

No-go theorem for concentration of nonclassicality

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Quantum technology has been growing rapidly, providing a variety of applications. Tasks in quantum information processing take advantages over their classical counterpart by exploiting quantum *resources*: e.g., nonclassicality has recently been introduced as a resource quantifying the metrological power [1–3]. Thus it is a crucial issue to identify quantum resources required to obtain quantum advantages. Quantum resource theories (QRTs) [4] provide a comprehensive framework for quantifying and manipulating quantum resources. In a realistic situation, quantum resources are contaminated by noisy environment. To overcome practical noise, we study *resource concentration* in which one aims to produce more resourceful output by consuming contaminated copies.

In this work, we study the limitation for the concentration of nonclassicality by looking into the properties of resource measures. Let us begin by briefly introducing the basic formalism of QRTs. A resource theory is determined by free states and free operations. The golden rule of QRTs is that free operations never create resources. Resource can be quantified by a proper resource measure R . One of the important properties of a resource measure is the monotonicity which states that R does not increase under free operations. We also introduce another important property called *tensorization property*, $R(\rho \otimes \sigma) = \max\{R(\rho), R(\sigma)\}$, used to derive our main result. The goal of resource concentration is to obtain output state with higher resource from noisy states. Consider that one is given noisy resource states $\rho_1, \rho_2, \dots, \rho_N$ and is to produce output state $\sigma = \Phi(\rho)$ using free operations Φ . One can say that (s)he successfully obtain concentrated resource state if $R(\sigma) > \max_i R(\rho_i)$. If R satisfies both monotonicity and tensorization property, one can readily show that achieving the above inequality is impossible.

In the continuous-variable system, an N -mode state ρ is called classical if it can be written as a convex mixture of coherent states. In the resource theory of nonclassicality, the free states becomes the set of states with positive P function. We choose the following operations as free operations: (1) passive linear unitaries and displacements, (2) adding classical ancilla modes, (3) projection onto coherent states, (4) classical mixing. The nonclassicality depth τ_m [5, 6] is defined by the minimum amount of added noise τ which makes the s -parametrized quasiprobability function positive. We here prove that the

nonclassicality depth satisfies the monotonicity and the tensorization property. Importantly, the monotonicity holds even for conditional operations which allows post-selection. Using these properties, we find that the nonclassicality depth can never be concentrated by classical operations even probabilistically.

Recently, nonclassicality has been studied as a resource quantifying the quantum advantage in metrological tasks [1–3]. The metrological power of nonclassicality, F_1 , is defined by the maximal advantage in displacement estimation among all directions in the phase space. The monotonicity under deterministic free operations is proved in [1, 2] with a slightly different set of free operations. For conditional operations $\Phi(\cdot) = \sum_j \Phi_j(\cdot)$, another form of monotonicity is proved which states that $p_j F_1(\Phi_j(\otimes_{i=1}^N \rho_i)) \leq F_1(\otimes_{i=1}^N \rho_i)$ under classical operations and destructive measurements. The metrological power obeys the tensorization property as well [2]. While the concentration using deterministic free operations is not allowed, a probabilistic concentration of F_1 is possible. When one tries to obtain the target state σ_T by probabilistic concentration, the success probability is upper bounded by $P_{\text{succ}} \leq \max_i F_1(\rho_i)/F_1(\sigma_T)$. It is worth noting the upper bound does not depend on the number of input copies.

It may seem that the no-go theorems for two different measures of nonclassicality contradict each other. We show that the nonclassicality depth and the metrological power are looking into different aspects of nonclassicality by examining practical concentration protocols for cat states and for noisy single-photon states. This work leads us to understand which operations are essentially required to obtain more resourceful states.

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