Implementation of arbitrary quantum measurements using classical resources and only single ancilla

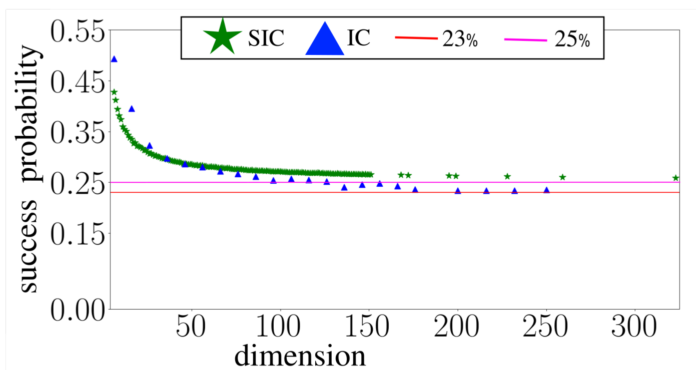
Tanmay Singal,[1,*] Filip B. Maciejewski,[1,†] and Michał Oszmaniec[1,‡]
[1] *Center for Theoretical Physics, Polish Academy of Sciences, Al. Lotników 32/46, 02-668 Warsaw, Poland*

The implementation of quantum operations becomes more noisy as the underlying system size increases. This has been observed, particularly, on near-term quantum devices. The implementation of general quantum measurements on a d dimensional system, using Naimark's dilation theorem, requires at least $\log_2 d$ ancillary qubits [1]. With this in mind, **we propose a scheme to implement a general quantum measurement using only a single ancillary qubit and classical resources**. To achieve this, we allow ourselves to exploit the following operations on measurements. (i) Randomisation: the probabilistic mixture of two or more measurements. (ii) Post-Processing: coarse-graining of measurement outcomes. (iii) Post-selection: choosing samples from certain measurement outcomes, and discarding remaining outcomes. We show that for any target measurement, we can find a set of other measurements with the following properties: (1) these other measurements can be implemented using only a single ancillary qubit, and (2) by using randomisation, post-processing and post-selection of these other measurements, we can simulate the target measurement. Since the probabilities appearing in randomisation and post-processing are inherently classical, they are classical resources, and are hence considered as free. But post-selection has the following cost: the target measurement can be implemented with a probability of success, which is less than one. Physically, the success probability, $\mathrm{p}_{\mathrm{succ}}$, signifies that the average number of trials needed obtain a sample of the target measurement is $1/\mathrm{p}_{\mathrm{succ}}$.

In an earlier work [2], a similar scheme to implement any measurement with no ancillary qubits, was proposed. The worst-case success probability for this scheme scales as $1/d$, making it infeasible for large dimension. **Our scheme generalizes the previous one, and the success probability scales significantly better. We expect that it is bounded below by a constant, which is independent of dimension. We obtain strong evidence for this**, which is as follows:

(i) Analytic results: for typical random measurements with d^2 outcomes, **we prove using concentration of measure, that for large but finite dimension, with overwhelming probability, the success probability is bounded below by a constant of 2.7%**. Separately, in the asymptotic limit of dimension, we have analytical backing of 25% using free probability theory.

(ii) Numerical evidence: we compute the success probability for symmetric informationally complete measurements (SIC-measurements) [3] for dimensions upto 323, and for asymmetric informationally complete measurements [4] (IC-measurements) for dimensions upto 250. As dimension increases, **the success probability of SIC-measurements and IC-measurements go to 25% and 23%** respectively, as shown in the figure below.



Conjecture: Based on this evidence, **we conjecture that the success probability is bounded from below by a constant for all measurements, independent of dimension**.

Finally, for the gate noise model used in the recent demonstration of quantum computational advantage [5], we prove that for typical random measurements, **noise compounding in circuits required by our scheme is substantially lower than in the scheme that directly uses Naimark's dilation theorem**.

* tanmaysingal@gmail.com
† filip.b.maciejewski@gmail.com
‡ michal.oszmaniec@gmail.com

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