An Efficient Generation of Arbitrary Superposition of Basis States

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Abstract. Quantum Amplitude Amplification is one of the useful quantum algorithms for finding solutions. The number of iterations for Quantum Amplitude Amplification can be reduced if a quantum state which meets solutions' requirements is prepared. In this paper, we propose a method for creating efficiently the quantum superposition of arbitrary computational basis states by H, X, cH, and cX gates.

Keywords: Quantum Amplitude Amplification

1 Introduction

In recent years, quantum computation, which is a computation model based on the quantum mechanics, has been attracting much attentions. Some classical problems are solved efficiently by quantum computation so that various quantum algorithms have been proposed so far. One of such algorithms is Quantum Amplitude Amplification [1], which is a method for finding solutions.

Quantum Amplitude Amplification needs to create the quantum superposition in the first step of the algorithm. In the remaining step, iteration is running to selectively amplify the amplitude of solutions' states in the prepared state. The number of those iterations is reduced if we prepare a quantum state which meets solutions' requirements. This paper proposes a method for creating a quantum superposition of arbitrary computational basis states efficiently by H, X, cH, and cX gates.

2 The proposed method

We propose a method to create quantum superposition of arbitrary computational basis states by H and X gate that are single-qubit gates, and cH and cX gate that are two-qubit gates. Our method is as follows.

- Input: $|0\rangle^{\otimes n}$
- Output: the quantum superposition of arbitrary n-qubit computational basis states. (targeted state)
- Quantum gate to use: H, X, cH and cX gates

Here is an example of creating a quantum superposition like this:

$$a\,|1010\rangle + b\,|1011\rangle + c\,|0100\rangle + d\,|0111\rangle + e\,|1000\rangle,$$

where a, b, c, d, $e \in \mathbb{R}$. The following is the proposed procedure to create the quantum superposition of arbitrary basis states if the solution's requirements are given.

Step 1 Applying H and/or cH gates

We apply H and/or cH gates until the number of the current computational basis states becomes the same as that of the target state.

The quantum superposition of the example has five basis states, so we apply one H gate and two cH gates and create the following state:

$$a\,|0000\rangle + b\,|0001\rangle + c\,|0101\rangle + d\,|0011\rangle + e\,|0111\rangle.$$

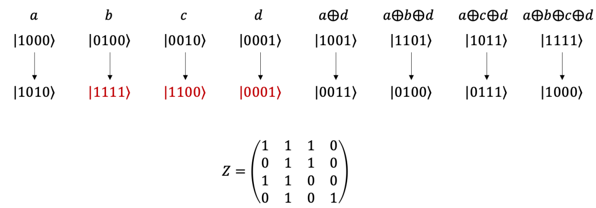

Figure 1: Generating matrix Z from I/O relationship

Step 2 Applying Gaussian elimination

(a) We apply X gates to transform the current state into a state which does not include $|0\rangle^n$. As an example, we apply one X gate to the 4th qubit of the state generated at Step 1.

(b) By Gaussian elimination, we can create a quantum circuit consisting of only cX gates [2]. We generate a matrix Z from the current I/O relationship (Figure 1). Then, We apply Gaussian elimination to matrix Z.

(c) If the quantum circuit cannot be created by Step 2 (b), we return to Step 2 (a) and try another combination of X gates.

(d) If Step 2 dose not find a solution, we return to Step 1 and try another combination of H and cH gates.

3 Conclusion

In this paper, we proposed the method for creating a quantum state to superpose arbitrary computational basis states by H, X, cH, and cX gates. Our future work is finding a method for creating a combination of I/O to which Gaussian elimination can be applied efficiently.

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