Characterizing incompatibility of quantum measurements via the Naimark extension

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The existence of incompatible measurements is a fundamental feature of quantum theory which has been well-studied, both from the point of view of quantum foundations as well as quantum information processing. Of central interest is the question of characterizing the incompatibility of a set of quantum measurements. It is well known that the incompatibility of a pair of projection-valued measurements (PVMs) is synonymous with their non-commutativity. Formally, a pair of PVMs is jointly measurable if and only if they pairwise commute. Furthermore, for a set of PVMs, it is known that the existence of a joint measurement for the whole set is equivalent to the existence of joint measurements for every pair of PVMs in the set. Commutativity implies compatibility, but compatibility does not imply commutativity for POVMs. If and only if condition for compatibility of POVMs is still unkmown.

In this work, We obtain a formal characterization of the compatibility or otherwise of a set of positive-operator-valued measures (POVMs) via their Naimark extensions. In general it is difficult to check whether an arbitrary set is compatible or not. Because it is difficult to find out the joint observable. We show that a set of POVMs is jointly measurable if and only if there exists a single Naimark extension, specified by a fixed ancilla state on the same ancilla Hilbert space, that maps them to a set of commuting projective measurements (PVMs). But all common Naimark extensions for a compatible set of observables may not commute. We use our result to obtain an easily checkable sufficient condition for the compatibility of a pair of dichotomic observables in any dimension. This in turn leads to a characterization of the compatibility regions for some important classes of observables including a pair of unsharp qubit observables. In future this result can be generalised for higher dimension and any number of observables with any number of outcomes. Finally, we also outline as to how our result provides an alternate approach to quantifying the incompatibility of a general set of quantum measurements. We have shown that any compatibility measure for PVMs can be extended to POVMs.

This article opens up scope for further research in this direction, which is not very much explored. This might help physicists to find out easier way to characterise the incompatibility of observables. This article will come to arxiv probably on tuesday.