Computational power of dual-unitary quantum circuits

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Abstract. Quantum circuits that are classically simulatable tell us when quantum computation becomes less powerful than or equivalent to classical computation. In this work, we propose a novel family of classically simulatable circuits by making use of the so-called dual-unitary gates and characterize its computational power.

Keywords: classically simulatable quantum circuits, dual-unitary quantum circuits

Quantum computation is widely believed to be intractable by classical computers. However, there also exists certain types of quantum circuits that can be efficiently simulated classically. Famous examples are quantum circuits which consist of Clifford gates [1] or matchgates [2]. Such classically simulatable circuits are of importance because they illustrate what makes universal quantum computation different from classical computers.

Recently, a new class of quantum gates called dual-unitary gates have been introduced [3]. They are unitary gates which preserve unitarity under a reshuffling of their indices. In Ref. [4], it has been shown that expectation values of local observables can be calculated analytically in the infinite system size limit for the so-called dual-unitary circuits, which consist of only nearest-neighbour dual-unitary gates with certain initial states. Despite the above property, dual-unitary gates contain arbitrary single-qubit gates and a certain class of two-qubit entangling gates. Note that these gates form a universal gate set if we can apply them freely [5]. Nevertheless, the carefully constructed initial states and the infinite size limit allow us to compute the expectation values efficiently.

In this work, we consider the case where the latter condition, that is the infinite sizedness, is removed, and discuss the computational aspects of such systems. Specifically, we consider the computational complexity of the problem of calculating expectation values of local observables and the sampling problem of one- and two-dimensinal dual-unitary circuits. Note that the generalization to the two-dimensional lattice is of interest not only from the viewpoint of quantum computing but also from the viewpoint that exactly solvable quantum dynamics in two spatial dimensions is limited.

As the first problem, we show that local observables of dual-unitary quantum circuits can be effciently calculated exactly in one- and two-dimensions until time $t \sim \frac{1}{2}N$ and $t \sim N$, respectively, where N is a system length. It illustrates that quantum circuits consisting of gates from a universal gate set cannot necessarily outperform classical computation. On the other hand, we find that the same task becomes BQP-complete for a sufficiently long time $t \geq \mathrm{poly}(N)$ by showing that dual-unitary quantum circuits of a sufficiently high depth is universal. This contrasts to conventional classically simulatable quantum circuits with a fixed gate set, such as Clifford or matchgate circuits, where computational capability does not change depending on the depth of circuits. As the second problem, we show that sampling of the output of one- and two-dimensional dual-unitary quantum circuits is intractable for classical computers after time $t \sim \frac{1}{2}(N - \sqrt{N})$ and $t \geq 4$, respectively, unless the polynomial hierarchy (PH) collapses to its third level. This result implies that, especially in the case of two dimensions, the sampling problem of constant-depth dual-unitary quantum circuits is as hard as that of constant-depth quantum circuits. We reveal that the computational power of dual-unitary quantum circuits make a transition between $\mathrm{O}(N)$ time and $\mathrm{poly}(N)$ time, and it provides new insights into a transition of computational complexity.

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References

[1] D. GOTTESMAN, "Stabilizer codes and quantum error correction", PhD thesis, California Institute of Technology (1997).

[2] L. G. Valiant, "Quantum circuits that can be simulated classically in polynomial time", SIAM Journal on Computing **31**, 1229–1254 (2002).

[3] B. Bertini, P. Kos, and T. Prosen, "Exact correlation functions for dual-unitary lattice models in 1+ 1 dimensions", Physical review letters **123**, 210601 (2019).

[4] L. Piroli, B. Bertini, J. I. Cirac, and T. Prosen, "Exact dynamics in dual-unitary quantum circuits", Physical Review B **101**, 094304 (2020).

[5] M. J. Bremner, C. M. Dawson, J. L. Dodd, A. Gilchrist, A. W. Harrow, D. Mortimer, M. A. Nielsen, and T. J. Osborne, "Practical scheme for quantum computation with any two-qubit entangling gate", Physical review letters **89**, 247902 (2002).