

Implementation of Multi-body Interaction for Quantum Annealing

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Quantum annealing [1, 2] is an algorithm for the combinatorial optimization problems, which aims at finding the minimum of a function called objective function. This algorithm has some difficulties. One of such difficulties is that it is hard to implement higher than third-order terms. The objective function includes arbitrary higher order terms in general. In quantum annealing machine such as the device produced by D-Wave Systems Inc., however, the objective function is limited up to quadratic terms because of their architecture [3]. Some solutions have been proposed so far [4, 5], but these solutions need many ancilla qubits.

In this paper, we explore the possibility of implementing efficiently higher-order terms in quantum annealing. Though they have not yet been implemented in quantum annealing machine [6], “non-stoquastic” terms are highly demanded by researchers in quantum annealing because these terms are known to make the device more powerful [7]. We prove that a combination of these terms and the “reverse annealing” technique [8] can realize the multi-body interaction in the spin system, which corresponds to the higher-order terms in the objective function. Moreover, we compare the efficiency of the proposed method with the conventional one.

Here, we illustrate the simplest case where the graph of the quantum annealer is all-to-all, and the objective function includes an N -body interaction:

$$H_P = \sum_{i=1}^N h_i \sigma_i^z + \sum_{i=1}^N \sum_{j=i+1}^N J_{ij} \sigma_i^z \sigma_j^z + K \bigotimes_{i=1}^N \sigma_i^z. \quad (1)$$

To find the ground state of H_P , we consider the time-dependent Hamiltonian:

$$H_S(t) = -A(t) \sum_{i=0}^N \sigma_i^z + B(t) \sigma_0^x \sum_{i=1}^N \sigma_i^x + C(t) \left(\sum_{i=1}^N h_i \sigma_i^z + \sum_{i=1}^N \sum_{j=i+1}^N J_{ij} \sigma_i^z \sigma_j^z + K_0 \sigma_0^z \right), \quad (2)$$

where we assume

$$0 \leq B(0) \sim C(0) \ll A(0), \quad (3)$$

$$0 \leq A(T/2) \sim C(T/2) \ll B(T/2), \quad (4)$$

$$0 \leq A(T) \sim B(T) \ll C(T), \quad (5)$$

are satisfied. This Hamiltonian (2) has a symmetry and we understand that the quantity $\bigotimes_{i=0}^N \sigma_i^z$ is a constant of motion [9]. Moreover, there exists a unitary operator \mathbb{W} transforming $\bigotimes_{i=0}^N \sigma_i^z$ into σ_0^z because the spectrum of $\bigotimes_{i=0}^N \sigma_i^z$ is the same as that of σ_0^z . The unitary operator is expressed explicitly as

$$\mathbb{W} = \prod_{i=1}^N C_{0,i} \otimes I_i, \quad C_{0,i} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad I_i = \bigotimes_{j(\neq i)} I_j, \quad (6)$$

in the standard ordered basis

$$\vec{B} = \{|00\rangle_{0,i}, |01\rangle_{0,i}, |10\rangle_{0,i}, |11\rangle_{0,i}\}. \quad (7)$$

Transforming $H_S(t)$ into $\tilde{H}_S(t) = \mathbb{W}^\dagger H_S(t) \mathbb{W}$, we get

$$\begin{aligned} \tilde{H}_S(t) &= \sum_{\lambda=0}^1 |\lambda\rangle_0 \langle \lambda| \otimes \tilde{H}_{S,\lambda}(t) \\ \tilde{H}_{S,\lambda}(t) &= -A(t) \left(\sum_{i=1}^N \sigma_i^z + (-1)^\lambda \bigotimes_{i=1}^N \sigma_i^z \right) + B(t) \sum_{i=1}^N \sigma_i^x \\ &\quad + C(t) \left(\sum_{i=1}^N h_i \sigma_i^z + \sum_{i=1}^N \sum_{j=i+1}^N J_{ij} \sigma_i^z \sigma_j^z + (-1)^\lambda K \bigotimes_{i=1}^N \sigma_i^z \right). \end{aligned} \quad (8)$$

As we impose $A(0) > 0$, the ground state of $H_S(0)$ is $\bigotimes_{i=0}^N |0\rangle_i$. Therefore, the ground state of $\tilde{H}_S(0)$ is

$$\mathbb{W} \bigotimes_{i=0}^N |0\rangle_i = \bigotimes_{i=0}^N |0\rangle_i, \quad (10)$$

and the state evolves in the subspace corresponding to $\lambda = 0$. Moreover, the initial state is also the ground state of $\tilde{H}_{S,0}(0)$ and we can get the ground state of (1) at $t = T$ when the system has evolved adiabatically.

Conventionally, the number of ancilla qubits to implement (1) is $O(N)$. Therefore, the proposed method is superior to the conventional one in this case, but the efficiency depends on the number of the higher-order terms in the objective function. We will also report such a dependency.

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