Certifying dimension of quantum systems by sequential projective measurements

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Abstract. This work analyses correlations arising from quantum systems subjected to sequential projective measurements to certify that the system in question necessarily has a quantum dimension greater than some dimension. We refine previous known methods and show that dimension greater than two can be certified in scenarios which are considerably simpler than the ones presented before. For the first time in this sequential projective scenario, we certify quantum systems with dimension strictly greater than three. With our method, we conclude that performing random projective measurements on random pure qutrit states allows a robust certification of quantum dimensions with very high probability.

Keywords: Dimension certification, dimension witness, SDP, NPA hierarchy.

With the recent development of quantum technologies and the different promising applications, it is primordial to guarantee the good functioning of the used apparatus through certification or benchmarking methods [1, 2]. Such methods can rely on fundamental properties of quantum physics to assert properties of quantum systems such as dimension. It has been proved that the usage of qudits instead of qubits is beneficial in a large range of applications in quantum information [3, 4].

In order to certify dimension of single quantum systems, one can use outcome statistics (that we call behaviours) from a realized experiment in a specific scenario relying on sequential measurements [5, 6, 7, 8]. However, it is difficult to extend these works to general cases due to their complexity. Another direction is to use the so-called Navascues-Pironio-Acin (NPA) hierarchy [9] which is a numerical method that gives an arbitrary close approximation to the measurement statistics of quantum systems. Such method has been used to characterize temporal correlations [10] and dimension [11]. However, this method is computationally expensive and does not provide insights about what scenario to use to certify what dimension.

In this work we are interested in addressing both issues. We show cases where it is not necessary to go at high level in the NPA hierarchy as the first level is already sufficient to certify dimensions. This give a substantial reduction of the computational cost. We found that even if the Leggett-Garg inequality [12] is already maximally violated by a qubit's behaviour, our results imply that this scenario is sufficient to certify dimension greater than three. So far, the only known way to certify qubit from the above dimensions was through the Peres-Mermin square [5]. Our results provide a drastic simplification of the previously known results by six measurements and shorten the length of the sequence of measurements by one which is much more favorable to experimental perspectives.

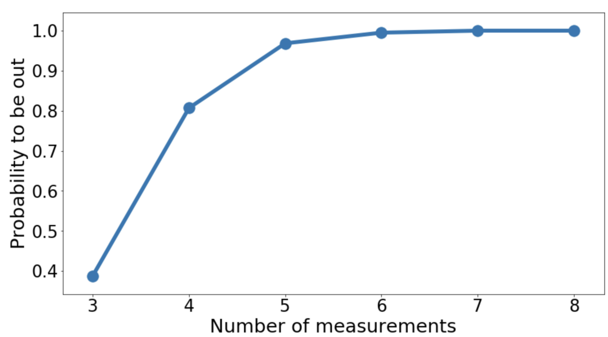

Figure 1: Probability of a random qutrit behavior not to be reproduce by a behavior in the first level of the hierachy of qubit behaviors in the m-2-2 scenario.

We also identify a way to classify scenarios which permit to identify good scenario for dimension certification and identify the different advantages to use more experimentally challenging scenarios in order to have a more accurate and robust dimension certifying. We provide an analysis and a characterization of scenarios in which dimension certification of dimension greater than two is possible. In particular, we use two metrics: the probability for a random quantum states with random measurements that does not admit any qubit quantum realization and the distribution of the visibility derived from the generalized robustness. The probability to find a behavior with no qubit quantum realization increases when the experiment is more complex (see Fig 1). When we increase the number of measurements or the length of the sequence we see that the distribution of the visibility shifts to lower values, this can be interpreted as a more robust certification.

In addition, our methods led us to certify, for the first time in this projective measurements scenario, quantum systems with dimensions strictly greater than three.

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