

Exact Solution of Error Probability for Quantum Illumination with Attenuation Using Quasi-Bell State

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Abstract. Quantum illumination is an application protocol of entanglement and used for target detection. The use of the Bell state and a two-mode Gaussian state is discussed by Lloyd and Tan *et al.*, respectively. However, the use of a quasi-Bell state has not been discussed. In this paper, we derive the exact solution of error probability for quantum illumination with attenuation when using the quasi-Bell state and clarify that the performance of quasi-Bell state is better than the others when attenuation is weak.

Keywords: Exact solution, Error probability, Quantum illumination, Quasi-Bell state

Quantum illumination proposed by Lloyd is a protocol that detects the existence of a target by using entangled light of two-mode[1]. The light corresponding to one mode is emitted towards the target and the reflected light is input to the receiver. The other one is directly input to the receiver. Finally, the receiver detects the existence of the target by performing a joint measurement on both lights. In general, the protocol offers a quantum advantage in strong attenuation and background noise.

Quantum illumination was first proposed using the Bell state[1], and then the use of a two-mode Gaussian state was proposed by Tan *et al.*[2]. On the other hand, there are many impact studies using the quasi-Bell state[3] such as quantum reading with error-free[4]. However, as far as we know, there are no studies that analyze quantum illumination using the quasi-Bell state. Therefore, this study aims to evaluate the performance when using the quasi-Bell state, and we continue the evaluation of quantum reading[5] and focus on the evaluation of quantum illumination in this paper.

The study by Tan *et al.* developed Lloyd's concept into a general model with the Heisenberg picture[2]. Practically, it is hard to calculate the error probability of an optimum quantum receiver by using the picture. Therefore, Tan *et al.* calculated the Chernoff bound instead of calculating the error probability. However, an instantaneous evaluation, such as the error probability of when one light pulse hits a moving target, is also required.

We first treat the attenuation channel in this paper. By using the Schrödinger picture, we can evaluate the error performance of quantum illumination. Practically, the quasi-Bell state we use is con-

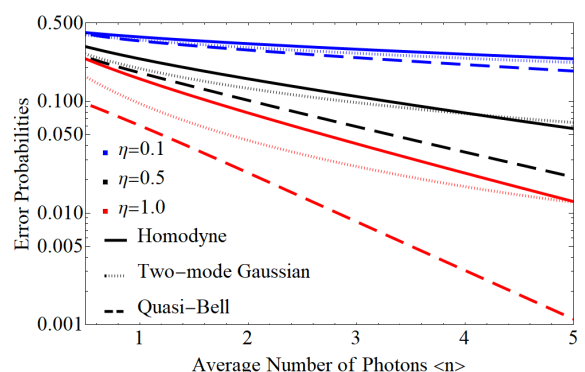


Figure 1: Error probabilities with respect to the average number of photons $\langle n \rangle$ where the transmissivity η equals to 0.1, 0.5, and 1

structed from coherent states, hence its density matrix is infinite dimensional. In general, it is hard to calculate, but we address this issue by using the optimum measurement states corresponding to the collection of coherent state signals as the orthonormal basis and obtain the two received quantum states $\{\Psi_0, \Psi_1\}$ represented by 6×6 density matrix. In order to calculate the error probability, the eigenvalues of $M(= \Psi_0 - \Psi_1)$ are required. Although there is no general solution, we address this issue by using the partial symmetry to divide M into two 3×3 matrices and obtain the exact solution for eigenvalues of M . Finally, we obtain the exact solution for error probability.

Figure 1 plots the error probabilities of using the quasi-Bell state, the two-mode Gaussian state, and the coherent state with homodyne measurement (i.e., a classical laser radar). From Fig. 1, we know that the quasi-Bell state offers a clear advantage over the others with increasing $\langle n \rangle$, when attenuation is weak, e.g., $\eta \geq 0.5$.

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