Robust quantum computational advantage using fermionic linear optics and magic input states

Michał Oszmaniec,[1] Ninnat Dangniam,[1] Mauro E.S. Morales,[2] and Zoltán Zimborás[3]

[1] *Center for Theoretical Physics, Polish Academy of Sciences*

[2] *Centre for Quantum Software and Information, University of Technology Sydney*

[3] *Wigner Research Centre for Physics*

Recent advances in NISQ technologies have led to an experimental demonstration of quantum (computational) supremacy of random circuit sampling (RCS) [1] and quantum chemistry calculations [2] on Google's 53-qubit superconducting quantum computer. RCS is the task of sampling from the output distribution of a randomly selected quantum circuit, a leading candidate for a task unreachable by classical simulations due to its rigorous hardness guarantees based on two ingredients available in random quantum circuits: a worst-to-average-case reduction [3, 4] and anticoncentration [5].

We propose a quantum advantage scheme based on a fermionic analogue of Boson Sampling: Fermion Sampling with magic input states. The scheme utilizes a restricted set of gates and layouts native to superconducting qubit architecture used in simulations of quantum chemistry [6] known as passive (or particle-number-preserving) and active fermionic linear optics (FLO). While FLO circuits with computational-basis input states can be efficiently simulated classically [7, 8], FLO circuits initialized in a special "magic" resource state leads to output distribution that is hard to classically simulate in the worst-case [9–11]. The magic states can be prepared using 3 entangling gates per each and every disjoint block of four qubits. (See Figure 1).

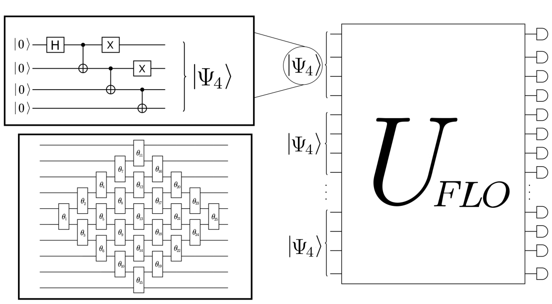

FIG. 1. A schematic of the Fermion Sampling task. The magic input states are prepared by a simple circuit and sent through a generic FLO circuit, which can be decomposed in the layout shown on the bottom left. One then samples from the computational-basis output distribution.

We prove two main technical results that underpin hardness of the proposed scheme:

(i) Robust worst-to-average-case hardness reduction for computation of probabilities for passive and activle FLO circuits initialized in magic states.

(ii) Anticoncentration of probability amplitudes in the output of the scheme for both passive and active FLO circuits initialized in magic states.

These results put Fermion Sampling at the same level as RCS [3, 4] in terms of state-of-the-art hardness guarantees, surpassing that of Boson Sampling. In addition, the advantage of our scheme compared to RCS is that FLO circuits can be efficiently certified due to its low-dimensional structural properties, while the disadvantage is the depth required to guarantee the worst-case hardness and slight weaker robustness. The scheme has the potential to be implementable in near-term quantum devices due to the compatiblity with existing superconducting qubit architecture used to demonstrate quantum chemistry simulations [2].

Instrumental to our proofs is the fact that active and passive FLO circuits are representations of the low-dimensional (of dimensions scaling polynomially with the number of qubits n) Lie groups $\mathrm{U}(n)$ and $\mathrm{SO}(2n)$ respectively. For the worst-to-average case reduction, we follows the state-of-the-art technique by Movassagh [4], which utilizes Cayley path to construct a low-degree rational interpolation between the worst-case and average-case circuits, while generalizing it in two significant directions. First, while the interpolation in [4] is performed directly at the level of physical circuits, ours is performed at the level of group elements which are then represented as circuits. Secondly, while [4] applies the interpolation to local one- and two-qubit gates that constitute the circuit, we directly apply it to a global circuit while maintaining the low-degree nature of the rational functions, which is required for the robust reduction. For the anticoncentration property, we do not use the 2-design property but instead relies on group-theoretic properties of the circuits.

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