Full simulation of fault-tolerant quantum error correction under general noise for near-term quantum devices

Mitsuki Katsuda[1] * Kosuke Mitarai[1 2 3] † Keisuke Fujii[1 2 4] ‡

[1] *Division of Advanced Electronics and Optical Science, Department of Systems Innovation, Graduate School of Engineering Science, Osaka University*

[2] *Center for Quantum Information and Quantum Biology, Institute for Open and Transdisciplinary Research Initiatives, Osaka University*

[3] *JST PRESTO, Japan*

[4] *Center for Emergent Matter Science, RIKEN, Wako Saitama 351-0198, Japan*

In recent years, many researchers has paid extensive effort for the realization of quantum computers. Quantum computers with over 50 qubits have been developed so far, for example, by Google and IBM [1, 2]. These are now considered as noisy-intermediate scale quantum devices, and their applications are explored. To execute quantum algorithms such as Shor's and Grover's algorithms, it is essential to construct reliable logical qubits, which are protected from noise by quantum error correction. A surface code, a sort of two-dimensional topological codes, has a high error threshold despite that the syndrome measurement for the error detection requires only nearest-neighbor two-qubit gates on a square array of physical qubits [3–5]. Specifically, the rotated surface code requires $2d^2 - 1$ qubits to achieve a code distance d [5]. For example, the required number of qubits with $d = 5$ is 49, and hence an experimental realization of one fault-tolerant logical qubit is now within a reach of the-state-of-the-art quantum devices [1]. However, it is still challenging to demonstrate that a logical error probability is suppressed sufficiently so that a logical lifetime of quantum information is longer than a coherence time of the physical qubit under a realistic situation. To this end, numerical analyses are indispensable along with experimental effort.

Regarding the numerical analyses of quantum error correction, most existing works assume stochastic Pauli errors. The error thresholds are estimated in various noise modes such as code capacity (10.3% [3]), phenomenological (2.97% [6]), and circuit-level (0.75% [7]) noise models. However, more general noise, including coherent errors, occurs in actual experiments. Numerical analyses for the coherent errors have been done for restricted cases, where the syndrome measurement error is ignored [8, 9] and the case for one-dimensional repetition code [10]. In both cases, the error thresholds are smaller than those for the stochastic Pauli errors, and hence it is vital to evaluate an error threshold under the coherent errors. Unfortunately, simulation of the coherent errors cannot be done efficiently in general and hence it will be challenging to do numerical analyses for fault-tolerant quantum error correction with existing or near-term quantum devices.

In this work, we perform a full simulation of fault-tolerant quantum error correction with a rotated surface code with $d = 5$ under general noise models. The number of the physical qubits is 49, and hence a straightforward simulation storing a full state vector requires memory of 8GB. While it is not impossible for the-state-of-the-art supercomputer, it is not so feasible if we think about the number of samples 10^4-10^6 required to achieve accuracy. To overcome this, we cleverly design the simulation of syndrome measurements so that the syndrome measurement qubit is reused through the simulation. This reduces the dimension of the state vector prepared for a simulation from 2^{49} to 2^{26}, and simulation becomes feasible even on a single-node workstation; one sample can be simulated within 8 secs on GPU (Nvidia Tesla V100). We numerically determine the error threshold with circuit-level noise under a mixture of coherent and incoherent noises and discuss whether or not the logical error probability is suppressed with experimentally realistic noise levels.

*u037351e@ecs.osaka-u.ac.jp

†mitarai@qc.ee.es.osaka-u.ac.jp

‡fujii@qc.ee.es.osaka-u.ac.jp

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