Nonlinear transformation of complex amplitudes via quantum singular value transformation

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Abstract. Due to the linearity of quantum operation, it is not straightforward to implement the nonlinear transformation on a quantum computer, making some practical tasks like a neural network hard to be achieved. In this work, we define a task called the *nonlinear transformation of complex amplitudes* and provide an algorithm based on quantum singular value transformation. We also provide a thorough discussion about how to implement practical tasks such as a neural network using complex amplitudes with this procedure.

Keywords: Quantum algorithm, Quantum singular value transformation, Quantum machine learning

Quantum algorithm has shown computational advantages in many tasks over the best classical algorithms. In recent years, ones for machine learning have attracted much attention due to their practical potentials. It has been shown that under certain assumptions, popular machine learning techniques like support vector machine [1] and linear regression [2] can gain exponential speedup from quantum computers using the Harrow-Hassidim-Lloyd algorithm [3] as a subroutine. However, there are still many problems waiting to be solved, if one wants to implement techniques like a neural network with a quantum computer.

A neural network, which can solve problems such as classification, is a method widely used in the machine learning field. It is based on linear transformations and element-wise nonlinear transformations $P(\vec{x})$ of data vectors $\vec{x}$. If we wish to translate it into a quantum task, its naive extension can be defined as follows. Assume we have an access to an oracle $U : U|0\rangle = \sum_{i=1}^{N} c_i |i\rangle$ which encodes a complex data vector $\vec{c} = \{c_i\}$, as well as its adjoint and their controlled versions. For given nonlinear functions P and Q, prepare a quantum circuit such that its output state is $\frac{\alpha}{N}\sum_{i=1}^{N}(P(x_i) + iQ(y_i))|i\rangle$, where $x_i + iy_i = c_i$ and α is a normalization factor. We call this task *nonlinear transformation of complex amplitudes* which can then be used to implement neural networks on a quantum computer. In the previous work that has considered a quantum implementation of neural network [4], they only perform linear operations on a quantum computer, and nonlinear operations are done classically. This approach avoids the above task but with the cost for input and readout of the amplitudes.

In this work, we provide an algorithm to solve this problem based on quantum singular value transformation [5]. We show that this task can be performed if the desired functions P and Q satisfies certain conditions that allows an efficient approximation by the quantum signal processing [6], and the oracle complexity depends on dimensions N, $\{c_i\}$, P, and Q. We will also provide a thorough discussion about how to implement certain practical nonlinear functions. For example, if our approach is used to implement the neural networks with activation function $\tanh(x)$, under certain conditions this algorithm can achieve quadratic quantum speedup with respect to the dimensions of data vectors. Since we do not need to perform repetitive measurements at the nonlinear transformation step, the proposed algorithm is advantageous compared with the previous work [4]. However, we point out that this method is not efficient for all the cases. We argue that this may connect to the fact that nonlinearity quantum computation allows us to solve some very hard problems efficiently [7, 8].

This work opens up a way to implement nonlinear operations on a quantum computer, and may contribute to the field of quantum machine learning and other physically motivated problems.

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References

[1] P. Rebentrost, M. Mohseni, and S. Lloyd, "Quantum support vector machine for big data classification", Physical Review Letters **113**, 10.1103/physrevlett.113.130503 (2014).

[2] M. Schuld, I. Sinayskiy, and F. Petruccione, "Prediction by linear regression on a quantum computer", Physical Review A **94**, 10.1103/physreva.94.022342 (2016).

[3] A. W. Harrow, A. Hassidim, and S. Lloyd, "Quantum algorithm for linear systems of equations", Physical Review Letters **103**, 10.1103/physrevlett.103.150502 (2009).

[4] J. Allcock, C.-Y. Hsieh, I. Kerenidis, and S. Zhang, *Quantum algorithms for feedforward neural networks*, 2019.

[5] A. Gilyén, Y. Su, G. H. Low, and N. Wiebe, "Quantum singular value transformation and beyond: exponential improvements for quantum matrix arithmetics", 10.1145/3313276.3316366 (2018).

[6] G. H. Low and I. L. Chuang, "Optimal hamiltonian simulation by quantum signal processing", Physical Review Letters **118**, 10.1103/physrevlett.118.010501 (2017).

[7] D. S. Abrams and S. Lloyd, "Nonlinear quantum mechanics implies polynomial-time solution for NP-complete and $\#P$ problems", Phys. Rev. Lett. **81**, 3992–3995 (1998).

[8] S. Aaronson, "Guest column: NP-complete problems and physical reality", SIGACT News **36**, 30–52 (2005).