Applying the Quantum Alternating Operator Ansatz to the Graph Matching Problem

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1 Introduction

Quantum Approximate Optimization Algorithms (abbreviated as QAOA) is a class of gate-model algorithms [1] which are used to find approximate solutions to combinatorial problems. Initially QAOA was designed to work on unconstrained problems, but was later modified [2] to work on constrained optimization problems. The authors termed this new framework as the Quantum Alternating Operator Ansatz (which we abbreviate as QAOA$^+$).

A matching is a set of edges which are vertex disjoint. Although finding a maximum matching in a graph is in P, counting problems with respect to matchings are #P-hard [3]. Hence, designing efficient algorithms to create a superposition over all distinct matchings with non-zero amplitudes in or all maximal matchings with non-zero amplitudes in polynomial time, is clearly a non-trivial task. We design and apply a QAOA$^+$ style algorithm to the input state to create a superposition over all maximal matchings with non-zero amplitudes, in polynomial time with respect to the input graph. Simple applications involve manipulating this output state using Grover style amplifications to obtain the maximum matching, or sampling efficiently to get a large matching. Designing "good" samplers on the output state itself has complexity theoretic importance, since hardness of sampling is closely related to hardness of counting [4].

We investigate and show how the output states of our QAOA$^+$ setup produces better matchings in expectation compared with a uniform distribution over all matchings. We also demonstrate how to modify the vanilla QAOA$^+$ setup in order to obtain better output states by using a superposition of feasible states as the initial state rather than using a single feasible state as the initial state.

2 Background

In a QAOA$^+$ algorithm we have two sets of operators. The phase separation operator $U_P(\gamma)$ encodes the objective function value, and the mixing operator $U_M(\beta)$ evolves the state from one feasible state to another. A QAOA$^+$ circuit consists of p alternating layers of $U_P(\gamma)$ and $U_M(\beta)$ applied to a suitable initial state $|s\rangle$ to produce the output state $|\gamma,\beta\rangle$. A computational basis measurement over the state $|\gamma,\beta\rangle$ returns a candidate solution state $|z\rangle$. In the case of a maximization problem, the goal of QAOA$^+$ is to obtain $|z\rangle$ with a high objective function value.

3 Our Work and Results

We design a QAOA$^+$ circuit for the matching problem as follows:

1. Encoding the edges as qubits, and creating a proper mixing unitary that preserves the feasibility of the initial state, using control clauses on the edges.
2. We demonstrate how a simple modification of the control clauses helps us harness the power of using a superposition of feasible states over a single feasible state as the initial state.

These improvements can also be used for harder problems like vertex cover, independent sets or cliques to derive results similar to what we list below.

Let the number of iterations of our QAOA$^+$ algorithm be p, and the number of edges of the input graph be $|E|$. We apply the Quantum Alternating Operator Ansatz to the Matching Problem and obtain the following main results:

1. Starting from the empty matching, we obtain a superposition over all possible distinct matchings with non-zero amplitude in $p=1$.
2. Starting from the $|W_1\rangle$ state, and using our modified definition of control clauses, we obtain a superposition over all possible distinct matchings of non-zero size with non-zero amplitude in $p=1$.
3. Using the W-state as the initial state, we can converge to a superposition over all maximal matchings with non-zero amplitude where $\mathbb{E}[p]\leq 2\cdot|E|$. Here $\mathbb{E}[p]$ means expected number of iterations.
4. For 2-regular graphs, using $|W_1\rangle$ as the initial state produces an output state which has a better expected matching size compared to using the empty matching as the initial state.
5. For 2-regular graphs, the expected matching size of the QAOA$^+$ output state is greater than the expected matching size obtained from a uniform distribution over all matchings.

References

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