</tr>
<tr>
<td>PC3 (Curvature)</td>
<td>0.17</td>
<td>7.95</td>
<td>90.74</td>
</tr>
<tr>
<td>PC4</td>
<td>0.69</td>
<td>0.14</td>
<td>88.71</td>
</tr>
<tr>
<td>PC5</td>
<td>3.84</td>
<td>7.19</td>
<td>17.23</td>
</tr>
<tr>
<td><strong>Panel B. 15 T-bond and 3 T-bill Yields (1, 3, and 6 Months)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PC1 (Level)</td>
<td>81.35</td>
<td>16.48</td>
<td>2.16</td>
</tr>
<tr>
<td>PC2 (Slope)</td>
<td>5.13</td>
<td>57.39</td>
<td>36.94</td>
</tr>
<tr>
<td>PC3 (Curvature)</td>
<td>0.68</td>
<td>8.48</td>
<td>75.67</td>
</tr>
<tr>
<td>PC4</td>
<td>0.06</td>
<td>0.40</td>
<td>30.87</td>
</tr>
<tr>
<td>PC5</td>
<td>0.46</td>
<td>2.72</td>
<td>0.35</td>
</tr>
</tbody>
</table>

The table shows that the slope factor (PC2)’s variations are largely driven by δt and rplt, each of which explains 70% (57%) and 27% (37%) according to Panel A (Panel B). This result is consistent with the factor loadings of T-bond yields in Figure 4 which shows that the slope becomes steeper when δt is low or rplt is high. Moreover, it also
explains why the slope factor per se is a poor proxy of risk premium. Not only it fails to outperform other benchmark risk premium factors in the predictive regression of bond excess returns, but also it loses all significance when monthly excess returns are used as a dependent variable. As the table shows, a large amount of the slope’s variations are determined by $\delta_t$, which is not related to risk premium.

The table also shows that higher-order principal components are dominated by the two risk premium factors, $rpl_t$ and $rps_t$. For example, according to Panel A, $rpl_t$ explains 89% of PC4’s variations and $rps_t$ does 72% of PC5’s variations. Interestingly, it is notable that PC5’s variations are dominantly determined by $rps_t$ even when T-bill yields are not used for the principal components’ estimation. This result also explains the literature that higher-order principal components are more informative of risk premium in the bond market than the first three principal components.

5 Relation to the Financial Market

5.1 Liquidity in the Bond Market

Treasury bills are special. Their values rise during a financial crisis since they are considered the safest collaterals in the world. For example, the yield on the three-month Treasury bill even turned negative on December 9, 2008, three months after the Lehman Brothers’ collapse, as investors had sought for a safe haven\footnote{http://blogs.wsj.com/marketbeat/2008/12/09/three-month-bill-yield-goes-negative/}. One-month Treasury bill yields briefly went negative again on August 4, 2011, since the European woes had cast a shadow over the market\footnote{http://blogs.wsj.com/marketbeat/2011/08/04/from-one-crisis-to-another-one-month-t-bill-yields-go-negative/}.

Even when compared to other types of credit-riskfree assets such as other government securities, Treasury bills still look different. For example, \cite{Duffee1996} describes...
Figure 6: **Fontaine and Garcia (2012)**’s Bond Liquidity Premium

Fontaine and Garcia (2012) estimate bond liquidity premium using the difference of yields between on-the-run (most recently issued) and off-the-run (seasoned) Treasury bonds. This figure compares its time series to the short-term risk premium factor ($r_{ps}$).

that the market segmentation between T-bills and T-bonds has increased since the early 1980s. **Krishnamurthy (2010)** also documents that the repo haircut rate of short-term Treasuries had been fixated at 2% in 2007-2009 meanwhile those of long-term Treasuries and investment-grade corporate bonds soared from 5% to 6% and 5% to 20% respectively.

Considering the role of T-bills as the safest haven and how its price is affected by $r_{ps}$, we can expect $r_{ps}$ to be related to liquidity premium. Motivated by the intuition, this section compares $r_{ps}$ to various liquidity measures in the bond market.

**Fontaine and Garcia (2012)** estimate bond liquidity premium as the difference of yields between on-the-run (most recently issued) and off-the-run (seasoned) Treasury bonds.\(^{11}\)

Since on-the-run bonds are more liquid and thus have higher values as collaterals than off-the-run bonds, the yield differential of on-the-run bonds over off-the-run ones is likely

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\(^{11}\)I am grateful to Jean-Sebastien Fontaine and Rene Garcia for generously sharing the data.
to increase when there are strong demands for liquidity. Figure 6 compares the time series of the bond liquidity premium to $r_{ps}$.

The figure shows that the two time series are remarkably overlapped with each other. They both peaked in 2008 after the Lehman Brothers collapsed and in 1987 after the infamous stock market crash on the Black Monday. They also soared during the Tequila crisis in 1994 and remained high throughout the Asian currency crisis in 1997 and the Russian moratorium and the subsequent demise of the Long Term Capital Management (LTCM) in 1998. The unconditional correlation between the two series is 0.768.

Figure 7 also compares $r_{ps}$ to two other measures of liquidity in the bond market. Panel (a) uses the 3-month AA financial commercial paper spreads over 3-month T-bill yields, and Panel (b) is based on the 3-month overnight index swap (OIS) spreads. Both panels display the log values of the spreads. The commercial paper spreads are downloaded from the FRED Economic Data,\textsuperscript{12} and the OIS spreads are from the Bloomberg. Note that the spreads are a close but not perfect liquidity measure since the spreads are determined not only by convenience yields (Grinblatt, 2001) but also by counterparty risk (Duffie and Singleton, 1997).

Again, the figure shows substantial comovement of $r_{ps}$ with the bond market spreads. They all had gradually increased since 2002, reached to the peak during the financial crisis, and dropped rapidly and stabilised thereafter. The correlations between $r_{ps}$ and the two log spreads are 0.402 and 0.365 respectively.

### 5.2 Risk Factors in the Stock Market

Risk premium in the bond market has been discussed so far. Naturally it raises a question whether it is also related to the stock market. The no arbitrage model imposes that all

\textsuperscript{12}http://research.stlouisfed.org/fred2/categories/120
Figure 7: Other Liquidity Measures in the Bond Market

This figure compares the short-term risk premium factor ($r_{ps}$) to two other liquidity measures in the bond market. Panel (a) uses 3-month AA financial commercial paper spread, and Panel (b) uses 3-month overnight index swap (OIS) spread. Both figures display the log values of those spreads. The scale of $r_{ps}$ is on the left axis and the log spreads are on the right axis.

(a) 3-Month AA Financial Commercial Paper Spread

(b) 3-Month Overnight Index Swap (OIS) Spread
risk premium should have come from the covariance between asset returns and innovation shocks to a pricing kernel. If an universal pricing kernel is able to price both bond and stock markets, risk premium factors in the bond market might have been related to the risk premium in the stock market.

This section is motivated by Koijen, Lustig, and Van Nieuwerburgh (2010), who show that “innovations to the nominal bond risk premium price the book-to-market sorted stock portfolios.” They find that the joint portfolios of stocks and bonds can be priced by three state variables: the level factor of yield curves, stock market returns, and Cochrane and Piazzesi (2005)’s tent-shaped risk premium factor. Interestingly, growth to value stock portfolios are found to have monotonically increasing loadings on the tent-shaped factor, implying that value and bond risk premium might have the same roots. Let me briefly describe its theoretical background below.

Let \( X_t \) denote a column vector of demeaned state variables, which are assumed to follow a VAR(1) process.

\[
X_{t+1} = \Phi X_t + \epsilon_{t+1}, \quad \Omega \equiv E \left[ \epsilon_{t+1} \epsilon_{t+1}^\top \right], \tag{12}
\]

One-period riskfree interest rate, \( r_t \), is given as a linear function of state variables.

\[
r_t = \delta_0 + \delta^\top X_t. \tag{13}
\]

The market price of risk, \( \lambda_t \), is a column vector which is also linearly proportional to state variables.

\[
\lambda_t = \lambda_0 + \Lambda X_t, \tag{14}
\]

where \( \lambda_0 \in \mathbb{R}^n \) and \( \Lambda \in \mathbb{R}^{n \times n} \). Thus, the log nominal pricing kernel can be derived as

\[
m_{t+1} = -r_t - \frac{1}{2} \lambda_t^\top \Omega \lambda_t - \lambda_t^\top \epsilon_{t+1}. \tag{15}
\]
Under the no arbitrage assumption, all risk premium should have come from the covariance of asset returns and the pricing kernel,

\[
E_t \left[ \text{exr}^{(j)}_{t+1} \right] = \text{cov}_t \left( \text{exr}^{(j)}_{t+1}, -m_{t+1} \right) \\
= \text{cov}_t \left( \text{exr}^{(j)}_{t+1}, \epsilon_{t+1} \right) \lambda_t \\
= \Sigma_j (\lambda_0 + \Lambda X_t), \quad (16)
\]

where \( \text{exr}^{(j)}_{t+1} \) denotes the excess returns from holding a risky asset \( j \), and \( \Sigma_j \) the covariance between the returns and the innovation shocks to state variables. By taking unconditional expectations on both sides of equation (16),

\[
E \left[ \text{exr}^{(j)}_{t+1} \right] = \Sigma_j \lambda_0. \quad (17)
\]

For example, suppose the covariance matrix, \( \Sigma_j \), is estimated for each of value portfolios. If the value premium were related to the \( i \)-th state variable, \( \Sigma_{ji} \) would have shown a monotonically increasing (or decreasing) pattern over \( j \).

Figure 8 shows the coefficients of regressing the excess returns of value, size, and momentum-sorted decile stock portfolios on the contemporary innovation shocks to the two risk premium factors, \( \tilde{r}_{pl} \) and \( \tilde{r}_{ps} \). The innovation shocks are measured as the residuals of estimating \( rpl \) and \( rps \)'s time series as two independent AR(1) processes. This figure is to illustrate the estimated covariance matrix, \( \Sigma_j \). Stock portfolio return data are downloaded from Kenneth French’s website.\(^{13}\)

Being consistent with Koijen, Lustig, and Van Nieuwerburgh (2010), the figure shows that the long-term risk premium factor (\( rpl \)) is related to all three cross-sectional stock risk premiums. \( \tilde{r}_{pl} \)'s coefficients show monotonic patterns in all panels, implying that \( rpl \) is positively related to value and size premiums and negatively to momentum. In

\(^{13}\)http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
Figure 8: Covariance with Stock Portfolio Returns

This figure shows the coefficients of regressing stock portfolio excess returns on the contemporary innovation shocks to two risk premium factors, $r_{pl}$ and $r_{ps}$. Portfolio 1 on the horizontal axis denotes growth / small / loser stocks meanwhile Portfolio 10 denotes value / large / winner stocks. Portfolio returns are downloaded from Kenneth French’s website.

(a) Value Portfolios

(b) Size Portfolios

(c) Momentum Portfolios
Table 6: Covariance with Cross-sectional Stock Return Factors

The dependent variables are the Fama-French three factors and the momentum factor, which are denoted on the first row. The explanatory variables are contemporary innovation shocks to the state variables. Panel A accounts for the shocks to risk premium factors only, and Panel B does all innovation shocks. Numbers in parentheses are Newey-West $t$-statistics with 12 lags. ***, **, and * denote significances at 1%, 5%, and 10% level respectively.

<table>
<thead>
<tr>
<th></th>
<th>MktRf</th>
<th>SMB</th>
<th>HML</th>
<th>MOM</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A. Innovation Shocks to Risk Premium Factors</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{r}pl_t$</td>
<td>-0.122</td>
<td>1.093***</td>
<td>0.734**</td>
<td>-1.636***</td>
</tr>
<tr>
<td></td>
<td>(-0.191)</td>
<td>(3.133)</td>
<td>(2.210)</td>
<td>(-2.886)</td>
</tr>
<tr>
<td>$\tilde{r}ps_t$</td>
<td>-0.965**</td>
<td>0.054</td>
<td>0.009</td>
<td>-0.290</td>
</tr>
<tr>
<td></td>
<td>(-2.488)</td>
<td>(0.334)</td>
<td>(0.036)</td>
<td>(-1.162)</td>
</tr>
<tr>
<td>obs</td>
<td>539</td>
<td>539</td>
<td>539</td>
<td>539</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.023</td>
<td>0.022</td>
<td>0.011</td>
<td>0.027</td>
</tr>
<tr>
<td><strong>Panel B. All Innovation Shocks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{\tau}_t$</td>
<td>-815.155</td>
<td>447.314</td>
<td>1.115</td>
<td>0.296</td>
</tr>
<tr>
<td></td>
<td>(-1.213)</td>
<td>(1.254)</td>
<td>(0.003)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$\tilde{\delta}_t$</td>
<td>-94.811*</td>
<td>11.850</td>
<td>-42.587*</td>
<td>54.610</td>
</tr>
<tr>
<td></td>
<td>(-1.728)</td>
<td>(0.516)</td>
<td>(-1.725)</td>
<td>(1.526)</td>
</tr>
<tr>
<td>$\tilde{r}pl_t$</td>
<td>-0.177</td>
<td>1.110***</td>
<td>0.720**</td>
<td>-1.618***</td>
</tr>
<tr>
<td></td>
<td>(-0.274)</td>
<td>(3.161)</td>
<td>(2.285)</td>
<td>(-2.839)</td>
</tr>
<tr>
<td>$\tilde{r}ps_t$</td>
<td>-0.752*</td>
<td>0.029</td>
<td>0.107</td>
<td>-0.416</td>
</tr>
<tr>
<td></td>
<td>(-1.664)</td>
<td>(0.170)</td>
<td>(0.419)</td>
<td>(-1.411)</td>
</tr>
<tr>
<td>obs</td>
<td>539</td>
<td>539</td>
<td>539</td>
<td>539</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.037</td>
<td>0.025</td>
<td>0.016</td>
<td>0.030</td>
</tr>
</tbody>
</table>

contrast, $\tilde{r}ps$’ coefficients seem to be flat, but they are all significantly different from zero. Thus, $rps$ can be considered closely related to stock market returns.

As a robustness check, Table 6 directly regresses the Fama-French three factors and the momentum factor on the contemporary innovation shocks to state variables. Panel A takes into account only the two risk premium factors meanwhile Panel B does all four
state variables. This table confirms the previous finding that \( rps \) is related to aggregate stock market returns meanwhile \( rpl \) is to value, size, and momentum premiums.

6 Macroeconomic Growth

The slope factor of yield curve has attracted not only the attentions of financial economists but also those of monetary policymakers since the slope is known to be the best predictor of recession. For example, Stambaugh (1988) explains that “inverted term structures precede recessions and upward-sloping structures precede recoveries,” and Estrella and Mishkin (1998) find that “the slope of the yield curve emerges as the clear individual choice” as a predictor of US recessions. Ang, Piazzesi, and Wei (2006) also document that “every recession after the mid-1960s was predicted by a negative slope—an inverted yield curve—within 6 quarters of the impending recession. Moreover, there has been only one ‘false positive’ (an instance of an inverted yield curve that was not followed by a recession) during this time period.” The Federal Reserve Bank also acknowledges that “the slope of the yield curve is a reliable predictor of future real economic activity.”

This paper shows that the slope factor is determined by two state variables: the transitory component of short interest rates \( \delta_t \) and the long-term risk premium factor \( rpl_t \). The question is which of them offers the forecastability of future macroeconomic activities.

To answer the question, Table 7 regresses future monthly macroeconomic growth rates on the state variables. Panel A and B measure the macroeconomic activities using the Industrial Productions which are provided by the FRED Economic Data. Their monthly growth rates are estimated as \( GR_{t+i} \equiv (X_{t+i}/X_{t+i-1} - 1) \times 100 \). Panel A is

\[ \text{http://www.ny.frb.org/research/capital_markets/ycfaq.html} \]
\[ \text{http://research.stlouisfed.org/fred2/categories/3} \]
Table 7: Forecast of Industrial Production Growth

The dependent variable is monthly growth rates of macroeconomic activities, which are shown on the first row. The growth rates of industrial productions are computed as \( GR_{t+i} \equiv (X_{t+i}/X_{t+i-1} - 1) \times 100 \). Numbers in parentheses are Newey-West \( t \) statistics with 12 lags. *** \( p \leq 0.01 \), ** \( p \leq 0.05 \), and * \( p \leq 0.10 \) denote significances at 1%, 5%, and 10% level respectively.

<table>
<thead>
<tr>
<th></th>
<th>Panel A. Industrial Production: Total</th>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( GR_{t+1} )</td>
<td>( GR_{t+3} )</td>
<td>( GR_{t+6} )</td>
<td>( GR_{t+12} )</td>
<td>( GR_{t+18} )</td>
<td>( GR_{t+24} )</td>
</tr>
<tr>
<td>( \tau )</td>
<td>0.483</td>
<td>0.726</td>
<td>1.226</td>
<td>1.593</td>
<td>1.825</td>
<td>1.751</td>
</tr>
<tr>
<td>( p )</td>
<td>0.167</td>
<td>0.279</td>
<td>0.509</td>
<td>0.687</td>
<td>0.025</td>
<td>0.572</td>
</tr>
<tr>
<td>( \delta )</td>
<td>-9.043***</td>
<td>-14.120***</td>
<td>-13.957***</td>
<td>-12.190***</td>
<td>-7.953***</td>
<td>-0.982</td>
</tr>
<tr>
<td>( p )</td>
<td>-3.694</td>
<td>-5.231</td>
<td>-5.299</td>
<td>-6.305</td>
<td>-2.668</td>
<td>-0.325</td>
</tr>
<tr>
<td>( rpl )</td>
<td>0.057</td>
<td>0.063*</td>
<td>0.049</td>
<td>0.069</td>
<td>0.107**</td>
<td>0.080*</td>
</tr>
<tr>
<td>( p )</td>
<td>1.485</td>
<td>1.849</td>
<td>1.491</td>
<td>1.551</td>
<td>2.174</td>
<td>1.672</td>
</tr>
<tr>
<td>( rps )</td>
<td>0.017</td>
<td>-0.012</td>
<td>0.023</td>
<td>0.102**</td>
<td>0.036</td>
<td>-0.072*</td>
</tr>
<tr>
<td>( p )</td>
<td>0.324</td>
<td>-0.237</td>
<td>0.519</td>
<td>2.351</td>
<td>0.855</td>
<td>-1.854</td>
</tr>
<tr>
<td>obs</td>
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<td>540</td>
<td>540</td>
<td>538</td>
<td>532</td>
<td>526</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.055</td>
<td>0.125</td>
<td>0.121</td>
<td>0.117</td>
<td>0.063</td>
<td>0.021</td>
</tr>
</tbody>
</table>

Panel B. Industrial Production: Consumer Goods

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( GR_{t+1} )</td>
<td>( GR_{t+3} )</td>
<td>( GR_{t+6} )</td>
<td>( GR_{t+12} )</td>
<td>( GR_{t+18} )</td>
<td>( GR_{t+24} )</td>
</tr>
<tr>
<td>( \tau )</td>
<td>1.877</td>
<td>1.962</td>
<td>2.261</td>
<td>2.566</td>
<td>2.631</td>
<td>2.552</td>
</tr>
<tr>
<td>( p )</td>
<td>1.049</td>
<td>1.144</td>
<td>1.382</td>
<td>1.595</td>
<td>1.566</td>
<td>1.501</td>
</tr>
<tr>
<td>( \delta )</td>
<td>-4.902***</td>
<td>-7.859***</td>
<td>-6.295***</td>
<td>-2.923</td>
<td>-1.117</td>
<td>3.860*</td>
</tr>
<tr>
<td>( p )</td>
<td>-2.609</td>
<td>-3.560</td>
<td>-3.010</td>
<td>-1.593</td>
<td>-0.455</td>
<td>1.649</td>
</tr>
<tr>
<td>( rpl )</td>
<td>0.098***</td>
<td>0.089***</td>
<td>0.093***</td>
<td>0.112***</td>
<td>0.104***</td>
<td>0.065*</td>
</tr>
<tr>
<td>( p )</td>
<td>3.180</td>
<td>2.766</td>
<td>2.817</td>
<td>2.777</td>
<td>2.805</td>
<td>1.841</td>
</tr>
<tr>
<td>( rps )</td>
<td>0.006</td>
<td>-0.031</td>
<td>-0.005</td>
<td>0.041</td>
<td>-0.013</td>
<td>-0.071**</td>
</tr>
<tr>
<td>( p )</td>
<td>0.153</td>
<td>-0.711</td>
<td>-0.123</td>
<td>0.937</td>
<td>-0.339</td>
<td>-2.403</td>
</tr>
<tr>
<td>obs</td>
<td>540</td>
<td>540</td>
<td>540</td>
<td>538</td>
<td>532</td>
<td>526</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.027</td>
<td>0.043</td>
<td>0.033</td>
<td>0.027</td>
<td>0.019</td>
<td>0.021</td>
</tr>
</tbody>
</table>

based on the total industrial production meanwhile Panel B is on the production of consumer goods.

The table shows that \( \delta \) dominates the forecast of total industrial production growth for the horizon of up to 18 months and the consumption goods growth up to 6 months. High \( \delta \) implies high short interest rates and flat slope, and it is likely to be followed by low economic growth in subsequent periods. This result is consistent with the previous
literature. It also supports the monetary policy to reduce the federal funds rate to boost the economic activity.

One interesting observation is that \( rpl_t \)'s forecastability is marginal for total growth but very significant for the growth of consumer goods. It is probably because \( rpl_t \) is related to investor sentiments. Obviously, high risk aversion leads to high consumption growth in subsequent periods due to today's suppressed consumption level. But, as implied by Panel A, \( rpl_t \) appears unrelated to non-consumption production such as investments or government spending. \( rpl_t \) may also be responsible for an uncontrollable movement of yield curve, reminding of Alan Greenspan’s conundrum in 2005\(^\text{16}\).

Lastly, \( rps_t \)'s lack of forecastability is consistent with Duffee (2011)'s finding that a hidden factor has little relevance to macroeconomic variables.

7 Conclusion

This paper finds that bond risk premium consist of two factors: one for a long term and the other for a short term. The long-term factor raises the slope of yield curve. In contrast, the short-term factor is completely hidden from Treasury bond yields but pulls down the yields of Treasury bills. The long-term factor predicts monthly excess returns over the horizon of even longer than one year meanwhile the short-term factor loses its predictability completely in one quarter. The two factors also have different economic implications. The long-term factor predicts future economic growth, and the short-term factor is closely related to bond liquidity premium. Lastly, the long-term factor is found to be related to value, size, and momentum premiums in the stock market whereas the short-term factor is to aggregate stock market returns.

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