Loss-tolerant concatenated Bell-state measurement with coherent-state qubits

Seok-Hyung Lee,[1] Seung-Woo Lee,[2] and Hyunseok Jeong[1]

[1] *Department of Physics and Astronomy, Seoul National University, Seoul 08826, Republic of Korea*

[2] *Center for Quantum Information, Korean Institute of Science and Technology, Seoul 02792, Republic of Korea*

Optical system is a promising candidate for quantum information processing due to its long coherence time and ease of long-distance transmission [1]. As well as encoding with vacuum / single-photon qubit and polarization qubit, encoding quantum information in a continuous-variable system such as coherent-state qubit has been also widely studied [2]. Coherent-state qubit, which encodes information in $\{|\alpha\rangle, |-\alpha\rangle\}$ basis, has an advantage that deterministic Bell-state measurement (BSM) is possible in lossless system [3]. However, it suffers two main obstacles: a BSM can fail due to the non-orthogonality of the basis, and the quantum state is dephased by photon loss. One may use coherent states with large α to reduce the effect of the non-orthogonality, but then the state gets more vulnerable to dephasing by photon loss.

To resolve the obstacles, we introduce parity encoding scheme modified for coherent-state qubit and suggest concatenated Bell-state measurement (CBSM) scheme, inspired by previous works on polarization qubit [4][5]. The basis set of the logical qubit in the modified parity encoding scheme, $\{|0_L\rangle, |1_L\rangle\}$, is defined by $|0_L(1_L)\rangle \propto \left[(|\alpha\rangle + |-\alpha\rangle)^{\otimes m} + (|\alpha\rangle - |-\alpha\rangle)^{\otimes m}\right]^{\otimes n}$ where n and m are odd integers. The parity encoding has an hierarchy structure of three different levels, which make it possible to perform BSM in concatenated manner.

We first suggest an optimized CBSM scheme minimizing the cost defined by the number of photon-number parity detectors used for a single CBSM. Then we demonstrate that the CBSM scheme is tolerant to photon loss by mitigating both failure and dephasing. We also show the expected fault-tolerance numerically with Monte-Carlo simulations, e.g., if the photon loss rate is 5%, the success rate is 79.6% without concatenation, but it reaches 98.9% when the average cost of CBSM is larger than 31. Also, we show that $|\alpha|$ does not need to be extremely high for CBSM, e.g., $|\alpha| \gtrapprox 0.8$ is needed for the photon loss rate of 1% to reach a success rate larger than 98%.

Next, we investigate quantum repeater as an application of CBSM, where CBSM is performed in each repeater station to correct errors. We evaluate the asymptotic key generation rate Rt_0 and the effective total cost Q_{tot}, the total cost of CBSM divided by Rt_0. We first show that arbitrary Rt_0 close to unity is obtainable with enough concatenation. Also, we find the optimal parameter settings minimizing Q_{tot} for different total distances L, e.g., for $L = 1000$ km, it reaches $Q_{tot} = 1.02 \times 10^5$ and $Rt_0 = 0.71$.

In conclusion, our work dramatically improves the BSM scheme with coherent-state qubit by simultaneously mitigating both failure and dephasing. Not only that, we show that the quantum repeater exploiting the CBSM scheme reaches arbitrary high key generation rate close to unity with enough concatenation. Finally, we demonstrate that the suggested schemes can be implemented with Schrödinger's cat state and elementary logical gates in the coherent-state basis.

[1] Pieter Kok and Brendon W Lovett, *Introduction to optical quantum information processing* (Cambridge university press, 2010).

[2] Samuel L. Braunstein and Peter van Loock, "Quantum information with continuous variables," Rev. Mod. Phys. **77**, 513–577 (2005).

[3] H. Jeong, M. S. Kim, and Jinhyoung Lee, "Quantum-information processing for a coherent superposition state via a mixedentangled coherent channel," Phys. Rev. A **64**, 052308 (2001).

[4] Sreraman Muralidharan, Jungsang Kim, Norbert Lütkenhaus, Mikhail D. Lukin, and Liang Jiang, "Ultrafast and fault-tolerant quantum communication across long distances," Phys. Rev. Lett. **112**, 250501 (2014).

[5] Seung-Woo Lee, Timothy C. Ralph, and Hyunseok Jeong, "Fundamental building block for all-optical scalable quantum networks," Phys. Rev. A **100**, 052303 (2019).