Low-depth quantum state preparation

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Exploring how a quantum state could be prepared with the minimal circuit depth is of fundamental interest with critical applications in quantum computing and quantum information processing. On one hand, the noise robustness of a quantum circuit is very sensitive to its depth [1, 2], especially for noisy intermediate-scale quantum (NISQ) devices [3–5]. On the other hand, a poly-logarithmic runtime state preparation method is the prerequisite of the quantum speedup for many algorithms, including HHL algorithm [6] and quantum machine learning [7–9].

Nevertheless, preparing an arbitrary n-qubit state with local unitary gates is generally hard [10], which requires a circuit with a depth at least $O(N/\log N)$ with $N = 2^n$ [11]. Based on uniformly controlled rotations that are decomposed to single qubit rotations and CNOT gates, Ref. [12] showed how to approximately achieve the lower bound with a circuit depth $O(N)$. While when a quantum random access memory (QRAM) [13] were available, the circuit depth could be significantly improved to $O(n)$. However, since a QRAM requires highly non-local interactions and the ability to control $O(N)$ routers simultaneously, it is still challenging for current quantum technologies.

In this work, we demonstrate several quantum algorithms—the sequential one and the parallel ones—to prepare an arbitrary N-level quantum state, with runtime $O(N^2 \log \varepsilon_{\text{th}}^{-1})$ and $O(\log(N)^2 \log \varepsilon_{\text{th}}^{-1})$, and the number of ancillary qubits $O(\log(N)^2)$ and $O(N^2)$, respectively. Here ε_{th} corresponds to the accuracy of the prepared state. A comparison of our algorithms to existing ones is summarized in Table I. We note that, by using ancillary qubits, we showed an exponential speed-up (compared to Refs. [11, 12]) for preparing an arbitrary N-dimensional state. Compared to QRAM, our method only requires gates to be acted on a constant number of qubits, which thus significantly simplifies its practical implementation. We expect our algorithms to have wide applications in both NISQ and universal quantum computing.

TABLE I: Comparison of the low-depth preparation to unitary [11, 12] and QRAM [13] preparation methods. Parameters from left to right are circuit depth, average runtime, number of ancillary qubits, and the number of qubits involved in each gate. Note that the success probability of the proposed algorithms is not unity, so we need to repeat the process several time. This is why the scaling of the runtime is generally larger than the circuit depth.

Schemes	Depth	Runtime	Qubits	Interaction
Parallel 1	$O\left(n^2\right)$	$O\left(n^2 \log \varepsilon_{\text{th}}^{-1}\right)$	$O\left(N^2\right)$	$O(1)$
Parallel 2	$O\left(n^2\right)$	$O\left(N^{1.52} \log \varepsilon_{\text{th}}^{-1}\right)$	$O\left(N\right)$	$O(1)$
Sequential	$O\left(n^2\right)$	$O\left(N^2 \log \varepsilon_{\text{th}}^{-1}\right)$	$O\left(n^2\right)$	$O(1)$
Unitary [11, 12]	$O(N)$	$O(N)$	$O(n)$	$O(1)$
QRAM [13]	$O(n)$	$O(n \log \varepsilon_{\text{th}}^{-1})$	$O(N)$	$O(N)$

[1] S. Wang, E. Fontana, M. Cerezo, K. Sharma, A. Sone, L. Cincio, and P. J. Coles, arXiv preprint arXiv:2007.14384 (2020).

[2] X. Gao and L. Duan, arXiv preprint arXiv:1810.03176 (2018).

[3] J. Preskill, Quantum **2**, 79 (2018).

[4] F. Arute, K. Arya, R. Babbush, D. Bacon, J. C. Bardin, R. Barends, R. Biswas, S. Boixo, F. G. Brandao, D. A. Buell, *et al.*, Nature **574**, 505 (2019).

[5] G. A. Quantum *et al.*, Science **369**, 1084 (2020).

[6] A. W. Harrow, A. Hassidim, and S. Lloyd, Phys. Rev. Lett. **103**, 150502 (2009).

[7] N. Wiebe, D. Braun, and S. Lloyd, Phys. Rev. Lett. **109**, 050505 (2012).

[8] S. Lloyd, M. Mohseni, and P. Rebentrost, Nat. Phy. **10**, 631 (2014).

[9] P. Rebentrost, M. Mohseni, and S. Lloyd, Phys. Rev. Lett. **113**, 130503 (2014).

[10] M. A. Nielsen and I. Chuang, *Quantum computation and quantum information* (Cambridge University Press, Cambridge, 2000).

[11] M. Plesch and Č. Brukner, Phy. Rev. A **83**, 032302 (2011).

[12] M. Möttönen, J. J. Vartiainen, V. Bergholm, and M. M. Salomaa, Quantum Information & Computation **5**, 467 (2005).

[13] V. Giovannetti, S. Lloyd, and L. Maccone, Phys. Rev. Lett. **100**, 160501 (2008).