Kraus Representation of Probabilistic Attenuated Channel

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Abstract. Toward the derivation of the Kraus representation of the probabilistic attenuated channel for infinite-dimensional systems, we consider the derivation from the Choi matrix. The obtained results in a qubit system are shown to be different from and more complicated than the results derived from the Stokes parameters.

Keywords: Quantum communication, Kraus representation, Choi matrix

1 Introduction

In the previous studies of the quantum fading channel, Personick's fading channel model[1] and Kita's probabilistic attenuated channel model[2] have been proposed. The purpose of this study is to clarify the relationship between the above channel models. Therefore, we consider that by deriving the Kraus representation of the probabilistic attenuated channel. In a qubit system, we derived the concise form of Kraus representation from the Stokes parameters[3]. However, the Stokes parameters cannot describe infinite-dimensional systems. Hence, we consider the derivation of Kraus representation from the Choi matrix in a qubit system.

2 Probabilistic attenuated channel

Let $\mathcal{F}$ be the probabilistic attenuated channel[2], and $\rho^{(\mathrm{in})}$ be the density operator of an input quantum state of the channel, then an output quantum state of the channel $\rho^{(\mathrm{out})}$ is

$$\rho^{(\mathrm{out})}=\mathcal{F}\left(\rho^{(\mathrm{in})}\right)=\int_0^1 P(\eta)\left(\sum_{k=0}^{\infty}E_k\rho^{(\mathrm{in})}E_k^{\dagger}\right)d\eta, \quad (1)$$

where $P(\eta)$ is the probability distribution of the transmissivity η and E_k ($k\in\mathbb{Z}_{\geq 0}$) is the Kraus operators of the attenuated channel[4]:

$$E_k=\sum_{n=0}^{\infty}\sqrt{\binom{n}{k}}\sqrt{\eta^{n-k}(1-\eta)^k}|n-k\rangle\langle n|. \quad (2)$$

3 Kraus representation of probabilistic attenuated channel

Here, we derive the Kraus representation from the Choi matrix in the qubit system. The Choi matrix of the probabilistic attenuated channel is

$$J(\mathcal{F}) = \begin{bmatrix}1 & 0 & 0 & \alpha\\ 0 & 0 & 0 & 0\\ 0 & 0 & 1-\beta & 0\\ \alpha & 0 & 0 & \beta\end{bmatrix}. \quad (3)$$

The Kraus operators are derived by the eigenvalue decomposition of the Choi matrix. Then, we obtain

$$F_0 = \begin{bmatrix}0 & \sqrt{1-\beta}\\ 0 & 0\end{bmatrix}, \quad (4)$$

$$F_1 = \frac{\alpha\sqrt{2(1+\beta-\gamma)}}{\sqrt{4\alpha^2+(1-\beta-\gamma)^2}}\begin{bmatrix}\frac{1}{2\alpha}(1-\beta-\gamma) & 0\\ 0 & 1\end{bmatrix}, \quad (5)$$

$$F_2 = \frac{\alpha\sqrt{2(1+\beta+\gamma)}}{\sqrt{4\alpha^2+(1-\beta+\gamma)^2}}\begin{bmatrix}\frac{1}{2\alpha}(1-\beta+\gamma) & 0\\ 0 & 1\end{bmatrix}, \quad (6)$$

where $\alpha=\int_0^1 P(\eta)\sqrt{\eta}d\eta, \beta=\int_0^1 P(\eta)\eta d\eta$, and $\gamma=\sqrt{(1+\beta)^2-4(\beta-\alpha^2)}$.

4 Summary and Discussion

Toward the derivation of the Kraus representation of the probabilistic attenuated channel for infinite-dimensional systems, we derived from the Choi matrix in the qubit system. The obtained results are more complicated than the results derived from the Stokes parameters[3]. Moreover, unlike the case of the latter results[3], the obtained operators do not include the standard Kraus operators of the attenuated channel[4]. That is, the reduction of obtained operators does not provide the standard operators by setting the probability distribution of the transmissivity to be the delta distribution. Hence, we will consider extensions of the form given in [3] to higher dimensional systems by utilizing the Choi matrix.

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