Consequences of preserving reversibility in quantum superchannels

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Similarly to quantum states, quantum operations can also be transformed by means of quantum superchannels, also known as process matrices. Quantum superchannels with multiple slots are deterministic transformations which take causally independent quantum operations as inputs and are enforced to respect the laws of quantum mechanics but the use of input operations may lack a definite causal order. While causally ordered superchannels admit a characterization in terms of quantum circuits, a similar characterization of general superchannels in terms of standard quantum objects with a clearer physical interpretation has been missing. In this paper we provide a mathematical characterization for pure superchannels with two slots (also known as bipartite pure processes), which are superchannels preserving the reversibility of quantum operations. We show that the reversibility preserving condition restricts all pure superchannels with two slots to be either a causally ordered quantum circuit only consisting of unitary operations or a coherent superposition of two pure causally ordered circuits. The latter may be seen as a generalization of the quantum switch, allowing a physical interpretation for pure two-slot superchannels. An immediate corollary is that purifiable bipartite processes cannot violate device-independent causal inequalities.

Similarly to states, quantum operations and the devices implementing the operations may also be subjected to transformations [1, 2]. Deterministic higher-order operations are named *quantum superchannels* or also as *process matrices* [4, 5].

Recently, it has been noted that the standard quantum circuit formalism has a strict notion of causal order between input operations [3, 4, 6]. Namely, quantum theory admits superchannels which make use of of input operations in an indefinite causal order. A seminal and illustrative example of a superchannel with indefinite causal order is the quantum switch [6].

Motivated by the role of reversibility in physical theories, Ref. [5] introduced the definition of pure superchannels as superchannels preserving the reversibility of input operations. More formally, a superchannel is said to be *pure* if it transforms causally independent unitary channels into a unitary channel.

We analyze the restrictions imposed by the reversibility preserving property of superchannels and present a simple decomposition for this class of processes. We show that pure quantum superchannels with two slots can be divided in two cases: 1) causally ordered superchannels that can be realized by standard quantum circuits only involving unitary operations; 2) coherent superposition of two pure causally ordered superchannels which may be seem as a generalization of the quantum switch.

This submission is based on a preprint [7].

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