Physical Implementability of Quantum Maps and Its Application in Error Mitigation

Jiaqing Jiang[1 2] Kun Wang[1] Xin Wang[1]

[1]*Institute for Quantum Computing, Baidu Research, Beijing 100193, China*
[2]*Computing and Mathematical Sciences, California Institute of Technology, Pasadena, CA USA*

Abstract. We introduce a systematic framework to characterize how well a quantum map, which transforms operators to operators, can be physically implemented. We decompose a quantum map into a linear combination of physically implementable operations and introduce the physical implementability measure as the least amount of negative portion that the quasiprobability must pertain. We show this measure satisfies many desirable properties and possesses an operational meaning within error mitigation, quantifying the ultimate sampling cost achievable using quantum devices. We further resolve the error mitigation cost for basic quantum channels and show that global error mitigation has no advantage over local error mitigation for parallel noise.

Keywords: quantum maps, physical implementability, error mitigation, quasiprobability

1 Introduction

The postulates of quantum mechanics prescribe that the evolution of a closed quantum system must be unitary [1]. The physically implementable quantum operations are obtained in the reduced dynamics of subsystems and are mathematically characterized by completely positive and trace-preserving maps (CPTPs). Nevertheless, many quantum maps, such as positive but not completely positive maps, which are impossible to be physically implemented, are also fundamental ingredients from theoretical and practical perspectives. This motivates us to study the problem of physically approximating such 'non-physical' quantum maps.

2 Physical implementability measure

In this paper, we introduce a systematic framework to resolve this task, using the powerful quasiprobability decomposition technique [2, 3, 4, 5, 6]. More specifically, we decompose a target quantum map $\mathcal{N}$ into a linear combination of physically implementable quantum operations, i.e. CPTPs. Then we define the *physical implementability* measure of $\mathcal{N}$ as the least amount of negative portion that the quasiprobability must pertain:

$$\nu(\mathcal{N}) := \log \min \left\{ \sum_\alpha |\eta_\alpha| \;\middle|\; \mathcal{N} = \sum_\alpha \eta_\alpha \mathcal{O}_\alpha \right\}, \quad (1)$$

where each $\mathcal{O}_\alpha$ is CPTP and η_α is real.

This measure bears interesting properties. It is efficiently computable via semidefinite programs. It satisfies the additivity property with respect to tensor products. This property ensures that parallel application of quantum maps cannot make its physical implementation 'easier' compared to implementing these quantum maps individually. It also satisfies the monotonicity property with respect to quantum superchannels. We derive bounds on this measure in terms of its Choi operator's trace norm. These bounds are tight in the sense that there exist quantum maps for which the bounds are saturated. What's more, we derive analytical expressions of this measure for some inverse maps of practically interesting CPTP maps.

3 Application in error mitigation

We endow ν an operational interpretation within the quantum error mitigation framework as it quantifies the *ultimate* sampling cost achievable using the full expressibility of quantum computers. Consider the task of estimating the expected value $\mathrm{Tr}[\rho A]$ for an observable A and a quantum state ρ. Preparation of ρ inevitably suffers from noise modeled by some CPTP $\mathcal{N}$. We can perform its invertible map $\mathcal{N}^{-1}$ to cancel the noise

$$\mathrm{Tr}\left[\mathcal{N}^{-1} \circ \mathcal{N}(\rho) A\right] = \mathrm{Tr}[\rho A]. \quad (2)$$

However, $\mathcal{N}^{-1}$ might not be implementable and (2) cannot be carried out physically. We propose a probabilistic error cancellation technique to deal with this issue and it turns out that the incurred sampling cost is quantified by ν. The mitigation procedure roughly goes as follows.

1. Decompose $\mathcal{N}$ into a combination of CPTPs as (1).
2. Iterate the following sampling procedure M times:
   (a) In the m-th iteration, sample $\mathcal{O}^{(m)}$ from distribution $\{\mathcal{O}_\alpha, |\eta_\alpha|/\sum_\alpha |\eta_\alpha|\}$. Let $\eta^{(m)}$ be the sampled coefficient.
   (b) Compute the expectation $\mathrm{Tr}\left[\mathcal{O}^{(m)} \circ \mathcal{N}(\rho) A\right]$.
3. Compute the *empirical mean value* $\xi := \frac{2^{\nu(\mathcal{N})}}{M} \sum_{m=1}^{M} \mathrm{sgn}\left(\eta^{(m)}\right) \mathrm{Tr}\left[\mathcal{O}^{(m)} \circ \mathcal{N}(\rho) A\right]$. Output ξ as an unbiased estimate of $\mathrm{Tr}[\rho A]$.

We further explore the operational properties of this error mitigation procedure based on the nice properties ν and its connection to the sampling cost. First, since $\nu(\mathcal{N})$ is efficiently computable, we can estimate the sampling cost of arbitrary quantum maps, yielding a feasible way to deal with quantum noise beyond the NISQ era [7]. Second, $\nu(\mathcal{N})$ gives the physical limits of error mitigation via the quasi-probability method. Notably, we find that certain Pauli channel can not be mitigated while it has a positive quantum capacity. Third, the additivity of ν implies that for parallel quantum noise, global error mitigation has *no advantage* over error mitigation locally and individually.

References

[1] Michael A. Nielsen and Isaac L. Chuang. *Quantum Computation and Quantum Information*. Cambridge University Press, 2011.

[2] Hakop Pashayan, Joel J Wallman, and Stephen D Bartlett. Estimating outcome probabilities of quantum circuits using quasiprobabilities. *Physical Review Letters*, 115(7):070501, 2015.

[3] Kristan Temme, Sergey Bravyi, and Jay M Gambetta. Error mitigation for short-depth quantum circuits. *Physical Review Letters*, 119(18):180509, 2017.

[4] Mark Howard and Earl Campbell. Application of a resource theory for magic states to fault-tolerant quantum computing. *Physical Review Letters*, 118(9):090501, 2017.

[5] Suguru Endo, Simon C Benjamin, and Ying Li. Practical quantum error mitigation for near-future applications. *Physical Review X*, 8(3):031027, 2018.

[6] Ryuji Takagi. Optimal resource cost for error mitigation. *arXiv preprint arXiv:2006.12509*, 2020.

[7] John Preskill. Quantum computing in the NISQ era and beyond. *Quantum*, 2:79, 2018.