Efficient Algorithms for Causal Order Discovery in Quantum Networks

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Quantum networks [1, 2] are the backbone of large-scale quantum communication and computation. The study of quantum networks provides tools to analyze interactions between users, quantum channels and devices, with applications to quantum key distribution [2], quantum distributed computation [3] and quantum cloud computing [4]. To support the applications, an efficient and robust quantum network is indispensable.

To ensure the robustness of quantum communication, we need to keep the communication channel stable from noise and errors, which could be solved by quantum error-correcting codes [5] and protocols resistant to misaligned reference frames [6, 7]. However, at a larger scale, other sources of instability emerge from the structure of the network. In a quantum communication network, parties are connected by repeaters and routers that serve as intermediate transmission nodes, whose availability may be frequently changing due to network traffic and environmental noise, causing dynamical changes in the structure of the network. To determine the optimal path to transmit data, one has to detect those structural changes frequently, and adaptively adjust the transmission paths.

Formally, the signalling of information can be modeled as cause-effect relations. The identification of cause-effect relations, i.e. causal order discovery, is crucial for a wide range of applications in science and society. This problem has been extensively studied in the classical scenario [8], while a quantum version of the causal order discovery problem has been formulated in Ref. [9]. Basic cases involving causal relations between a few inputs and outputs has been studied in Refs. [10, 11]. Ref. [12] deals with the case of many inputs and outputs. It formulates quantum causal orders as quantum combs [13], and proposes a classical algorithm based on the full classical description of the process. However, in a quantum network that is frequently changing, the classical description is not known in advance, and obtaining such a description is practically difficult, often requiring a process tomography [14] which can take exponential time.

In this article, we adopt the formulation of quantum causal orders as quantum combs, and propose the first efficient quantum causal order discovery algorithm for many-system quantum processes with black-box queries to the process. Our algorithm searches for the last input and the last output in the causal order, removes them, and iteratively repeats the above procedure until we get the order of all inputs and outputs. Our method guarantees a polynomial running time for quantum combs with a low Kraus-rank, namely processes with low noise and little information loss. We also propose algorithms with a lower query complexity for cases where the causal order can be inferred from local observations, for example, when each input has a non-trivial influence on all outputs after it, and when the comb is a tensor product of single-system channels. These algorithms only require local state preparation and local measurements, and the number of uses of the process could grow logarithmically with the number of input and output systems.

The applications of causal order discovery are not limited to quantum networks. Our independence tests will also be applicable to quantum devices and circuits for testing whether they have correct links and input-output correlations. This will be an important quality assurance procedure for the production of reliable components in quantum computers and networks. Causal order discovery can be also used to detect the latent structure of quantum systems, by applying the causal order discovery algorithms to detect the correlations in multipartite states. Knowing the causal structure of the system may allow us to ignore unnecessary correlations and represent states efficiently with, for example, tensor networks [15, 16], which allow efficient simulation [16–18] and compression [19] of multipartite states.

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