Modeling and mitigation of realistic readout noise with applications to Quantum Approximate Optimization Algorithm

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Introduction

Imperfect measurements contribute greatly to errors in currently available quantum devices. Recently, methods to mitigate measurement errors were proposed, relying on classical post-processing of experimental statistics, preceded by the suitable device characterization [1–11]. Those techniques suffer from the curse of dimensionality due to characterization cost, sampling complexity and the complexity of post-processing, all scaling exponentially with the number of qubits N. Fortunately, some interesting problems in quantum computing, including Quantum Approximate Optimization Algorithms (QAOA) [12], require simultaneous estimation of a number of a few-particle *marginals*. This suggests that error mitigation techniques can be efficiently applied in this setting. In this work we report a measurement noise model which captures cross-talk between measurement errors, can be efficiently described and characterized, and which admits effective noise-mitigation on the level of marginal probability distributions. We test error-mitigation on experiments on 15 qubits on IBM's superconducting quantum device and conclude good performance. Furthermore, we study effects of readout noise and its mitigation on QAOA, together with analysis of sampling complexity of energy estimation.

Result 1a – readout noise model

As a model for readout noise, we consider stochastic map Λ which has a specific structure imposed by the locality of the cross-talk interactions in the readout noise, namely

$$\Lambda_{X_1\dots X_N|Y_1\dots Y_N} = \prod_{C_i} \Lambda^{\mathbf{Y}_{\mathcal{N}(C_i)}}_{\mathbf{X}_{C_i}|\mathbf{Y}_{C_i}} \ , \tag{1}$$

where we label matrix elements by classical bit-strings, with $X_1 \dots X_N$ denoting the classical state of N qubits which is the (possibly erroneous) output of the measurement device provided that input was the state $|Y_1 \dots Y_N\rangle$. In Eq.(1), we consider a set $\{C_i\}_{i=1}^{\mathcal{K}}$ of disjoint partitions (clusters) of qubits, together with the indices of their neighbors $\mathcal{N}(C_i)$. Each partition (cluster) characterizes qubits for which measurement noise is strongly correlated, while the exact form of of the noise depends on the input state $\mathbf{Y}_{\mathcal{N}(C_i)}$ of the neighbors. The structure of correlations is inferred from the noise characterization procedure (described below), and does not necessarily correspond to physical connectivity of a device.

Result 1b – noise mitigation on marginals

We show that if the Eq. (1) holds for global distribution, then the marginal distributions are affected by stochastic noise. This allows us to perform the noise-mitigation individually on marginal distributions up to some error which depends on the level of correlations and number of gathered samples. Crucially, our noise-mitigation procedure exhibits low sampling and computational complexity, contrary to the naive procedure performed globally.

Result 1c – efficient noise characterization

To characterize noise matrices, we use generalization of Quantum Overlapping Tomography [13] to probe diagonal elements of the detector's POVM efficiently. This allows to capture k-qubit correlations in N-qubit device using only $\propto \exp(k) \log N$ quantum circuits consisting of only 1-qubit gates. To benchmark our error-mitigation, we experimentally implement ground states of diagonal Hamiltonians and perform noise-mitigation on the level of marginals. In Fig. 1 we present results of such experiments on 15 qubits (IBM's *Melbourne* device) for 600 Hamiltonians encoding random MAX-2SAT problems with clause density 4 and 600 fully-connected Hamiltonians with random interactions. We conclude a good performance of our noise model compared to no mitigation and to the simple uncorrelated noise model.

Result 2 – readout noise in QAOA

We study how readout noise and mitigation procedure affect the performance of QAOA. We observe that for the numerically studied instances the realistic readout noise can affect QAOA procedure in two ways – first by reducing the quality of energy estimators, and second by slowing down the convergence of the algorithm. Importantly, we also observe that employing the error-mitigation procedure allows to circumvent these undesirable effects. To back these statements we perform exhaustive numerical experiments of noisy QAOA on 8 qubits using classical optimizer known as Simultaneous Perturbation Stochastic Approximation [14]. In Fig. 2 we present exemplary results of numerical optimization simulations, with measurement noise model based on results of IBM's device characterization. We conclude effectiveness of the error-mitigation scheme compared to no mitigation.

Result 3 – statistical errors for local Hamiltonians

We present arguments why in the task of energy estimation of local Hamiltonians, one can expect that local terms will effectively behave as uncorrelated variables in a sense of statistical fluctuations. This holds for a broad classes of quantum states, which include Haar-random states, typical states in local random circuits, and states at the beginning and at the end of the QAOA optimization. Observed effects drastically lower the sampling complexity of the energy estimation task in QAOA, and, as a result, lower the errors in noise-mitigation on the marginal distributions. The arguments are based on random matrix theory, theory of random quantum circuits [15], information spreading [16] in shallow circuits, and analysis of correlations propagation in QAOA [17].

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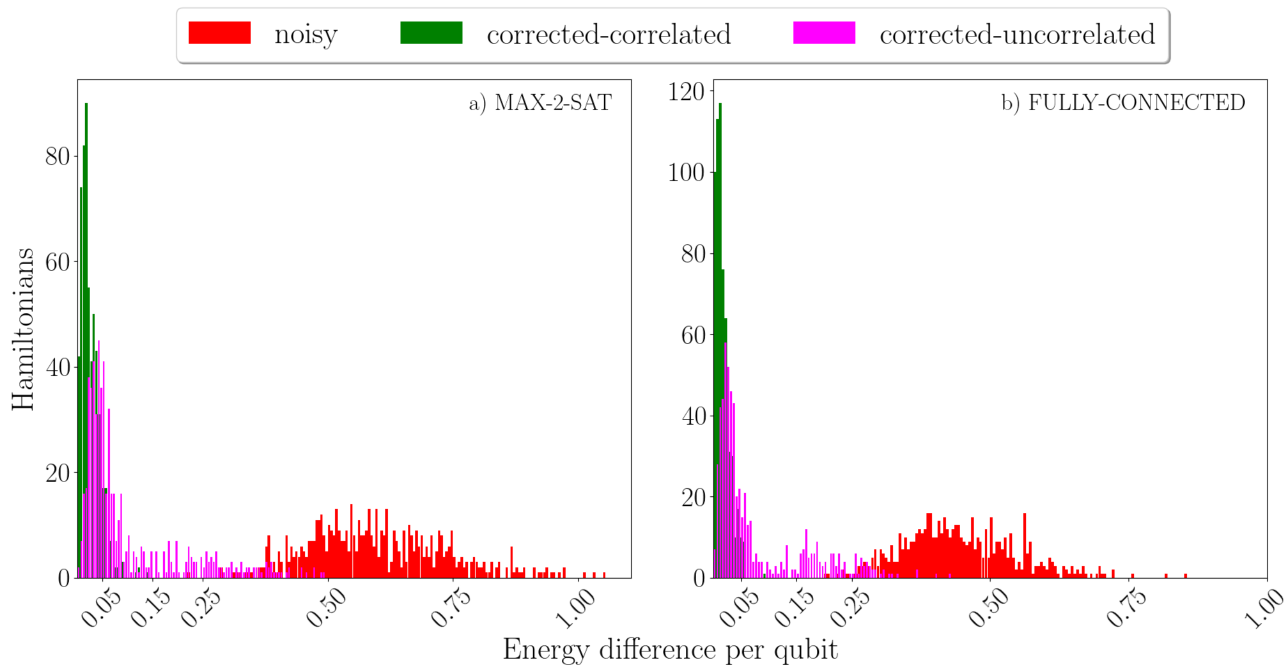

FIG. 1. Noise mitigation benchmark for 15q experiments on IBM's Melbourne device. Shown are results for estimation of energy of the ground states of 600 Hamiltonians encoding a) MAX 2-SAT problem with clause density 4, and b) fully-connected graph with random interactions. Vertical axis presents the absolute value of the difference between true and estimated state energies, divided by the number of qubits (hence error per single qubit). Red bars correspond to unmitigated results, magenta bars correspond to noise-mitigation based on simple uncorrelated readout noise model, and green bars correspond to noise-mitigation based on our correlated noise model from Eq. (1). Each estimator was obtained from ≈ 41000 samples.

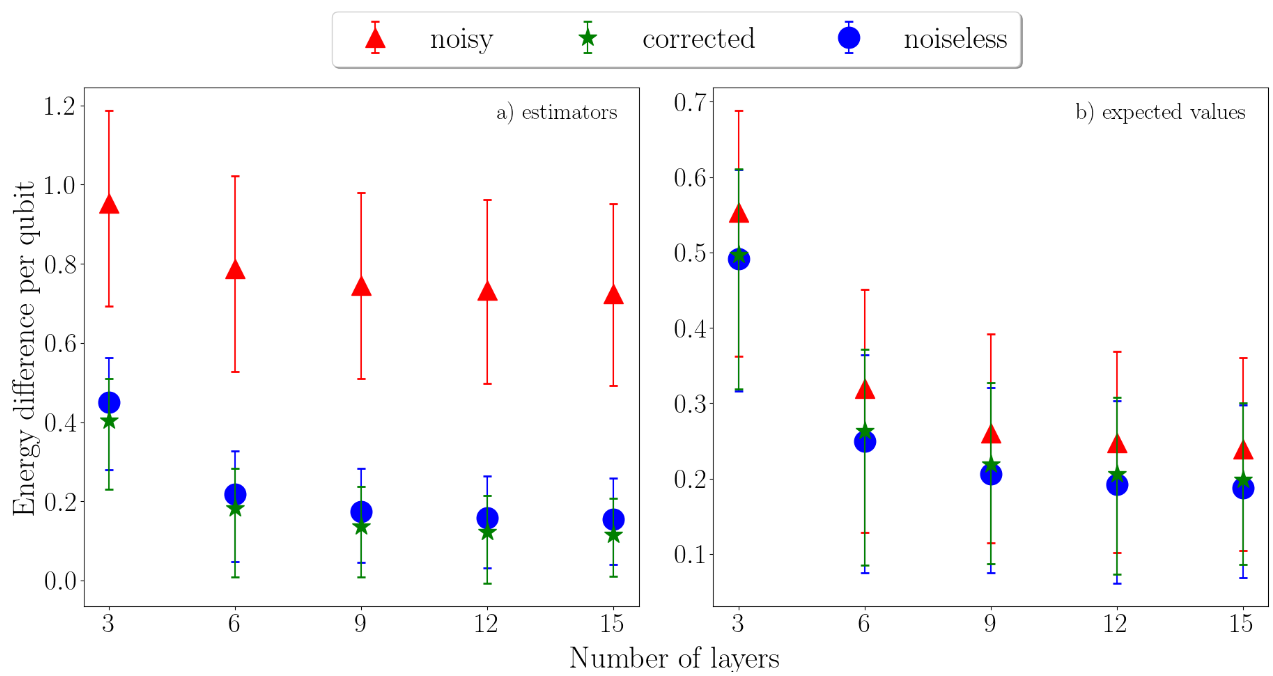

FIG. 2. Numerical simulation of QAOA for 8 qubits for Hamiltonians encoding random MAX-2-SAT instances with clause density 4. Each data point is an average over 92 Hamiltonians. Vertical axis shows energy difference per qubit calculated for a) *estimators of energy* (this represents error in energy estimation), and b) *expectation values* on optimized states (this represents error in obtained QAOA parameters). Plotted are three types of optimization – the optimization guided by noisy estimators (red triangles), the optimization guided by noise-mitigated estimators (green stars), and noiseless optimization (blue dots) given for reference. Each estimator was calculated from ≈ 1000 samples. Shown error bars are 1σ.