

Quantum error correction with adaptive syndrome measurements

Shoya Takagi^{1 *}

Kosuke Mitarai^{1 2 3 †}

Keisuke Fujii^{1 2 4 ‡}

¹ Graduate School of Engineering Science, Osaka University

² Center for Quantum Information and Quantum Biology, Institute for Open and Transdisciplinary Research Initiatives, Osaka University

³ JST PRESTO, Japan

⁴ Center for Emergent Matter Science, RIKEN, Wako Saitama 351-0198, Japan

Abstract. While the syndrome measurement is essential for quantum error correction, it also causes an error in a fault-tolerant setting, where any operations, including those for the syndrome measurements, have imperfection. Here we propose a method to implement the syndrome measurements adaptively conditioned on the previous syndrome measurement outcomes. We construct a decoding algorithm using a decoder graph depending on a defective syndrome resulting from the adaptive measurements. As a demonstration, we numerically show that the adaptive syndrome measurements successfully reduce the logical error probability in a one-dimensional repetition code.

Keywords: quantum error correction, adaptive syndrome measurement

A challenge for constructing viable applications on quantum devices is noise accumulated during quantum computation. In order to handle this issue, quantum error correction is applied. Recently, quantum error correction using the stabilizer codes [1] has been studied, especially aiming its practical implementations. For example, the codes suitable for biased noise [2, 3], correlated errors [4], and reducing connectivity of qubits [5] have been developed. Since quantum computers of a few tens of qubits are now available, one of the near-term milestones in quantum computing is an experimental demonstration of suppression of the logical error probability sufficiently to achieve a lifetime of quantum information much longer than a coherence time of physical qubits. However, the noise level in experiments is still high around the noise threshold, we should improve the error correction protocol as well as experimental effort to reduce the noise.

In this work, we propose a quantum error correction with adaptive syndrome measurements, where the syndrome measurements are done adaptively depending on the previous syndrome measurement outcomes. If the outcome of the previous syndrome measurements suggests any existence of an error, certain stabilizer operators relevant to the error are measured in the following cycle. Otherwise, the syndrome measurements are skipped to avoid an unnecessary error introduction. If the operational error introduced by the syndrome measurements is dominant against the data storage error, the adaptive syndrome measurements are expected to improve

the logical error probability, since the error events are reduced while detecting possible errors.

To this end, we construct a decoding algorithm for a defective error syndrome, where the error syndrome is not necessarily obtained for all stabilizer operators at every cycle. Note that there have been earlier works relevant to quantum error correction under incomplete syndrome measurements. These mainly address the loss of qubits [6–8], so some stabilizers are not measured permanently. On the other hand, in this work, the pattern of the defects of the error syndrome changes in time.

In order to verify that the proposed scheme improves the logical error probability with a realistic situation, we perform a numerical simulation on the one-dimensional repetition code. For example, if we take the code distance $d = 5$, the number of syndrome measurement cycles $r = 25$, and two-qubit error rate $p_2 = 10^{-3}$ and single-qubit error rate $p_1 = 10^{-4}$, then the logical error probability with the adaptive syndrome measurements is $(0.8 \pm 0.2) \times 10^{-6}$, which achieves an improvement of 50% against the case of the conventional syndrome measurements.

The proposed scheme is straightforwardly applicable to the surface codes [9, 10]. Furthermore, the proposed decoding algorithm with defective syndrome measurements can also be applied for the case of non-deterministic syndrome measurements possibly caused by a failure of a special ancilla preparation [11, 12]. Therefore the proposed scheme can also be used to improve the noise threshold for the stabilizer codes with a higher weight such as color codes.

*shoya.takagi@qc.ee.es.osaka-u.ac.jp

†mitarai@qc.ee.es.osaka-u.ac.jp

‡fujii@qc.ee.es.osaka-u.ac.jp

References

- [1] D. GOTTESMAN, “Stabilizer codes and quantum error correction”, PhD thesis, California Institute of Technology (1997).
- [2] D. K. Tuckett, S. D. Bartlett, and S. T. Flammia, “Ultrahigh error threshold for surface codes with biased noise”, *Physical review letters* **120**, 050505 (2018).
- [3] J. P. Bonilla-Ataides, D. K. Tuckett, S. D. Bartlett, S. T. Flammia, and B. J. Brown, “The xzzx surface code”, *arXiv preprint arXiv:2009.07851* (2020).
- [4] N. H. Nickerson and B. J. Brown, “Analysing correlated noise on the surface code using adaptive decoding algorithms”, *Quantum* **3**, 131 (2019).
- [5] C. Chamberland, G. Zhu, T. J. Yoder, J. B. Hertzberg, and A. W. Cross, “Topological and subsystem codes on low-degree graphs with flag qubits”, *Physical Review X* **10**, 011022 (2020).
- [6] T. M. Stace, S. D. Barrett, and A. C. Doherty, “Thresholds for topological codes in the presence of loss”, *Physical review letters* **102**, 200501 (2009).
- [7] T. M. Stace and S. D. Barrett, “Error correction and degeneracy in surface codes suffering loss”, *Physical Review A* **81**, 022317 (2010).
- [8] S. Nagayama, A. G. Fowler, D. Horsman, S. J. Devitt, and R. Van Meter, “Surface code error correction on a defective lattice”, *New Journal of Physics* **19**, 023050 (2017).
- [9] A. Kitaev, “Anyons in an exactly solved model and beyond”, *Annals of Physics* **321**, 2–111 (2006).
- [10] A. G. Fowler, M. Mariantoni, J. M. Martinis, and A. N. Cleland, “Surface codes: towards practical large-scale quantum computation”, *Physical Review A* **86**, 032324 (2012).
- [11] D. P. DiVincenzo and P. W. Shor, “Fault-tolerant error correction with efficient quantum codes”, *Physical review letters* **77**, 3260 (1996).
- [12] D. P. DiVincenzo and P. Aliferis, “Effective fault-tolerant quantum computation with slow measurements”, *Physical review letters* **98**, 020501 (2007).