Constructing linear distance quantum codes in the ground space of local translation-invariant spin chains

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We construct explicit quantum error correcting codes with linear distance that reside in the ground space of a new semi-exactly solvable local quantum spin chain. Our quantum codes go beyond the stabilizer formalism, which we systematically construct as follows: We identify a classical code with the desired properties, which our quantum code construction inherits. We then cast the quantum error correction criteria into an under-constrained set of linear equations, and show that the kernel of the linear system can be used to construct quantum codes that encode a single logical qubit. This is illustrated by an explicit 8-qudit code example. The parent Hamiltonian whose ground space contains the code is a translation invariant quantum integer-spin chain. We prove that it is frustration free, and analytically characterize its ground space completely. We then show that the quantum code is supported on a subspace of the ground space whose dimension is a constant fraction of the full dimension of the ground space. The new quantum spin chain may be of independent interest.

Since topological models of quantum computation, it has been recognized that quantum codes may naturally appear in the ground space of physical systems [3]. Most physical are 2-local interactions, and many have investigated the encoding of quantum codes in their ground spaces. The most celebrated example is Kitaev's toric code that resides in the ground space of a 4-local Hamiltonian, which is an effective Hamiltonian of a perturbed 2-local Hamiltonian [4]. The Toric code does not lie within the full ground space of the original 2-local Hamiltonian. Nevertheless Kitaev's Toric code is an example of topological order and paved the way for the topological model of quantum computation. The compass model [2] is 2-local on a lattice, and has recently been proposed as a candidate to encode quantum codes in the eigenbasis of the Hamiltonian [5]. However, the performance of these quantum codes are not well understood and have mainly been numerically investigated. The advantage of our work over the aforementioned works is that we construct a 2-local Hamiltonian, whose quantum error correcting properties we analytically prove.

In a nice recent result, Brandao *et al.* gave a non-constructive proof of the existence, with high probability, of *approximate* quantum error correcting codes (AQECCs) within the low-energy sector of translation invariant quantum spin chains [1]. What is remarkable about their result is that they proved the existence of AQECCs for a multitude of translation invariant models [1]. The challenges that remained were that the codes were not explicit, the quantum error correction criteria was only approximately satisfied (hence AQECCs), errors had to be on consecutive set of spins, and the codes were in a low-energy sector of the local Hamiltonian (i.e., not the ground space). Moreover, the distance of the code grows logarithmically with the number of spins.

Our work overcomes these challenges. We construct explicit codes with linear distance that encode one logical qubit. We write down a new and explicit 2-local quantum integer spin-s chain parent Hamiltonian, analytically prove its ground space, and show that the aforementioned codes reside in a subspace of its ground space. Lastly, the quantum error correction condition is satisfied exactly (not approximately).

References

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