Selling Substitute Goods to Loss-Averse Consumers: Limited Availability, Bargains and Rip-offs

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Abstract

This paper characterizes the profit-maximizing pricing and product-availability strategies for a retailer selling two substitute goods to loss-averse consumers, showing that limited-availability sales can manipulate consumers into an ex-ante unfavorable purchase. Consumers have unit demand, are interested in buying only one good, and their reference point is given by their recent rational expectations about what consumption value they would receive and what price they would pay. If the goods are close substitutes, the seller maximizes profits by raising the consumers' reference point through a tempting discount on a good available only in limited supply (the bargain) and cashing in with a high price on the other good (the rip-off), which the consumers buy if the bargain is not available to minimize their disappointment. The seller might prefer to offer a deal on the more valuable product, using it as a bait, because consumers feel a larger loss, in terms of forgone consumption, if this item is not available and are hence willing to pay a larger premium to reduce the uncertainty in their consumption outcomes. I also show that the bargain item can be a loss leader, that the seller's product line is not welfare-maximizing and that she might supply a socially wasteful product. The results of the model suggest that the current FTC Guides Against Bait Advertising, by allowing retailers to employ limited-availability sales, could reduce consumer and social welfare.

JEL classification: D11; D42; L11.

Keywords: Retail Pricing; Reference-Dependent Preferences; Loss Aversion; Limited Availability; Bait and Switch; Loss Leaders.

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1 Introduction

Retailers frequently use low prices and offer deals to attract consumers. In many cases, these deals apply only to a subset of a store's product line and are often subject to "limited-availability". Some shops, for example, offer deals that are valid only "while supplies last," or they might offer price reductions on sale items only to the very first customers of the day. Consider the two following examples:

Example 1 *A retailer in Berkeley California has offered the following:*
Converse All Star high-top in black for just \$24.99 (offer valid while supplies last).
Any other color for \$54.99.[1]

Example 2 *On Black Friday 2011, Best Buy offered, among other items, the following:*
Panasonic 50" Class / Plasma / 1080p / 600Hz / Smart HDTV for \$599.99.
Panasonic 50" Class / Plasma / 720p / 600Hz / HDTV for \$799.99.[2]

In the first example, the store is offering a deal on black shoes — \$20 less than the regular price. There is, however, no deal on other colors; indeed their price is \$10 higher than the regular price. The \$30 difference between the price of black and non-black shoes is unlikely to be explained by differences in cost or demand. Furthermore, the deal on black shoes is valid only while current supplies last and the price could well be higher once the store restocks. In the second example, the store is selling two very similar TVs for very different prices; moreover, somewhat puzzlingly, the TV with the higher-resolution screen, universally preferred, is offered at a lower price. The original Best Buy ad specified that the one on the superior TV was an online-only deal, that availability was "limited to warehouse quantity," and no rainchecks would be offered to consumers. Notice also that the goods in these examples are substitutes and consumers normally buy at most one unit. Why, then, do stores discount only a few items heavily, and why is there so much dispersion, within the same store, in the price of similar goods? How do stores select which products to offer for a discount?

Traditional search-theoretic models of sales based on costly information acquisition are not well-equipped to answer these questions, as they pertain mainly to retailers supplying only one product. Moreover, they are concerned with explaining price dispersion either across different stores (as in Salop and Stiglitz, 1977) or across different time periods (as in Varian, 1980), not with the issue of within-store price dispersion across similar items, nor they look at the role of product availability in retailing.[3]

[1] At the same retailer, Bancroft Clothing Co., the regular price during the "non-deal" weeks is \$44.99, independent of color. The manufacturer online price is \$50 plus shipping fees.

[2] Black Friday is the day following American Thanksgiving and traditionally marks the beginning of the Christmas shopping season. The 1080p TV first appeared at Best Buy on March 20, 2011 for \$1,000 and its price has been constant until Thanksgiving Day of the same year. The 720p TV first appeared at Best Buy on March 28 for \$719.99 and its price was reduced to \$649.99 on August 9, 2011 and raised again up to \$799.99 on November 10, 2011, two weeks before Thanksgiving. These data have been collected using camelbuy.com, a website that provides a price tracker and price history charts for products sold online at Amazon.com and Best Buy.com.

[3] For an extensive survey of the search theory literature in IO, see Baye, Morgan and Scholten (2006). Rhodes (2011) and Zhou (2012) study multi-product search models with complements. A notable exception is provided by Konishi and Sandfort (2002). In their paper a multi-product store can increase its profits by discounting only some of its products, even when they are substitutes. However, consumers in this model shop for a "search good" and hence they learn their tastes only once they arrive at the store and discounts on few items are a way to increase store traffic. The logic in my model is quite different.

In this paper, I show that retailers' use of limited-availability sales can be understood as a method to exploit consumers' loss-aversion and prompt them to willfully engage in ex-ante unfavorable trade. I do so by introducing consumer loss aversion into an otherwise classical model of linear pricing: a risk-neutral profit-maximizing monopolist sells two substitutable goods to homogeneous consumers who demand at most one unit altogether and whose reference point for evaluating a purchase, following the model of Kőszegi and Rabin (2006), is given by their recent rational expectations about the purchase itself. With these preferences, a consumer's willingness to pay for a good is determined not only by his intrinsic value for it, but also endogenously by the market conditions and his own anticipated behavior. Moreover, the monopolist can directly affect consumers' expectations by making announcements regarding prices or availability. For example, if a consumer expects to buy with high probability, he experiences a loss if he fails to buy. This, in turn, increases his willingness to pay. On the other hand, compared to the possibility of getting a deal, paying a high price is assessed by the consumer as more of a loss, which in turn decreases his willingness to pay. Since expectations are the reference point and because expectations are (also) about own future behavior, the reference point is determined endogenously in the model by requiring that the (possibly stochastic) outcome implied by optimizing behavior conditional on expectations coincides with expectations.

The main result of the paper is that, when two goods are close substitutes, the monopolist maximizes profits by offering a limited-availability deal on one of the goods to lure consumers and then cashes in with a high price on the other. Consumers perceive this limited-availability sale as equivalent to a lottery on both which good they will end up with and how much they will pay. The price of the good on sale (the *bargain*) is chosen such that it is not credible for the consumers to expect not to buy it. Thus, the limited-availability deal works as a bait in luring consumers into the store.[4] Then, because the consumers expect to make a purchase with positive probability and dislike the uncertainty in their consumption outcomes, in the event that the bargain is not available, they prefer to buy the substitute good, even at a higher price (the *rip-off*). In other words, consumers go to the store enticed by the possibility of the bargain, but if it is not there they buy a substitute good as a means of reducing their disappointment.[5]

The limited-availability nature of the deal is critical for this strategy to work, and the degree of availability of each item is publicly announced by the seller. On the one hand, a high likelihood of availability for the bargain makes the consumers more attached to the idea of buying. This allows the seller to charge a higher price on the rip-off. On the other hand, a greater availability of the bargain necessarily means fewer sales of the rip-off. When choosing the supply level of the bargain item, the seller optimally trades off these two effects. I also show that if the bargain is the product with the smaller social surplus, its availability is bounded above by 50%, implying that less than half of the consumers actually end up buying the item on sale.[6]

[4] There is a reason why in Black Friday jargon these deals are called "doorbusters."

[5] Because of loss aversion, consumers are willing to pay a premium in order to avoid the feeling of loss resulting from not getting the bargain. So, the seller is not exploiting a cognitive bias of the consumers. This is in contrast to several models with boundedly rational or naïve consumers, as in DellaVigna and Malmendier (2004), Eliaz and Spiegler (2006, 2008, 2011b), Gabaix and Laibson (2006), Grubb (2009), Rubinstein and Spiegler (2008), and Spiegler (2006). See Spiegler (2011) for a textbook treatment.

[6] Besides Black Friday, other examples of limited-availability sales that take place in the U.S. are: (i) Cyber Monday, the first Monday after Thanksgiving Day, which mainly pertains to online shopping; (ii) Back-to-School Sales taking place at

According to the current FTC regulation, it is not a bait-and-switch if the store communicates up-front that availability is limited.[7] Nevertheless, the popular press and various consumers' associations seem to perceive limited-availability deals as being of an exploitative nature, as suggested by the following quotes:

> One of the biggest problems during significant sale days like Black Friday is the deceptive practice of offering a popular, expensive item for a great sale price, but only stocking a very limited number of these products. This is somewhat of a bait-and-switch because even if that product is unavailable, you are likely to stay at the store and take advantage of other, less valuable sales. (Denver Better Business Bureau, http://denver.bbb.org)
>
> Know why they call it "Black Friday?" It isn't because those sale items push retailers into the "black" (accounting speak for profitability). Those sale items are almost always loss leaders — items sold at a loss in order to lure you into the store in the hope you'll buy other, more profitable items. What really pushes retailers into the black are the profitable items you buy because you showed up at 4am and everything you hoped to buy was sold out and you HAD to buy SOMETHING. *(*http://www.thewisdomjournal.com/Blog/beware-of-black-friday-bait-and-switch/)

The above quotes seem to imply that among the consumers who go shopping during sales with the intention of getting a deal, some fulfill their goal and get a bargain; others, however, might not find what they were looking for and might end up buying a different and often not-on-sale item.[8] But, if they know in advance that the chance of getting a deal is small, why do consumers go shopping anyway?

Interestingly, by exploiting the time inconsistency of the consumers' preferences, with a limited-availability strategy the seller is able to push the consumers' reservation utility below zero. This is possible because with expectations-based reference-dependent preferences, the consumers' participation constraint is belief-dependent — and therefore endogenous — and the seller can manipulate the consumers' beliefs with her own strategy.[9] The intuition is as follows: if a consumer wakes up believing he will find a product he likes available for a very low price, he will definitely plan to buy it. The attachment to the good induced by realizing that he will do so, however, changes his attitudes toward the purchasing decision. If the store runs out of the good on sale for a low price, but still has a similar one available for a higher price, the consumer must now choose between a loss of money from paying

the end of summer when most schools and colleges begin their school year; and (iii) the The Running of the Brides, which was a one-day sale of wedding gowns that used to take place in many Filene's Basement stores (in December 2011 Filene's Basement declared bankruptcy and went out of business). Moreover, many big national retailers, like Target and Toys R Us, have begun to hold Black Friday-style sales during the summer as well (see http://www.washingtonpost.com/wp-dyn/content/article/2010/07/22/AR2010072206101.html)

[7] The current FTC Guides Against Bait Advertising require retailers "to have available at all outlets listed in the advertisement a sufficient quantity of the advertised product to meet reasonably anticipated demands, unless the advertisement clearly and adequately discloses that supply is limited and/or the merchandise is available only at designated outlets" (16 C.F.R. Part 238.3).

[8] Empirical studies in marketing and psychology reveal indeed that consumers are likely to buy substitute items when their preferred product is out of stock, and even more so if the product they were planning to buy was on sale or if the seller had announced up-front that quantities were limited. I review the evidence about consumers' response to stockouts in Section 2.

[9] Spiegler (2012b) studies the problem of incentivizing participation for agents with expectations-based reference-dependent preferences in more general environments.

a higher price and a loss of consumption from returning home empty-handed. While, in equilibrium, buying the expensive substitute is indeed the best response to his expectations, it is still worse than if he could have avoided the feeling of loss by avoiding the expectation of getting the limited-availability bargain in the first place. More generally, because an expectations-based loss-averse consumer does not internalize the effect of his ex-post behavior on ex-ante expectations, the strategy that maximizes ex-ante expected utility is often not a credible plan. Moreover, consumers are hurt also by the uncertainty about which item they will get to consume and how much they will pay. Thus, despite the fact that, with some probability, they get a good deal, on average consumers are made worse off by this combination of limited availability, bargains, and rip-offs. Hence, the current FTC Guides Against Bait Advertising, by allowing stores to credibly announce that they have limited supplies for bargain items, might have the perverse effect of reducing consumers' welfare.

Despite the products being substitutes, loss aversion creates positive demand spillovers between products so that the higher a consumer's intrinsic valuation for a product, the higher his willingness to pay is for a substitute of that product as well. When the goods are vertically differentiated, the seller tends to use the more valuable item as the bargain. This may, at first, seem odd, given that consumers are (intrinsically) willing to pay a higher price for the superior good. Yet, exactly because consumers value the superior item more, the possibility of a bargain causes them to feel a larger loss, in terms of forgone consumption, when this item is not available; hence, they are willing to pay an even bigger premium to reduce the uncertainty over their consumption outcome, which, in turn, allows the seller to charge an even higher price for the rip-off. So my model predicts that more valuable items should be more likely to be used as baits, as in Example 2 above.

A related implication is that the monopolist, in order to effectively induce uncertainty into the consumers' purchasing plans, might introduce a less socially desirable or, worse, socially wasteful product and the profit-maximizing product line could differ from the socially optimal one.[10] Although this implication appears also in models of second-degree price discrimination via quality distortion (i.e., Deneckere and McAfee, 1996), the motive in this case is not to screen the consumers, but rather to exploit the aforementioned positive spillover effect by selling a less valuable product at a higher price.

Furthermore, the bargain item can be a "loss leader" (i.e., being priced below cost). Traditional models of consumer behavior in industrial organization can explain the use of loss leaders for complementary goods (see Ambrus and Weinstein, 2008); my model instead can rationalize the use of loss leaders for substitutes. With classically assumed reference-free preferences, the scope for using loss leaders is to increase store traffic; however, for this increase in store traffic to be profitable, consumers must buy other items *in addition to* the loss leader. In my model, instead, loss leaders lure consumers into the store, but their profitability stems from the fact that, if the seller has run out of the loss-leading product, consumers will buy another item *instead of* the loss leader in order to minimize their disappointment. Moreover, while traditional models — like the one of Lal and Matutes (1994) — suggest that products with lower reservation prices are more natural candidates to be loss leaders, my model

[10] Klemperer and Padilla (1997) obtain a similar result in an oligopoly model where consumers have classical preferences and multi-unit demand. For this environment they show that a firm might want to introduce an additional, socially wastful variety, because of a profitable business stealing effect.

can explain the use of highly valuable products as loss leaders.[11] This is consistent with the observation that, on Black Friday, Best Buy offers a below-cost large-screen flat TV to the first ten people who buy one.

My paper is related to, and builds upon, the analysis in Heidhues and Kőszegi (2012), which provides an explanation for why regular prices are sticky, but sales prices are variable, based on expectations-based loss aversion. In their model, a single-product monopolist maximizes profits by committing to a stochastic-price strategy made of low, variable sales prices and a high, sticky regular price. Their result and mine share a similar intuition: low prices work as baits to attract the consumers who, once in the store, are willing to pay a price even above their intrinsic valuation to avoid the loss resulting from going home empty-handed. The key-difference in my paper is that I consider a monopolist who sells two goods and uses one of them as a bait to attract the consumers and the other one to exploit them. My result on the optimality of limited-availability sales can be seen as a foundation as well as a more plausible re-interpretation of their result about the optimality of random-price sales.[12]

Katz and Nelson (1990) also study product availability and price dispersion for the case of a monopolist selling two substitutable goods to consumers with downward sloping and continuous multi-unit demand, who can choose whether to enter the market and have type-dependent outside options. They show that if the monopolist can credibly commit to have stockouts, there exists a two-price equilibrium in which the lower-price brand is understocked. However, they study only the case of perfect substitutes and their main result relies on the assumption that once a consumer enters the store, he forfeits his outside option and if faced with a stockout of the low-priced brand, he must buy the expensive one. In my model, instead, the consumers' behavior in the event of a stockout is not assumed, but it arises endogenously in equilibrium because consumers have expectations-based reference-dependent preferences and prefer to buy the expensive substitute instead of leaving the store empty-handed.

There are also a few papers focusing on the role of product availability as a strategic variable in various oligopoly settings (see Daughety and Reinganum, 1991; Chakravarty and Ghose, 1994; Balachander and Farquhar, 1994; Dana, 2001b; Watson, 2009). In these models, firms supply only one product and by competing (also) in availability, they are able to charge higher prices. However, how availability interplays with a firm's other strategic variables (quantity or price) varies considerably between the papers depending on the specific details of each model.

The remainder of this paper proceeds as follows. Section 2 briefly summarizes the key empirical evidence on sales and limited availability. Section 3 describes the baseline model with homogeneous consumers and the features of market demand when consumers have expectations-based reference-dependent preferences. Section 4 presents the main result about the seller's optimal pricing and availability with homogeneous consumers. Section 5 deals with three extensions of the baseline model: endogenous product lines, heterogeneous consumers' tastes and consumers' naïvete. Section 6 relates the paper to the literatures on firms' response to consumers' loss aversion, loss leaders, bait-and-switch, price discrimination, and other topics. Section 7 concludes by recapping the results of the model and pointing out some of its limitations as well as possible avenues for future research.

[11] Kamenica (2008) proposes a model of contextual inference from product lines where a firm may try to manipulate consumers' beliefs by introducing premium loss leaders — expensive goods of overly high quality that increase the demand for other goods.

[12] I discuss in more detail the similarities and differences with respect to Heidhues and Kőszegi (2012) in Section 7.

2 Evidence on Sales and Stock-Outs in Retailing

This section summarizes empirical evidence that points to three main facts: (1) sales are frequent but affect a small fraction of items, (2) products on sale are more likely to be out of stock and (3) consumers are willing to buy substitute products when their preferred item is sold out. These facts frame the importance and relevance of the analysis of this paper in understanding why and how retailers use limited-availability sales, and how consumers react when facing alternatives for a product that is sold out.

Sales, in the sense of periodic, temporary price reductions, are a ubiquitous feature of retail pricing (see Hosken and Reiffen, 2004a and Nakamura and Steinsson, 2008).[13] However, among all the items supermarkets and other retailers carry, usually only a small fraction each week are offered at a low sale and, within categories, retailers seem to systematically place some products on sale more often then others, with more popular items — those appealing to a wider range of customers — being more likely to go on sale (Hosken and Reiffen, 2004b). Relatedly, Nakamura (2008) finds that only a small fraction (19%) of price variation is common to all products in a category at a given retail store. According to a recent study by ShopAdvisor, a deferred shopping service used by independent websites and tablet magazines, in the 54 days from Nov. 1^{st} through Dec. 24^{th} 2011, the day with the lowest percentage (46%) of products on sale below their initial holiday season price was indeed Black Friday, Nov. 25^{th}. In fact, Black Friday is also the day on which shoppers begin to see prices spike on selected items: on Black Friday itself, 24% of the toys on ShopAdvisor's list were priced above their initial holiday season price.[14] Strausz (2007) reports that the largest German discounters, Aldi and Lidl, weekly advertise limited-availability bargain sales on products that do not belong to their usual selling stock. Chevalier, Kashyap and Rossi (2003) find that the majority of sales are not caused by changes in wholesale pricing, implying therefore that sales are primarily due to changes in retailers' margins. Similarly, Anderson, Nakamura, Simester and Steinsson (2012) report that while regular prices react strongly to costs and wholesale price movements, the frequency and depth of sales is largely unresponsive.

While not as ubiquitous as sales, stockouts are also prevalent in retailing. Gruen, Corsten and Braradwaj (2002) report an 8.3% out-of-stock rate worldwide, rising to even 25% for some promoted items. Hess and Gerstner (1987) sampled two general merchandise stores and found that stockouts occurred more often for products on sale than for similar products not on sale. Using data from a supermarket chain in Spain, Aguirregabiria (2005) documents a significant amount of heterogeneity across items in the frequency of stockouts; most of this heterogeneity is within-product (i.e., among brands of the same product line) and not among products. Grant-Worley, Saltford and Zick (1982) surveyed five major non-food chains in Syracuse, New York and found that the average rate of unavailability for advertised products was 12%. Similarly, Taylor and Fawcett (2001) investigated availability of advertised products for three large national mass merchants, four category killers involved in the office supplies and electronics subcategories and three retail grocers in the Mid-West, and found that the stock-out ratio for advertised items was twice as high as that of comparable, non-advertised items.

[13] Sales might also refer to systematic reductions in the price of fashion items; see Lazear (1986), Pashigian (1988) and Pashigian and Bowen (1991).

[14] See http://www.prnewswire.com/news-releases/shopadvisor-cautions-buyers-on-black-friday-not-worth-the-early-morning-rise-174618581.html

Bils (2004) presents evidence on temporary stockouts for durable consumer goods using data from the CPI *Commodities and Services Survey* and finds that from January 1988 to June 2004 the temporary stockout rate averaged between 8.8% and 9.2%.

Several marketing and psychology studies on consumers' response to product unavailability (Emmelhainz, Stock and Emmelhainz, 1991; Anupindi, Dada and Gupta, 1998; Verbeke, Farris and Thurik, 1998; Fitzsimons, 2000; Campo, Gijsbrechts and Nisol, 2000, 2003; Zinn and Liu, 2001) show that consumers are often willing to buy substitute items when faced with stockouts: depending on the specific characteristics of the product and store under study, the percentage of consumers who is willing to buy a substitute — within the same store — ranges from 30% to 80%. Through a post-purchase questionnaire, Zinn and Liu (2001) find also that consumers are more likely to leave a store empty-handed if they are surprised by the stockout; this finding suggests that prior expectations of product availability may be an important predictor of out-of-stock response. Relatedly, Anderson, Fitzsimons and Simester (2006) and Ozcan (2008) find that consumers are more willing to buy a substitute if the stockout product was on sale or if limited supplies were announced up-front. Conlon and Mortimer (2011) conducted a field experiment by exogenously removing top-selling products from a set of vending machines and tracking subsequent consumer responses. Their results show that most consumers purchase another good when a top-selling product is removed. Moreover, some product removals increase the vendor's profits as consumers substitute toward products with higher margins. Ozcan (2008) ran a survey study in a grocery store where the manager had previously agreed to create stockouts artificially by removing some items entirely from the shelves. Of all the consumers who replied to the survey saying that they had experienced a stockout, 11% said they cancelled or postponed the purchase, 49% decided to switch store (there are two other supermarkets within a 4 minute walking distance from the treated store), and 40% said they bought a substitute item for the one that was not available.[15]

3 Model

In this section, I first introduce the consumers' preferences and outline the timing of the interaction between the monopolist and the consumers. Then, I describe the consumers' strategies and illustrate the logic behind the solution concepts. I end this section with a simple example that shows how the monopolist can achieve higher profits by strategically manipulating product availability.

3.1 Environment

There is a unit mass of identical consumers whose intrinsic valuation for good i is v_i, $i = 1, 2$. Assume $v_1 \geq v_2 > 0$. The goods are substitutes and each consumer is interested in buying at most one unit of one good. The goods could be two different brands of a consumer durable, such as a household appliance.[16]

[15] Although product availability is probably more relevant for traditional brick and mortar stores than for online retailers, some more recent papers show that limited-availability sales and stockouts pertain to online shopping as well; see Breugelmans, Campo and Gijsbrechts, (2006), Jing and Lewis (2011) and Kim and Lennon (2011).

[16] Alternatively, this situation can be thought as one of vertical differentiation in which there are two versions of the same item, with good 2 being the "basic" version and good 1 being the "advanced" version. All consumers agree on the vertical ranking of the two goods.

Consumers have expectations-based reference-dependent preferences as formulated by Kőszegi and Rabin (2006). In this formulation, a consumer's (his) utility function has two components. First, when buying item i at price p_i, a consumer experiences *consumption utility* $v_i - p_i$. Consumption utility can be thought of as the classical notion of outcome-based utility. Second, a consumer also derives utility from the comparison of his actual consumption to a reference point given by his recent expectations (probabilistic beliefs).[17] For a riskless consumption outcome (v_i, p_i) and riskless expectations $(\widetilde{v}_i, \widetilde{p}_i)$, a consumer's total utility is given by

$$U\left[(v_i, p_i) \mid (\widetilde{v}_i, \widetilde{p}_i)\right] = v_i - p_i + \mu\left(v_i - \widetilde{v}_i\right) + \mu\left(\widetilde{p}_i - p_i\right) \tag{1}$$

where

$$\mu(x) = \begin{cases} \eta x & \text{if} \quad x \geq 0 \\ \eta \lambda x & \text{if} \quad x < 0 \end{cases}$$

is *gain-loss utility.*[18]

I assume $\eta > 0$ and $\lambda > 1$. By positing a constant marginal utility from gains and a constant, but larger marginal disutility from losses, this formulation captures prospect theory's (Kahneman and Tversky 1979, Tversky and Kahneman 1991) loss aversion, but without its diminishing sensitivity. The parameter η can be seen as the relative weight a consumer attaches to gain-loss utility, and λ can be seen as the coefficient of loss aversion.

According to (1), a consumer assesses gains and losses over product's quality and payment, separately. For instance, if his reference point is that he will not get the product (and thus pay nothing), then he evaluates getting the product and paying for it as a gain in the item dimension and a loss in the money dimension rather than as a single gain or loss depending on total consumption utility relative to his reference point. This feature of the Kőszegi-Rabin's model is what allows the monopolist to extract more than the consumer's intrinsic valuation for the good.[19] Furthermore, this is consistent with much of the experimental evidence commonly interpreted in terms of loss aversion.[20]

Because in many situations expectations are stochastic, Kőszegi and Rabin (2006) extend the utility function in (1) to allow for the reference point to be a pair of probability distribution $F = (F^v, F^p)$ over the two dimensions of consumption utility. In this case a consumer's total utility from the outcome

[17] Recent experimental evidence lends support to Kőszegi and Rabin's (2006, 2007) expectations-based model of reference-dependent preferences and loss aversion; see for instance Abeler, Falk, Goette, and Huffman (2011), Ericson and Fuster (2011), Gill and Prowse (2012), Karle, Kirchsteiger and Peitz (2012), Song (2012) and Sprenger (2011).

[18] The model of Kőszegi and Rabin (2006) assumes that the gain-loss utility function μ is the same across all dimensions. In principle, one could also allow for this function to differ across the item and the money dimension. For example, Novemsky and Kahneman (2005) and Kőszegi and Rabin (2009) argue that reference dependence and loss aversion are weaker in the money than in the item dimension.

[19] The other crucial feature of these preferences, which is that the reference point is determined by the decision maker's forward-looking expectations, is implicit in disappointment-aversion models of Bell (1985), Loomes and Sugden (1986), and Gul (1991). However, because in these models gains and losses are assessed along only one dimension, the consumer's intrinsic utility ($v_i - p_i$, in this paper), they are unable to predict the type of pricing schemes that is the subject of this paper.

[20] This feauture is able to predict the endowment effect observed in many laboratory experiments (see Kahneman, Knetsch, and Thaler 1990, 1991). The common explanation of the endowment effect is that owners feel giving up the object as a painful loss that counts more than money they receive in exchange, so that they demand a lot of money for the object. But if gains and losses were defined over the value of the entire transaction, owners would not be more sensitive to giving up the object than to receiving money in exchange. Heffetz and List (2011), however, find no evidence that expectations alone play a part in the endowment effect.

(v_i, p_i) can be written as

$$U\left[(v_i, p_i) \,|\, (F^v, F^p)\right] = v_i - p_i + \int_{\widetilde{v}_i} \mu\left(v_i - \widetilde{v}_i\right) dF^v\left(\widetilde{v}_i\right) + \int_{\widetilde{p}_i} \mu\left(\widetilde{p}_i - p_i\right) dF^p\left(\widetilde{p}_i\right) \tag{2}$$

In words, when evaluating (v_i, p_i) a consumer compares it to each possible outcome in the reference lottery. For example, if he had been expecting to buy good 1 for \$15, then buying good 2 for \$10 feels like a loss of $v_1 - v_2$ on the quality dimension and a gain of \$5 on the money dimension.[21] Similarly, if a consumer had been expecting to buy good 1 for either \$10 or \$20, then paying \$15 for it feels like a loss of \$5 relative to the possibility of paying \$10, and like a gain of \$5 relative to the possibility of paying \$20. In addition, the weight on the loss (gain) in the overall experience is equal to the probability with which he had been expecting to pay \$10 (\$20).

To complete this theory of consumer behavior with the above belief-dependent preferences, Kőszegi and Rabin (2006) assume that beliefs must be consistent with rationality: a consumer correctly anticipates the implications of his plans, and makes the best plan he knows he will carry through. Notice that any plan of behavior — which in my setting amounts simply to a price-contingent strategy of which item to buy — induces some expectations. If, given these expectations, the consumer is not willing to follow the plan, then he could not have rationally formulated the plan in the first place. Hence, a credible plan must have the property that it is optimal given the expectations it generates. Following the original definitions in Kőszegi and Rabin (2006) and Kőszegi (2010), I call such a credible plan a *personal equilibrium* (PE). If there exist multiple credible plans, a rational consumer chooses the one that maximizes his expected utility from an ex-ante perspective. I call such a favorite credible plan a *preferred personal equilibrium* (PPE).[22]

The seller (she) is a monopolist supplying good 1 and good 2 at a unit cost of $c_1 \geq 0$ and $c_2 \geq 0$, respectively (these could be the wholesale prices). The seller does not experience economies of scale or scope in supplying these goods. For $i = 1, 2$, let q_i denote the amount or degree of availability of good i offered by the monopolist. If $q_i < 1$, then good i is subject to "limited availability" so that only a fraction q_i of the consumers can purchase it. I assume that, in the event of a stockout, rationing is proportional: each consumer has the same ex-ante probability of obtaining the good, which is allocated to consumers on a random first-come, first-serve basis.[23]

The interaction between the monopolist and the consumers lasts two periods, 0 and 1. In period 0, the seller announces (and commits to) a price pair $(p_1, p_2) \in \mathbb{R}_+^2$ and a quantity pair $(q_1, q_2) \in [0, 1]^2$; after observing the seller's choice of quantities and prices, consumers pick the plan that is consistent and that maximizes their expected utility (PPE). I assume also that consumers cannot commit ex-ante not to go to the store. In period 1, consumers execute their purchasing plans and payments are made. The assumption about the seller announcing both prices in period 0 is not very realistic because while stores frequently advertise their good deals, it is rather uncommon to see a store publicizing its high

[21] Therefore, the two goods are substitutes not only in the usual sense, but also in the sense of being evaluated along the same hedonic dimension.

[22] In the simple environment considered in this paper, a PPE always exists and is generically unique. Kőszegi (2010) discusses conditions for existence and uniqueness of PPE in more general environments.

[23] Gilbert and Klemperer (2000) show that rationing can be a profitable strategy if consumers must make sunk investements to enter the market, and Nocke and Peitz (2007) show that rationing across periods can be profitable in a model of intertemporal monopoly pricing under demand uncertainty.

prices. However, in Appendix C I show that the main results of the paper are unchanged if the seller commits only to the price and availability of the bargain.[24] Finally, I assume that when indifferent between a plan that involves buying and another plan that involves not buying, consumers always break the indifference in favor of the first of these plans.

3.2 Consumers' Demand

Let $H \in \Delta\left([0,1]^2 \times \mathbb{R}_+^2\right)$ denote a consumer's expectations, induced by the seller's strategy, about the degree of availability and the prices he might face. For a given seller's choice of prices and degree of availability, a consumer chooses among five possible plans: (i) "never buy," (ii) "buy item 1 if available and don't buy otherwise," (iii) "buy item 2 if available and don't buy otherwise," (iv) "buy item 1 if available and otherwise buy item 2 if available" and (v) "buy item 2 if available and otherwise buy item 1 if available."[25] Let $\sigma \in \{\{\varnothing\}, \{1,\varnothing\}, \{2,\varnothing\}, \{1,2\}, \{2,1\}\}$ denote a consumer's plan and let $\Gamma_{H,\sigma}$ denote the distribution over final consumption outcomes induced jointly by H and σ. In a personal equilibrium the behavior generating expectations must be optimal given the expectations:

Definition 1 *σ is a Personal Equilibrium (PE) if*

$$U\left[\sigma|\Gamma_{H,\sigma}\right] \geq U\left[\sigma'|\Gamma_{H,\sigma}\right]$$

for any $\sigma' \neq \sigma$.

Utility maximization in period 0 implies that the consumer chooses the PE plan that maximizes his expected utility:

Definition 2 *σ is a Preferred Personal Equilibrium (PPE) if it is a PE and*

$$EU_{\Gamma_{H,\sigma}}\left[\sigma|\Gamma_{H,\sigma}\right] \geq EU_{\Gamma_{H,\sigma'}}\left[\sigma'|\Gamma_{H,\sigma'}\right]$$

for any σ' such that σ' is a PE.

In the remainder of this section, I analyze the conditions for when plans (i), (ii) and (iv) constitute a PE or a PPE.[26] This allows me to both illustrate the logic of PE and PPE, as well as to start developing the intuition for my main result on the optimality of limited-availability schemes. Specifically, a central element of the seller's strategy is to make sure that plan (i) is not a PE and I start by analyzing conditions for this.

Conditions for plan (i) to be a PE For never buying, to be a PE, the consumer must expect never to buy. Suppose a buyer enters the store with the expectation of not buying; in this case his

[24] Since consumers have rational expectations, they would correctly infer the price of the rip-off even if it was not publicly advertised.

[25] Mixing between plans on the consumers' side can easily be ruled out by the fact that the seller would never choose a price-pair inducing a buyer to buy with probability less than 1.

[26] The relevant conditions for plans (iii) and (v) are analogous to the ones for plans (ii) and (iv), respectively; hence, I do not show them here. In appendix A, I thoroughly derive the conditions for when each plan constitutes a PE.

reference point is to consume nothing and pay nothing. Let the price of good 1 be p_1 and suppose the consumer sticks to his plan. Then, his overall utility is

$$U\left[(0,0)\,|\,\{\varnothing\}\right] = 0.$$

What if instead the consumer decides to deviate from his plan and buys item 1? In this case his overall utility is

$$U\left[(v_1,p_1)\,|\,\{\varnothing\}\right] = v_1 - p_1 + \eta v_1 - \eta\lambda p_1,$$

where $v_1 - p_1$ is his intrinsic consumption utility from buying item 1 at price p_1, ηv_1 is the gain he feels from consuming item 1 when he was expecting to consume nothing, and $-\eta\lambda p_1$ captures the loss he feels from paying p_1 when he was expecting to pay nothing. Thus, the consumer will not deviate in this way from the plan to never buy if

$$U\left[(0,0)\,|\,\{\varnothing\}\right] > U\left[(v_1,p_1)\,|\,\{\varnothing\}\right] \Leftrightarrow \; p_1 > \frac{1+\eta}{1+\eta\lambda} v_1.$$

A similar threshold can be derived for the case in which the consumer considers deviating from his original plan and buy item 2 at price p_2. Therefore, the plan to never buy is a PE if and only if $p_1 > \frac{1+\eta}{1+\eta\lambda} v_1 \equiv p_1^{\min}$ and $p_2 > \frac{1+\eta}{1+\eta\lambda} v_2 = p_2^{\min}$ because otherwise the consumers would not follow through their intended plan of not buying. The expected utility associated with the plan to never buy is

$$EU\left[\{\varnothing\}\,|\,\{\varnothing\}\right] = 0$$

as the expected utility from planning to consume nothing and pay nothing and expecting to follow this plan is of course zero.

Therefore, if either $p_1 \leq p_1^{\min}$ or $p_2 \leq p_2^{\min}$ plan (i) cannot be a PE and consumers must select a plan that involves buying at least one item with positive probability. As I will show in the next section, it turns out that (unsurprisingly) it is optimal for the seller to induce consumers to select plan (iv) and thus to expect to never leave the store empty-handed whenever an item is available; however, (less obviously) it is not optimal for that to be the only PE plan. Hence, the seller would like the consumer to prefer plan (iv) over plan (ii) ex-ante.

Conditions for plan (ii) to be a PE Suppose a buyer enters the store expecting to buy item 1 if available and not to buy otherwise. In this case his reference point on the product dimension is to enjoy v_1 with probability q_1 and to consume nothing with probability $1 - q_1$; similarly, on the price dimension he expects to pay p_1 with probability q_1 and to pay nothing with probability $1 - q_1$. If the consumer follows this plan his realized utility if item 1 is indeed available is

$$U\left[(v_1,p_1)\,|\,\{1,\varnothing\}\right] = v_1 - p_1 + \eta\left(1-q_1\right)v_1 - \eta\lambda\left(1-q_1\right)p_1,$$

where $v_1 - p_1$ is his intrinsic consumption utility from buying item 1 at price p_1, $\eta\left(1-q_1\right)v_1$ is the gain he feels from consuming item 1 when he was expecting to consume nothing with probability $1-q_1$, and $-\eta\lambda\left(1-q_1\right)p_1$ is the loss he feels from paying p_1 when he was expecting to pay nothing with probability $1-q_1$. Suppose that item 1 is available but the buyer instead deviates and does not buy.

In this case his overall utility is

$$U\left[(0,0)\,|\,\{1,\varnothing\}\right] = 0 - \eta\lambda q_1 v_1 + \eta q_1 p_1,$$

where 0 is his intrinsic consumption utility, $-\eta\lambda q_1 v_1$ is the loss he feels from consuming nothing when he was expecting to consume item 1 with probability q_1, and $\eta q_1 p_1$ is the gain from paying nothing instead of p_1 which he was expecting to pay with probability q_1. Thus, the consumer will not deviate in this way from his plan if

$$U\left[(v_1,p_1)\,|\,\{1,\varnothing\}\right] \geq U\left[(0,0)\,|\,\{1,\varnothing\}\right] \Leftrightarrow \; p_1 \leq \frac{1+\eta\left(1-q_1\right)+\eta\lambda q_1}{1+\eta q_1+\eta\lambda\left(1-q_1\right)} v_1. \tag{3}$$

Next, consider the case in which item 1 is not available. If the buyer follows his plan, his overall utility is $U\left[(0,0)\,|\,\{1,\varnothing\}\right]$. If instead he deviates and buys item 2, for $p_1 \geq p_2$ his overall utility is

$$U\left[(v_2,p_2)\,|\,\{1,\varnothing\}\right] = v_2 - p_2 + \eta\left(1-q_1\right)v_2 - \eta\lambda q_1\left(v_1-v_2\right) + \eta q_1\left(p_1-p_2\right) - \eta\lambda\left(1-q_1\right)p_2,$$

where $v_2 - p_2$ is the intrinsic consumption utility from buying item 2 at price p_2, $\eta\left(1-q_1\right)v_2$ is the gain he feels from consuming item 2 compared to the expectation of consuming nothing with probability $\left(1-q_1\right)$, $-\eta\lambda q_1\left(v_1-v_2\right)$ is the loss he feels from consuming item 2 instead of item 1 when he was expecting to consume item 1 with probability q_1 (recall that $v_1 \geq v_2$), $\eta q_1\left(p_1-p_2\right)$ is the gain from paying p_2 instead of p_1 which he was expecting to pay with probability q_1, and $-\eta\lambda\left(1-q_1\right)p_2$ is the loss from paying p_2 when he was expecting to pay nothing with probability $1-q_1$. Thus, the consumer will not deviate in this way from his plan if

$$U\left[(0,0)\,|\,\{1,\varnothing\}\right] > U\left[(v_2,p_2)\,|\,\{1,\varnothing\}\right] \Leftrightarrow \; p_2 > \frac{1+\eta\left(1-q_1\right)+\eta\lambda q_1}{1+\eta q_1+\eta\lambda\left(1-q_1\right)} v_2. \tag{4}$$

Notice that conditions (3) and (4) together imply that $U\left[(v_1,p_1)\,|\,\{1,\varnothing\}\right] > U\left[(v_2,p_2)\,|\,\{1,\varnothing\}\right]$, so that there is no need to check that a consumer does not want to deviate and buy item 2 when item 1 is available. Therefore, for $p_1 \geq p_2$, $\{1,\varnothing\}$ is a PE if and only if $p_2 > \frac{1+\eta(1-q)+\eta\lambda q}{1+\eta q+\eta\lambda(1-q)} v_2$ and $p_1 \leq \frac{1+\eta(1-q)+\eta\lambda q}{1+\eta q+\eta\lambda(1-q)} v_1$. Similarly, for $p_1 < p_2$, $\{1,\varnothing\}$ is a PE if and only if $p_1 < \frac{1+\eta(1-q_1)+\eta\lambda q_1}{1+\eta q_1+\eta\lambda(1-q_1)} v_1$ and $p_2 > \frac{v_2[1+\eta(1-q_1)+\eta\lambda q_1]+q_1\eta(\lambda-1)p_1}{1+\eta\lambda}$. The expected utility associated with this plan is

$$EU\left[\{1,\varnothing\}\,|\,\{1,\varnothing\}\right] = q_1\left(v_1-p_1\right) - q_1\left(1-q_1\right)\eta\left(\lambda-1\right)\left(v_1+p_1\right). \tag{5}$$

The first term in (5), $q_1\left(v_1-p_1\right)$, is standard expected consumption utility. The second term, $-q_1\left(1-q_1\right)\eta\left(\lambda-1\right)\left(v_1+p_1\right)$, is expected gain-loss utility and it is derived as follows. On the product dimension, the consumer compares the outcome in which with probability q_1 he consumes item 1 and enjoys v_1 with the outcome in which with probability $1-q_1$ he does not consume and gets 0. Similarly, on the price dimension he compares paying price p_1 with probability q_1 with paying 0 with probability $1-q_1$. Notice that the expected gain-loss utility is always negative as, since $\lambda > 1$, losses are felt more heavily than equal-size gains. Also, notice that uncertainty in the product and uncertainty in money are "added up" so that the expected gain-loss term is proportional to $v_1 + p_1$.

Conditions for Plan (iv) to be a PE For the plan to buy item 1 if available and otherwise buy item 2, a consumer's reference point in the product dimension is to consume item 1 and enjoy v_1 with probability q_1, to consume item 2 and enjoy v_2 with probability q_2 and to consume nothing with probability $1-q_1-q_2$; similarly, in the price dimension, a consumer expects to pay p_1 with probability q_1, p_2 with probability q_2 and to pay nothing with probability $1-q_1-q_2$. Then, if he follows his plan and buys item 1, for $p_1 \geq p_2$, his realized utility is

$$U\left[(v_1,p_1)\,|\,\{1,2\}\right] = v_1 - p_1 + \eta q_2\left(v_1-v_2\right) + \eta\left(1-q_1-q_2\right)v_1 - \eta\lambda q_2\left(p_1-p_2\right) - \eta\lambda\left(1-q_1-q_2\right)p_1.$$

If instead he deviates and buys item 2, his utility is

$$U\left[(v_2,p_2)\,|\,\{1,2\}\right] = v_2 - p_2 - \eta\lambda q_1\left(v_1-v_2\right) + \eta\left(1-q_1-q_2\right)v_2 + \eta q_1\left(p_1-p_2\right) - \eta\lambda\left(1-q_1-q_2\right)p_2.$$

Thus, the consumer will not deviate in this way from his plan if

$$U\left[(v_1,p_1)\,|\,\{1,2\}\right] \geq U\left[(v_2,p_2)\,|\,\{1,2\}\right] \Leftrightarrow\; p_1 \leq p_2 + \frac{1+\eta\left(1-q_1\right)+\eta\lambda q_1}{1+\eta q_1+\eta\lambda\left(1-q_1\right)}\left(v_1-v_2\right). \tag{6}$$

Suppose now that once a consumer arrives at the store, item 2 is everything that is left. If he follows his plan and buys item 2 his overall utility is $U\left[(v_2,p_2)\,|\,\{1,2\}\right]$. If instead he deviates and does not buy his utility is

$$U\left[(0,0)\,|\,\{1,2\}\right] = 0 - \eta\lambda q_1 v_1 - \eta\lambda q_2 v_2 + \eta q_1 p_1 + \eta q_2 p_2.$$

Thus, the consumer will not deviate in this way from his plan if

$$U\left[(v_2,p_2)\,|\,\{1,2\}\right] \geq U\left[(0,0)\,|\,\{1,2\}\right] \Leftrightarrow\; p_2 \leq \frac{1+\eta\lambda\left(q_1+q_2\right)+\eta\left(1-q_1-q_2\right)}{1+\eta\left(q_1+q_2\right)+\eta\lambda\left(1-q_1-q_2\right)}v_2. \tag{7}$$

Notice that conditions (6) and (7) together imply that $U\left[(v_1,p_1)\,|\,\{1,2\}\right] > U\left[(0,0)\,|\,\{1,2\}\right]$. Hence, for $p_1 \geq p_2$, $\{1,2\}$ is a PE if and only if $p_1 \leq p_2 + \frac{1+\eta(1-q_1)+\eta\lambda q_1}{1+\eta q_1+\eta\lambda(1-q_1)}\left(v_1-v_2\right)$ and $p_2 \leq \frac{1+\eta\lambda(q_1+q_2)+\eta(1-q_1-q_2)}{1+\eta(q_1+q_2)+\eta\lambda(1-q_1-q_2)}v_2$. Similarly, for $p_1 < p_2$, $\{1,2\}$ is a PE if and only if $p_2 \leq \frac{1+\eta\lambda(q_1+q_2)+\eta(1-q_1-q_2)}{1+\eta q_2+\eta\lambda(1-q_2)}v_2 + \frac{\eta(\lambda-1)q_1}{1+\eta q_2+\eta\lambda(1-q_2)}p_1$. The expected utility associated with this plan is

$$\begin{aligned} EU\left[\{1,2\}\,|\,\{1,2\}\right] \;=\;& q_1\left(v_1-p_1\right)+q_2\left(v_2-p_2\right) \\ & -q_1\left(1-q_1-q_2\right)\eta\left(\lambda-1\right)\left(v_1+p_1\right) \\ & -q_2\left(1-q_1-q_2\right)\eta\left(\lambda-1\right)\left(v_2+p_2\right) \\ & -q_1q_2\eta\left(\lambda-1\right)\left(v_1-v_2\right) \\ & -q_1q_2\eta\left(\lambda-1\right)\left(\max\{p_1,p_2\}-\min\{p_1,p_2\}\right). \end{aligned} \tag{8}$$

The first and second terms in (8), $q_1\left(v_1-p_1\right)+q_2\left(v_2-p_2\right)$, are the standard expected consumption utility terms. The third term, $q_1\left(1-q_1-q_2\right)\eta\left(\lambda-1\right)\left(v_1+p_1\right)$, is always negative and captures expected gain-loss utility in both the product and the money dimensions from comparing the outcome in which the consumer buys item 1 and pays p_1 with the outcome of returning home empty-handed. Similarly, the fourth term captures expected gain-loss utility in both dimensions from comparing the outcome of buying item 2 at price p_2 with the outcome of returning home empty-handed. The fifth term, $-q_1 q_2 \eta (\lambda - 1)(v_1 - v_2)$, captures expected gain-loss utility in the consumption dimension when comparing the two outcomes in which he buys something: with probability q_1 the consumer expects to buy good 1 and with probability q_2 he expects to buy good 2. Notice again that this term is negative, but it is proportional to $(v_1 - v_2)$. This is because with this plan, the consumer is "guaranteeing" himself to enjoy at least the item he values v_2 and the expected gain-loss utility is therefore related only to by how much more he would prefer to consume the other good (or, the degree of substitutability between the two goods). The sixth term, $-q_1 q_2 \eta (\lambda - 1)(\max\{p_1, p_2\} - \min\{p_1, p_2\})$, captures expected gain-loss utility in the money dimension when comparing the two outcomes in which he buys and can be explained in a similar fashion.

Conditions for Plan (iv) to be the PPE Suppose that $p_1 > p_2$. When both plan (ii) and (iv) are Personal Equilibria, a consumer will select plan (iv) rather than plan (ii) if and only if

$$EU[\{1,2\} \mid \{1,2\}] \geq EU[\{1,\varnothing\} \mid \{1,\varnothing\}] \Leftrightarrow v_2 - p_2 \geq \eta (\lambda - 1)(1 - 2q_1 - q_2)(v_2 + p_2). \tag{9}$$

Notice, crucially, that condition (9) might hold even if $p_2 > v_2$, provided that $q_1 > \frac{1-q_2}{2}$. Therefore, a consumer might prefer, from an ex-ante perspective, to plan to always buy even if $p_2 > v_2$. This happens because, by planning to always buy, the consumer is essentially insuring himself against extreme fluctuations in his consumption outcome.[27]

3.3 An Illustrative Example

Consider a monopolist supplying two goods, 1 and 2, to a unit mass of consumers who have expectations-based reference-dependent preferences with $\eta = 1$ and $\lambda = 3$. Let $v_1 = v$, $v_2 = \frac{2}{3}v$, $c_1 = \frac{3}{5}v$ and $c_2 = \frac{v}{3}$. If she had to provide full availability, the seller would supply only item 1 and price it at v, obtaining a profit of $\frac{2}{5}v$.

Consider instead the following limited-availability scheme: $q_1 = \frac{1}{4}$, $q_2 = \frac{3}{4}$, $p_1 = \frac{v}{2}$ and $p_2 = v$. Since $p_1 < p_1^{\min}$, it is not a PE for consumers to never buy: the price of item 1 is so low that if consumers had planned not to buy it, then if item 1 is indeed available, they would like to surprise themselves and buy it, and since the price is very low, the gain on the item dimension more than outweighs the loss on the money dimension.

The plan to buy item 1 if available and nothing otherwise is a PE because $p_1 < \frac{5}{7}v_1$ and $p_2 > \frac{5v_2+p_1}{8}$. Intuitively, if consumers enter the store with the expectation of consuming item 1 with positive probability and item 1 is available, they are willing to follow their plan since the price of item 1 is relatively low compared to its intrinsic value; however, they are not willing to buy item 2 if they were not expecting to do so, since the price of item 2 is relatively high compared to its intrinsic value.

Similarly, the plan to buy item 1 if available and item 2 otherwise is a PE because $p_2 < \frac{8v_2+p_1}{5}$. The intuition is that, by planning to always buy something, consumers expect to enjoy at least v_2 for sure; because of this attachment effect, therefore, they are willing to buy item 2 if they were expecting

[27] More generally, as shown in Kőszegi and Rabin (2007), a decisionmaker with expectations-based loss aversion dislikes uncertainty in consumption utility because he dislikes the possibility of a resulting loss more than he likes the possibility of a resulting gain.

to do so even if its price is relatively high. Furthermore, this plan is the PPE since

$$\begin{aligned} EU\left[\{1,2\}\mid\{1,2\}\right] &= \frac{1}{4}\left(v-\frac{v}{2}\right)+\frac{3}{4}\left(\frac{2}{3}v-v\right)-\frac{9}{16}\left(\frac{v}{3}+\frac{v}{2}\right) \\ &> \frac{1}{4}\left(v-\frac{v}{2}\right)-\frac{9}{16}\left(v+\frac{v}{2}\right)=EU\left[\{1,\varnothing\}\mid\{1,\varnothing\}\right]. \end{aligned}$$

The reason why, from an ex-ante point of view, consumers prefer the plan to always buy is that this plan reduces the magnitude of the fluctuations of their consumption outcomes and, therefore, makes them subject to a smaller expected gain-loss disutility. Finally, notice that with this limited-availability scheme the seller's profit equals $\frac{19}{40}v$, which is higher than the profit under full availability.

This example illustrates already many of the key insights of the general model. First, with a limited-availability scheme the seller is able to obtain a higher profit than what she can obtain with perfect availability. The prices of the bargain and the rip-off are chosen by the seller in a way such that (i) not buying is not a PE for the consumers and (ii) planning to always buy is the consumers' PPE. Furthermore, the superior item is chosen as the bargain and it is priced below its marginal cost. The purpose of the next section is to formalize and generalize these insights.

4 Optimal Availability and Pricing

For given prices (p_1, p_2) and "quantities" (q_1, q_2), the monopolist's profit is

$$\pi(p_1, p_2, q; c_1, c_2) = q_1(p_1 - c_1) + q_2(p_2 - c_2).$$

If consumers were not loss-averse, the profit-maximizing strategy for the seller would be to just set $p_i = v_i$, for $i = 1, 2$, and $q_1 = 1$ (resp. $q_2 = 1$) if $v_1 - c_1 \geq v_2 - c_2$ (resp. if $v_1 - c_1 < v_2 - c_2$). Consumers would get zero surplus and the seller's profit would be exactly $v_1 - c_1$ (resp. $v_2 - c_2$).

The first lemma of this section shows that with loss-averse consumers, if restricted to supply one good with certainty, the above mentioned strategy remains the monopolist's profit-maximizing one.[28]

Lemma 1 *With perfect availability the monopolist cannot extract more than v_1 from the consumers.*

In general, however, this strategy need not be the profit-maximizing one when consumers are loss-averse as the seller instead can achieve a higher profit by reducing the availability of some goods and thus inducing uncertainty into the buyers' plans.

The next lemma states that even though she might reduce the degree of availability of some goods, it is in the seller's best interest that all consumers get to buy a good for sure, and the uncertainty is only about which good they will buy.[29] The intuition for this result relies on the seller's intent to mitigate the "comparison effect" and simultaneously magnify the "attachment effect" for the consumers (Kőszegi and Rabin, 2006). An increase in the likelihood of buying increases a consumer's sense of loss if he does not buy, creating an "attachment effect" that increases his willingness to pay. On the other

[28] All proofs are relegated to Appendix B.

[29] A similar result is provided by Pavlov (2011) and Balestrieri and Leao (2011) for the case of a monopolist selling substitutes to risk-neutral consumers.

hand, for a fixed probability of buying, a decrease in the price a consumer expects to pay makes paying a higher price feel like more of a loss, creating a "comparison effect" that lowers his willingness to pay the high price.

Lemma 2 *The market is fully covered:* $q_1 + q_2 = 1$*.*

With $q_1+q_2 = 1$, if a consumer plans to always buy, he is guaranteed to get at least the less preferred item (v_2) and thus he is not exposed anymore to the possibility of returning home empty-handed; this increases the consumer's willingness to pay through the attachment effect. At the same time, because the possibility of buying nothing has disappeared, the consumer expects to always spend some money; this also increases the consumer's willingness to pay through reducing the comparison effect.

Given Lemma 2, from this point forward I am going to use q and $1-q$ to denote the quantities of good 1 and 2, respectively. The lemma below shows that with limited availability, the monopolist must offer at least one good at a discounted price.

Lemma 3 *If* $q \in (0,1)$ *then either* $p_1 < v_1$ *or* $p_2 < v_2$*.*

With limited availability, a consumer faces uncertainty about his consumption outcome before arriving at the store and because losses are felt more heavily than gains, if he expects to buy with positive probability, his expected gain-loss utility is negative. Therefore, for a consumer to be willing to plan to buy, the seller must guarantee him a strictly positive intrinsic surplus on at least one item, otherwise he would be better off by planning to not buy and this plan would be consistent for $p_1 \geq v_1$ and $p_2 \geq v_2$.

Having established that the monopolist can sell a strictly positive quantity of both goods only if one of them is priced at a discount, the next question is how big this discount must be. The next lemma states that the seller must offer a bargain on this good; in other words, its price must be so low that it is not credible for consumers to plan on not buying.

Lemma 4 *If* $q \in (0,1)$ *the seller chooses prices such that the plan to never buy is not a PE.*

Since, for a given product i, the highest price the seller can charge to make not buying a non credible plan is $p_i^{\min} \equiv \frac{1+\eta}{1+\eta\lambda} v_i$, then it must be that if the seller is producing both goods in strictly positive quantity, one of them is priced at this "forcing price."[30]

What about the price of the other item? If she produces a strictly positive quantity of both goods, the seller wants the buyers to plan to always buy. However, as the lemma below shows, it is not optimal for the seller to choose the other price such that always buying is the unique consistent plan. Instead, the optimal price pair is such that consumers are indifferent, ex-ante, between the plan of always buying and the plan of buying only the discounted item.

Lemma 5 *For* $q \in (0,1)$*, if the seller uses item 2 as the bargain (i.e.,* $p_2 = p_2^{\min}$*), then the optimal price for item 1 is*

$$p_1^* = v_1 + \frac{2(1-q)\eta(\lambda-1)\left[v_2(2+\eta+\eta\lambda) - v_1(1+\eta\lambda)\right]}{(1+\eta\lambda)\left[1+\eta(\lambda-1)(1-q)\right]} > v_1.$$

[30] This result is akin to the single-product one in Heidhues and Kőszegi (2012) from whom I borrowed the term "forcing price."

If instead she uses item 1 as the bargain (i.e., $p_1 = p_1^{\min}$), then the optimal price for item 2 is

$$p_2^* = v_2 + \frac{2qv_1\eta(\lambda-1)(1+\eta)}{(1+\eta\lambda)[1+\eta(\lambda-1)q]} > v_2.$$

This last lemma implies that consumers are willing to pay a premium, in the form of a higher price on the item that is not on sale (and therefore in the form of a higher expected expenditure), to avoid ex-ante the disappointment of leaving the store empty-handed. Furthermore, p_i^* is the highest price such that consumers (weakly) prefer, from an ex-ante point of view, the plan of buying item j if available and item i otherwise to the plan of buying item j if available and nothing otherwise, when item j is sold at its "forcing price." To gain intuition on why a consumer might find it optimal to plan to buy at $p_i^* > v_i$, suppose the seller uses item 1 as the bargain, by pricing it at $p_1^{\min}$. If a consumer plans to buy only item 1 and nothing otherwise his expected utility is equal to

$$q\left(v_1 - p_1^{\min}\right) - \eta(\lambda-1)q(1-q)\left(v_1 + p_1^{\min}\right).$$

While the term relating to consumption utility in the above expression is strictly positive, the expected gain-loss utility term is strictly negative. If instead the consumer plans to always buy, then his expected utility is

$$q\left(v_1 - p_1^{\min}\right) + (1-q)(v_2 - p_2^*) - \eta(\lambda-1)q(1-q)\left(v_1 - v_2 + p_2^* - p_1^{\min}\right).$$

In the above expression the expected gain-loss utility is still negative, but now its magnitude is $(v_1 - v_2 + p_2^* - p_1^{\min})$. Therefore, as long as $p_2^* - v_2 < 2p_1^{\min}$, by planning to always buy a consumer is subject to a smaller expected gain-loss disutility and this allows the seller to raise p_2^* above v_2. Furthermore, the closer v_2 is to v_1, the more freedom the seller has in raising p_2^*, implying that dispersion in prices and dispersion in valuations are inversely related.

Both rip-off prices p_1^* and p_2^* are increasing in the degree of availability of their respective bargain item — $1-q$ and q — implying that the attachment effect (see Kőszegi and Rabin, 2006 and Heidhues and Kőszegi, 2012) carries over to the case of multiple goods evaluated along the same hedonic dimension.

Similarly, notice that $\frac{\partial p_i^*}{\partial v_j} > 0$, for $i, j = 1, 2$, $i \neq j$. Thus, expectations-based loss aversion produces a kind of positive demand spillover across products, despite these being substitutes. Indeed, both p_1^* and p_2^* are written as the sum of two components: the direct effect, which simply equals the consumers' intrinsic valuation for the product, and the spillover effect due to loss aversion. Notice that while the spillover effect for p_2^* depends only on v_1 and is increasing in it, the spillover effect for p_1^* depends both on v_1 and v_2 and is increasing in the former and decreasing in the latter. Intuitively, increasing consumers' intrinsic value for item 1 makes item 2 is not such a good substitute for it. This, however, does not affect p_2^* because when item 1 is the bargain, a higher v_1 increases consumers' expected gain-loss disutility when planning to buy only the bargain and when planning to always buy by the same amount.

Having derived the optimal prices for the bargain and the rip-off, the next step for the seller is to choose the optimal degree of availability for each item. For example, consider the case in which the seller uses item 2 as the bargain. Then, she is going to choose the q that solves the following maximization problem:

$$\max_q q\left(p_1^* - c_1\right) + (1-q)\left(p_2^{\min} - c_2\right).$$

The first-order condition yields

$$p_1^* - c_1 - \left(p_2^{\min} - c_2\right) + q\frac{\partial p_1^*}{\partial q} = 0. \tag{10}$$

Notice that $q\frac{\partial p_1^*}{\partial q} < 0$ because of the attachment effect: the higher the degree of availability of the bargain, the more optimistic the consumers' beliefs about making a deal. This in turn, allows the seller to charge a higher mark-up on the rip-off. On the other hand, a greater availability of the bargain necessarily means fewer sales of the rip-off and hence reduces the seller's profits, as captured by $p_1^* - c_1 - \left(p_2^{\min} - c_2\right) > 0$. At the optimal degree of availability these two effects offset each other.

Lemma 6 *If the seller uses item 2 as the bargain, the optimal degree of availability of item 1 is* $\overline{q} = \arg\max_q \pi\left(p_1^*, p_2^{\min}, q; c_1, c_2\right)$, *with* $\overline{q} \in \left(\frac{1}{2}, 1\right)$. *If instead she uses item 1 as the bargain, the optimal degree of availability of item 1 is* $\underline{q} = \arg\max_q \pi\left(p_1^{\min}, p_2^*, q; c_1, c_2\right)$, *and* $\underline{q} \in \left(0, \frac{1}{2}\right)$ *if* $v_2 - c_2 \geq v_1 - c_1$ *or if* $v_2 - c_2 < v_1 - c_1$ *and* $\eta \leq 1$. *Furthermore,* $\overline{q} > 1 - \underline{q}$.

When the bargain is the product with the lower social surplus, the seller always supplies more units of the rip-off item than the bargain. So, even if a high degree of availability for the bargain allows her, via the attachment effect, to increase the price of the rip-off, the effect is not strong enough for the seller to be willing to sell the bargain more often than the rip-off. This can be seen most easily when the two items are perfect substitutes ($v_1 = v_2 = v$) and have zero costs. In this case, (10) reduces to:

$$1 + \frac{2\eta(\lambda-1)(1-q)}{1+\eta(\lambda-1)(1-q)}\frac{1+\eta}{1+\eta\lambda} = \frac{1+\eta}{1+\eta\lambda} + \frac{2\eta(\lambda-1)q}{\left[1+\eta(\lambda-1)(1-q)\right]^2}\frac{1+\eta}{1+\eta\lambda}. \tag{11}$$

The left-hand-side of (11) captures the seller's marginal gain from an increase in q; similarly, the right-hand-side captures the seller's marginal loss. The following is necessary for (11) to hold:

$$\frac{2\eta(\lambda-1)q}{\left[1+\eta(\lambda-1)(1-q)\right]^2} > \frac{2\eta(\lambda-1)(1-q)}{1+\eta(\lambda-1)(1-q)}$$

$$\Leftrightarrow \quad \frac{q}{1-q} > 1 + \eta(\lambda-1)(1-q).$$

The above inequality can be satisfied only for $q > \frac{1}{2}$. Then, for $v_i - c_i > v_j - c_j$, $i,\ j \in \{1,2\}$, $i \neq j$ if item j is the bargain it follows

$$p_i^* - c_i > v_i - c_i > v_j - c_j > p_j^{\min} - c_j,$$

so that the seller's margins on the two items are even further apart if the items are not perfect substitutes and have different costs. Hence, the seller wants to reduce the availability of the bargain below $\frac{1}{2}$ even more.

On the other hand, suppose that $v_2 - c_2 < v_1 - c_1$ but the seller uses item 1 as the bargain (as shown in the lemma below, this can be a profit-maximizing strategy for the seller). In this case we have that $p_2^* > v_2$ and $p_1^{\min} < v_1$, yet the difference $p_2^* - c_2 - \left(p_1^{\min} - c_1\right)$ could be relatively small. Then, as λ tends to 1 $p_1^{\min}$ approaches v_1 and for $\eta > 1$ the attachment effect could be strong enough for the seller to choose $\underline{q} > \frac{1}{2}$.

Furthermore, as $\overline{q} > 1 - \underline{q}$, the seller chooses a higher degree of availability for the bargain when this is the superior item. Intuitively, when the seller uses the superior item as the bargain, some consumers will end up paying a very high price for the item they like the least; in order to convince them to do so, the seller must compensate the consumers with a higher ex-ante chance of making a deal.

The above analysis does not specify which item the seller would prefer to use as the bargain. To determine whether the seller would prefer to use item 1 or 2, we must compare her profits in the two cases. Unfortunately, these are complex non-linear functions of v_1 and v_2, which are difficult to sign even in the simplest cases and are intractable in general. To overcome this difficulty, I employ comparative statics techniques based on the envelope theorem; but the downside of this approach is that some of the results in the following lemma apply only for small changes in the relevant parameters.[31]

Lemma 7 *If the two goods are perfect substitutes (i.e., $v_1 = v_2$) the seller prefers to use as the bargain the one with the higher marginal cost and is indifferent if the two goods have the same marginal cost (i.e., $c_1 = c_2$). For $v_1 > v_2$, the seller uses item 2 as the bargain only if $v_1 - c_1 + c_2 > v_2 > \frac{2(1+\eta\lambda)(c_1-c_2)}{1+2\eta}$ and $v_1 \geq \widetilde{v}_1$, where $\widetilde{v}_1$ is implicitly defined by:*

$$\left[\frac{1-\eta(\lambda-1)(1-\overline{q})}{1+\eta(\lambda-1)(1-\overline{q})}\overline{q} - \frac{1+\eta}{1+\eta\lambda}\underline{q} - \left(1-\underline{q}\right)\frac{1+\eta}{1+\eta\lambda}\frac{2\underline{q}\eta(\lambda-1)}{1+\underline{q}\eta(\lambda-1)}\right](v_1 - v_2) \geq \left(\overline{q} - \underline{q}\right)(c_1 - c_2).$$

Otherwise, she prefers using item 1 as the bargain.

So, if $v_1 - c_1 \leq v_2 - c_2$, the seller always uses item 1 as the bargain. Arguably more interesting, however, is the fact that the seller might prefer to use item 1 as the bargain even when this is the item with the greater social surplus (i.e., $v_1 - c_1 > v_2 - c_2$). The intuition for this result can be seen in two steps. First, as $v_1 > v_2$ it follows that $p_1^{\min} > p_2^{\min}$ and this in turn implies that $p_2^*\left(\underline{q}\right) > p_1^*(\overline{q})$ through both the attachment effect and the comparison effect. So both prices are higher when the seller uses item 1 as the bargain. However, from this we cannot yet conclude that the seller's revenue is higher when she supplies item 1 at a discount because the weights, $\overline{q}$ and $\underline{q}$, are different. Indeed, we know from lemma 6 that the seller supplies more units of the rip-off when this is the superior good. Nevertheless, for $v_1 - v_2$ small enough the difference in the weights is a second-order one and the seller prefers to use item 1 as the bargain even if $c_1 = c_2$. Second, if $c_2 < c_1$, by using the superior item as the bargain, the seller is able to reduce her average marginal cost by more, compared to the case in which she uses item 2 as the bargain.

Figure 1 shows how the profitability of different schemes changes with v_1 for the case in which $v_1 - c_1 > v_2 - c_2$ and the difference in marginal costs is small. The black line represents the seller's profits when supplying only item 1 at price $p_1 = v_1$, whereas the green and red curves depict the seller's

[31] The results apply only for small changes because comparative statics techniques linearize profits around the maximum. Klemperer and Padilla (1997) use the same approach in a similar context.

profits with limited availability when either item 1 or 2 is used as the bargain item, respectively (notice that the seller's overall profit is given by the upper envelope of these three curves). Concerning the choice of the bargain item, in the graph we can distinguish three different regions, delimited by the two dashed vertical lines. For relatively low values of v_1, the profit-maximizing strategy for the seller is to use a limited-availability deal and use item 1 as the bargain. As v_1 increases, the difference between the green and the red curve becomes smaller and eventually the two cross. Then, for intermediate values of v_1, the seller maximizes profits by using item 2 as the bargain item. Finally, for high values of v_1 the seller prefers to supply just item 1 and price it at its intrinsic value.

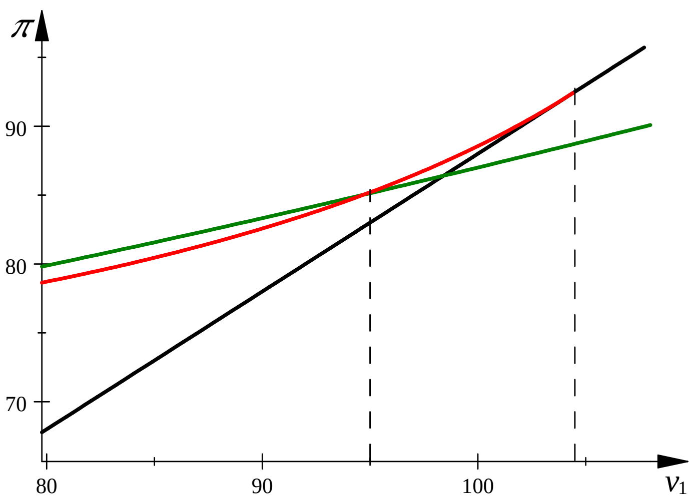

Figure 1: Profits as a function of v_1, for $\eta = 1$, $\lambda = 3$, $v_2 = 80$, $c_1 = 12$, $c_2 = 10$.

When the difference in marginal costs is larger, however, the seller prefers to use item 1 as the bargain item for low as well as intermediate values of v_1. This is shown in Figure 2 where the green curve is always above the red one. In this case item 1 is more valuable to the consumer and it has a larger social surplus; yet it is never used as a rip-off item.

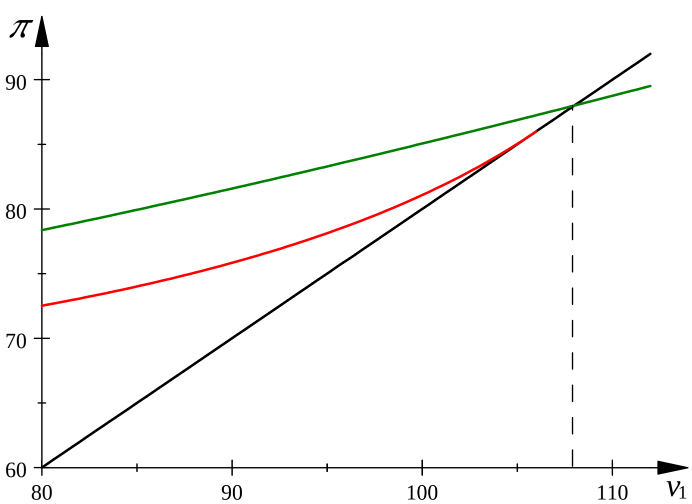

Figure 2: Profits as a function of v_1, for $\eta = 1$, $\lambda = 3$, $v_2 = 80$, $c_1 = 20$, $c_2 = 10$.

The following proposition, which constitutes the main result of this section, identifies the necessary and sufficient conditions for a limited-availability scheme to be profit-maximizing.

Proposition 1 *Fix any $\eta > 0$ and $\lambda > 1$. The seller's profit-maximizing strategy is as follows:*

(i) *for $v_1 \leq v_2 - c_2 + c_1$ there exists a $\alpha(v_2, c_1, c_2, \eta, \lambda)$ such that if $v_1 \geq \alpha$ the seller uses item 1 as the bargain and item 2 as the rip-off, and if $v_1 < \alpha$ she supplies only item 2;*

(ii) *for $\widetilde{v}_1 > v_1 > v_2 - c_2 + c_1$ there exists a $\beta(v_2, c_1, c_2, \eta, \lambda)$ such that if $v_1 \leq \beta$ the seller uses item 1 as the bargain and item 2 as the rip-off, and if $v_1 > \beta$ she supplies only item 1;*

(iii) *for $v_1 > \widetilde{v}_1$ there exists a $\gamma(v_1, c_1, c_2, \eta, \lambda)$ such that if $v_2 \geq \gamma$ the seller uses item 2 as the bargain and item 1 as the rip-off, and if $v_2 < \gamma$ she supplies only item 1.*

Furthermore, $\pi(p_1, p_2, q; c_1, c_2) \geq \max\{v_1 - c_1, v_2 - c_2\}$ and the inequality is strict if both items are supplied.

The exact expressions for α, β and γ are derived in the proof of the proposition in Appendix B. What they imply is that, if the two goods are close substitutes, the seller's profit-maximizing strategy consists of luring the consumers with a tempting discount on one good which is available only in limited supply ($p_i^{\min} < v_i$) and cashing in with a high price on the other ($p_j^* > v_j$). Moreover, by offering both products and inducing uncertainty into the buyers' plans through this type of limited-availability deals, the seller is able to achieve a profit higher than $\max\{v_1 - c_1, v_2 - c_2\}$.

The limited-availability scheme described in Proposition 1 cannot be rationalized by introducing a shopping (or search) cost into a model where consumers have traditionally assumed reference-free preferences. The reason is that, although shopping (or search) costs that are sunk once the consumers reach the store induce an ex-post boost in consumers' willingness to pay, this boost (i) is independent of a good's intrinsic consumption value, (ii) is always smaller than the intrinsic value itself — otherwise consumers would not go to the store, even if the price were to be zero — and, crucially, (iii) because randomization does not affect a risk-neutral consumer's reservation utility, any profit the seller can achieve with randomization could be also achieved with a single price.[32] Therefore, in this case the seller would simply supply the product with the larger social surplus and price it at its intrinsic value minus the shopping (or search) cost.

It is possible for the seller to find this limit-availability strategy profit-maximizing even if the bargain is a loss leader, as the following example shows.

Example 3 ***(Loss Leader)*** *Let $\eta = 1$, $\lambda = 3$, $v_1 = 60$, $v_2 = 40$, $c_1 = 35$ and $c_2 = 22$. For these parameters' values the seller profit-maximizing strategy is given by: $\underline{q} = \frac{3\sqrt{5}}{\sqrt{83}} - \frac{1}{2}$, $p_1^{\min} = 30$ and $p_2^* = \frac{200\underline{q}+40}{2\underline{q}+1} = 59.26$. Item 1 is used as a loss leader and the seller's profit is 27.27.*

By combining the results in Proposition 1 with the condition for the bargain item to be a loss leader (i.e., $p_i^{\min} < c_i$) we immediately obtain the following result.

[32] If consumers are risk-averse in the sense of Expected Utility Theory, then randomization in prices yields always lower profits than committing to a single price since consumers must be compensated for the ex-ante risk they face about the price.

Corollary 1 *Item 1 is a loss leader if either* $\frac{1+\eta\lambda}{1+\eta}c_1 > v_1 \geq \alpha$ *or* $v_1 < \min\left\{\left(\frac{1+\eta\lambda}{1+\eta}\right)c_1,\ \beta\right\}$. *Similarly, item 2 is a loss leader if* $\frac{1+\eta\lambda}{1+\eta}c_2 > v_2 \geq \gamma$.

As shown by Ambrus and Weinstein (2008), classical models of consumers behavior can rationalize the use of loss leaders when the goods are complements but not when they are substitutes. The reason is that with classical preferences a store might benefit from using a loss-leading strategy only if consumers buy other items together with the loss leader. In my model, instead, the presence of loss leaders still attracts consumers into the store but, because the loss-leading product is in shortage, in equilibrium some consumers end up buying a different, more expensive product.

Despite the consumers being homogeneous in terms of tastes for both items, the bargains and rip-offs strategy described above endogenously separates them. Some consumers end up purchasing the good that is offered at a discount, making a bargain indeed. Others, instead, end up purchasing the other good and paying for it even more than their intrinsic valuation. The next result shows that in expectation consumers are hurt by this strategy.

Proposition 2 *For any* $\eta > 0$ *and* $\lambda > 1$ *a consumer's expected surplus is at most zero and therefore he would be better off if he could commit to a strategy of never buying rather than following through his actual equilibrium strategy of always buying.*

As with the similar result obtained in Heidhues and Kőszegi (2012), Proposition 2 suggests that firms' sales are "manipulative" in the sense that they lead the consumers to go to the store even though ex-ante they would prefer not to. Consumers enter the store with the expectations — induced by the seller — of making a bargain by purchasing a good on sale and then might end up buying something else at an even higher price. Of course, this rather extreme result relies on the assumption that the seller knows the consumer's preferences perfectly. Nevertheless, Proposition 5 below shows that even with consumer heterogeneity, some consumers who buy would be better off making and following through a plan of never buying. Notice also that the assumption about the seller being able to credibly commit in advance to a given degree of availability is crucial. In fact, she has a strong incentive to always claim, ex-post, that the bargain item is sold-out and to try to sell only the rip-off. Having rational expectations, however, the consumers would correctly anticipate this and would never plan to buy to begin with and this plan would be consistent. Hence, the current FTC Guides Against Bait Advertising, by allowing to advertise limited-availability deals, provide the stores exactly with the commitment power they need to implement this exploitative scheme. Abolishing the role of limited-supply claims as a disclaimer for bait-and-switch or mandating retailers to issue rainchecks when advertised products are out of stock, would therefore improve consumers' welfare.

In addition to the consumers being worse off with limited availability, the monopolist's product line is sub-optimal:

Remark 1 *With limited availability, if* $v_1 - c_1 \neq v_2 - c_2$, *the monopolist's profit-maximizing product mix differs from the socially optimal one.*

Therefore, except for the non-generic case in which the two goods contribute equally to social surplus ($v_1 - c_1 = v_2 - c_2$), by employing a limited-availability strategy, the seller is reducing welfare compared to first-best, according to which only the item with the larger social value should be supplied. The monopolist, however, can make matters even worse and bring into the market a socially wasteful product, as the following examples show.

Example 4 ***(Wasteful Product 1)*** *Let $\eta = 1$, $\lambda = 3$, $v_1 = 20$, $v_2 = 15$, $c_1 = 21$ and $c_2 = 10$. For these parameters' values the seller profit-maximizing strategy is given by: $\underline{q} = \frac{\sqrt{15}-3}{6}$, $p_1^{\min} = 10$ and $p_2^* = \frac{70\underline{q}+15}{2\underline{q}+1} = 35 - 4\sqrt{15}$ for a total profit of* 6.52.

Example 5 ***(Wasteful Product 2)*** *Let $\eta = 1$, $\lambda = 3$, $v_1 = 30$, $v_2 = 24$, $c_1 = 28$ and $c_2 = 25$. For these parameters' values the seller profit-maximizing strategy is given by: $\underline{q} = \frac{\sqrt{105}-7}{14}$, $p_1^{\min} = 15$ and $p_2^* = \frac{108\underline{q}+24}{2\underline{q}+1} = 54 - 2\sqrt{105}$ for a total profit of* 3.52.

In fact, none of the results required $v_i > c_i$, $i \in \{1, 2\}$. The intuition in Example 4 is that, albeit socially wasteful, item 1 is highly valuable to the consumers and this makes it an ideal candidate for a bait. The intuition is somewhat different for Example 5 because the seller is now introducing an item that is socially wasteful as well as inferior for the consumers; the key here is that item 2 has a lower marginal cost than item 1 and therefore the seller can reduce her average marginal cost by introducing such a wasteful item. Average revenue also decreases, but as the example shows the cost-saving effect might outweigh the decrease in revenue. Furthermore, by comparing Example 4 with Example 5, we see also that the socially wasteful product can be either the bargain or the rip-off.

By combining the results in Proposition 1 with the condition for an item to be socially wasteful (i.e., $v_i < c_i$) we immediately obtain the following result.

Corollary 2 *The seller supplies a socially wasteful product only if item 1 is used as the bargain. She supplies a socially wasteful item 1 if and only if $v_2 - c_2 \geq 0 > v_1 - c_1$ and $v_1 \geq \alpha$. She supplies a socially wasteful item 2 if and only if $v_1 - c_1 \geq 0 > v_2 - c_2$ and $\beta \geq v_1$.*

Moreover, with limited availability the seller could even supply two socially wasteful products and still obtain strictly positive profits.[33]

Example 6 ***(Two Wasteful Products)*** *Let $\eta = 1$, $\lambda = 3$, $v_1 = 20$, $v_2 = 9$, $c_1 = 21$ and $c_2 = 10$. For these parameters' values the seller profit-maximizing strategy is given by: $\underline{q} = \frac{\sqrt{2}-1}{2}$, $p_1^{\min} = 10$ and $p_2^* = \frac{58\underline{q}+9}{2\underline{q}+1} = 29 - 10\sqrt{2}$ for a total profit of* 1.57.

Example 6 shows how the seller can simultaneously exploit the aforementioned effects and supply two socially wasteful products at the same time: item 1 is highly valuable and thus allows the seller to increase her revenue whereas item 2 has a strong cost-saving effect. Unlike other models where consumers buy socially wasteful products (i.e., Gabaix and Laibson, 2006 and Heidhues,Kőszegi and Murooka, 2012), consumers are rational in my model and it is the combination of reference dependence and lack of ex-ante commitment that makes them buy socially wasteful products.

I end this section with the comparative statics with respect to the products' social value for the seller's profits under limited availability.

[33] A similar implication arises also in the paper of Heidhues and Kőszegi (2012), where a single-product monopolist sells an item valued at $v > 0$ by the consumers. Because the monopolist is able to extract, in expectation, more than v from the consumer, she can still attain strictly positive profits for $c > v$.

Proposition 3 *Let* $\pi_1 \equiv \pi\left(p_1^*, p_2^{\min}, \overline{q}; c_1, c_2\right)$ *and* $\pi_2 \equiv \pi\left(p_1^{\min}, p_2^*, \underline{q}; c_1, c_2\right)$ *and assume* $\eta \leq 1$. *Then, we have:* $\frac{d\pi_1}{dv_2} > \left|\frac{d\pi_1}{dc_2}\right| > 0$, $\frac{d\pi_2}{dv_1} > \left|\frac{d\pi_2}{dc_1}\right| > 0$, $\frac{d\pi_2}{dv_2} = \left|\frac{d\pi_2}{dc_2}\right| > 0$ *and* $\left|\frac{d\pi_1}{dc_1}\right| > \frac{d\pi_1}{dv_1} > 0$.

When consumers have classically-assumed reference-free preferences, increasing their valuation for a product from v to $v + \varsigma$, with $\varsigma > 0$, by making it more appealing, or reducing the product's marginal cost by the same amount, would have the same effect on the seller's profit. Proposition 3 implies that this is no longer the case if consumers have reference-dependent preferences.

Intuitively, since the bargain item is a bait that lures consumers into the store and that the seller does not want to sell more often than necessary, her profits rise by more if this product is made more appealing than if its marginal cost is reduced. Indeed, as previously highlighted, expectations-based loss-averse preferences induce a positive demand spillover across products since the more valuable the bargain item is, the higher the price the seller can charge the consumers for the rip-off.

Things are different, however, for the rip-off. Since this is the item the monopolist sells more often, she has a bigger incentive to reduce its marginal cost. When item 2 is the rip-off, the two effects go in opposite directions, but have the same magnitude and end up offsetting each other. When instead item 1 is the rip-off, the gain from reducing its marginal cost is strictly larger than the one from increasing its appeal to consumers. In fact, if item 1 becomes more valuable by ς, consumers' ex-ante uncertainty in the product dimension also increases by ς so that the seller can raise p_1^* by less than ς. This can be easily seen by recalling that the spillover effect for p_1^* is decreasing in v_1.

5 Extensions

In this section I analyze three extensions of the baseline model. In the first subsection, I consider the case in which the seller is able to create perfect substitutes of a given product through a cosmetic change at no additional cost. In this case, the profit-maximizing strategy is always a limited-availability one. Moreover, if item 2 is the socially superior item, the seller might want to introduce the socially inferior item 1, even if she can create a perfect substitute for item 2 at no additional cost.

In the second subsection, I consider a model in which consumers have heterogeneous tastes. I first analyze a case with single-dimension heterogeneity and I show that even in this more general case the seller's profit-maximizing strategy is to reduce availability and use a combination of bargains and rip-offs. Interestingly, with limited availability, the seller is able to serve a larger portion of the potential demand. Then, I look at a situation with multi-dimension heterogeneity and I show that the profit-maximizing scheme is a limited-availability one only if the seller serves all potential demand.

In the third and last subsection I relax the assumption of rational expectations and derive the profit-maximizing strategy for a monopolist selling to overly optimistic loss-averse consumers. For moderate levels of optimism, the seller's profit-maximizing strategy is qualitatively similar to the one with rational consumers. However, when consumers are extremely optimistic, there is no need for the seller to offer a tempting deal on one item to make not buying not a credible plan. Instead, she can simply induce the consumers to believe that they will find the bargain item available for sure at a price equal to its intrinsic value and then charge for the rip-off the highest price consumers are willing to pay *ex-post*.

5.1 Endogenous Product Line

In the model of the previous section, the seller was exogenously endowed with two different products that the consumers regarded as imperfect substitutes. However, retailers can often create almost-perfect substitutes of a given product through a small cosmetic change that does not affect consumers' valuations. For example, two TVs might share the same technology and have the same screen-size and number of pixels, thus providing consumers with the same picture quality, and just differ in their frame's color. An alternative interpretation is that the seller is able to charge different prices for some units of the same product. This happens, for example, when a retailer offers a price reduction on a particular product only for the first units sold on a day.

To formally model this idea, consider a situation in which the seller can create a perfect substitute for a product without incurring any additional cost and suppose she is allowed to price these *de facto* identical products differently. Therefore, the seller now has the choice between supplying two substitutable but distinct items or just supplying two slightly different versions of the same item. In either case, the seller has the option of reducing the availability of one of the items, just like in the model of the previous section.

Assume $v_1 > v_2$, $c_i \geq 0$, $i \in \{1,2\}$ and, let $p_i^{\min}$, $\overline{q}(\eta, \lambda, v_1, v_2, c_1, c_2)$ and $\underline{q}(\eta, \lambda, v_1, v_2, c_1, c_2)$ be defined as in the previous section. Because now the seller can supply two different versions of the same product, let $p_{i,j}^*$ be the price of the rip-off item i, when item j is the bargain. The following proposition characterizes the seller's profit-maximizing strategy.

Proposition 4 *Fix any $\eta > 0$ and $\lambda > 1$. If $v_1 - c_1 > v_2 - c_2$, the seller maximizes profits by supplying two different versions of item 1: the bargain version is priced at $p_1^{\min}$, with degree of availability $1 - \overline{q}(\eta, \lambda, v_1, v_1, c_1, c_1)$ and the rip-off version is priced at $p_{1,1}^*$, with degree of availability $\overline{q}(\eta, \lambda, v_1, v_1, c_1, c_1)$. If $v_1 - c_1 \leq v_2 - c_2$, there exists a $\widetilde{v}_2 < v_1$ such that: (i) for $v_2 \leq \widetilde{v}_2$ the seller maximizes profits by using item 1 as a bargain, with price $p_1^{\min}$ and degree of availability $\underline{q}(\eta, \lambda, v_1, v_2, c_1, c_2)$ and item 2 as a rip-off, with price $p_{2,1}^*$ and degree of availability $1 - \underline{q}(\eta, \lambda, v_1, v_2, c_1, c_2)$; (ii) for $v_2 > \widetilde{v}_2$ the seller maximizes profits by supplying two different versions of item 2: the bargain version is priced at $p_2^{\min}$, with degree of availability $\underline{q}(\eta, \lambda, v_2, v_2, c_2, c_2)$ and the rip-off version is priced at $p_{2,2}^*$, with degree of availability $1 - \underline{q}(\eta, \lambda, v_2, v_2, c_2, c_2)$.*

Proposition 4 delivers several interesting results. First, if the seller can easily create perfect substitutes of the same item that are valued equally by consumers, the profit-maximizing strategy is always a combination of limited availability, bargains and rip-offs.[34] This result can be interpreted as a foundation for the analysis in Heidhues and Kőszegi (2012): although it might not be possible for the seller to credibly commit to a stochastic pricing strategy, she could achieve the same goal by introducing many slightly different — but equivalent from the consumers' point of view — versions of the same product. Second, if the socially superior product is the most preferred by the consumers, the seller prefers to create perfect substitutes of this product instead of introducing another, inferior, one. On the other hand, if the socially superior item is the one consumers value the least, the seller might want

[34] The results would be the same if the seller had to incur a positive cost k to create the artificial substitute, as long as k is not too large.

to supply both products, even if she could create a perfect substitute for either product at no additional cost. The intuition is that, albeit socially inferior, item 1 is highly valuable to the consumers and this makes it an ideal candidate for a bait because it allows the seller to charge an even higher price for the rip-off, therefore increasing average revenue; although average cost also increases, the former effect might dominate. In this case the consumers' most preferred item is used as a bargain and the seller's product line is not welfare-maximizing. Finally, it is easy to see that the results from the previous section about loss leaders and socially wasteful products still apply in this context.

5.2 Heterogeneous Values

In the model analyzed in Section 4 the seller did not face any trade-off between margins and quantities due to the homogeneity assumption about the consumers' preferences. In this section, I consider a more general and realistic environment in which the monopolist faces a classical downward-sloping demand curve and I show that she can still make higher profits by using a limited-availability scheme with a bargain item and a rip-off item. The key insight for this result is that although the seller must choose between serving a large share of the demand with a low price or a small share of the demand with a high price, she can still extract from the marginal consumer more than his intrinsic value for the product.

Consider a seller supplying item 1 at a constant marginal cost $c_1 \geq 0$ to a unit mass of consumers who differ in their intrinsic value, v_1, for the seller' product. From the seller's point of view v_1 is a random variable with distribution F. Assume F is strictly increasing, weakly convex and differentiable, with positive density f everywhere on the support $\left[v_1^l, v_1^h\right]$ with $v_1^h > c_1 \geq v_1^l \geq 0$.[35]

Without loss aversion the seller would just choose the price $\widehat{p}_1$ that solves the following maximization problem:

$$\max_{p_1} \ (p_1 - c_1)\left[1 - F(p_1)\right].$$

Taking FOC and re-arranging yields

$$\widehat{p}_1 - c_1 = \frac{1 - F(\widehat{p}_1)}{f(\widehat{p}_1)}.$$

The consumer with value $v_1 = \widehat{p}_1$ is the "marginal" type; that is, the type who is exactly indifferent between buying or not. The seller's profit is equal to

$$(\widehat{p}_1 - c_1)\left[1 - F(\widehat{p}_1)\right]$$

and consumers' surplus is equal to

$$\int_{\widehat{p}_1}^{v_1^h} (v_1 - \widehat{p}_1) \ dF(v_1).$$

As before, this perfect-availability strategy constitutes a feasible option for the seller also when consumers are expectations-based loss-averse. To see why, notice that, given the price announced by the seller, types below $\widehat{p}_1$ can just plan not to buy and this plan is not only consistent but it maximizes

[35] The assumptions on F ensure that, for deterministic prices, the demand curve is decreasing and weakly concave (a property that is typically assumed in models of industrial organization).

their expected utility; similarly, types above $\widehat{p}_1$ prefer the plan of buying for sure at price $\widehat{p}_1$. Since $q_1 = 1 - F(\widehat{p}_1)$, the measure of types who plan to buy coincides with the amount the seller is supplying and there is no uncertainty in the outcome that each type is expecting; therefore, gain-loss utility is zero in equilibrium. Yet, the seller can attain a higher profit through the introduction of a limited-availability deal. In this case the seller must induce some uncertainty in the buyers' plans otherwise, as argued above, gain-loss utility would be irrelevant.

Suppose that the seller can create an artificial perfect substitute for item 1 without incurring any additional cost and suppose she can price these *de facto* identical products differently. A type-v consumer will plan to buy with positive probability only if $p_1^{\min} \leq \frac{1+\eta}{1+\eta\lambda} v \equiv p_1^{\min}(v)$. From Section 4 we also know that, for given $1-q$ (the degree of availability of the bargain item), this consumer will be indifferent between the plan of buying only the bargain item and the plan of buying the bargain item if available and the rip-off item otherwise if and only if

$$p_1^* = v\left[1 + \frac{2\eta(\lambda-1)(1-q)}{1+\eta(\lambda-1)(1-q)}\frac{1+\eta}{1+\eta\lambda}\right] \equiv p_1^*(v).$$

In order to maximize how much surplus she can extract from this consumer, the monopolist chooses the following degree of availability:

$$\overline{q} = \arg\max_q \, qp_1^*(v) + (1-q)\, p_1^{\min}(v) - c_1.$$

Notice that $\overline{q}$ does not depend on either v or c_1 (see appendix B for the details).

With heterogeneous values there is an additional difficulty in characterizing the optimal limited-availability scheme because different types might select different PPEs. The lemma below describes the PPEs for all consumers' types.

Lemma 8 *Suppose the seller plays the limited-availability strategy that makes a type-v consumer indifferent between buying only the bargain item and always buying. Then, for types in $\left[v_1^l, v\right)$ the PPE plan is to never buy whereas for types in $\left[v, v_1^h\right]$ the PPE plan is to always buy. Furthermore, a consumer's equilibrium expected utility is weakly increasing in his type.*

In order to identify the profit-maximizing marginal type, the seller solves the following program:

$$\max_v \left[\overline{q}p_1^*(v) + (1-\overline{q})\, p_1^{\min}(v) - c_1\right][1 - F(v)],$$

which can be re-written as

$$\max_v \, (\Phi v - c_1)[1 - F(v)],$$

where $\Phi \equiv \frac{4-2\eta^2+\eta^2\lambda^2+4\lambda\eta+\eta^2\lambda-2\sqrt{2(2+\eta+\eta\lambda)(1+\eta)(1+\eta\lambda-\eta)}}{\eta(\lambda-1)(1+\eta\lambda)} > 1$. Let $\widehat{v}_1$ be the solution to the above program. It is immediate to see that $\widehat{v}_1 < \widehat{p}_1$, implying that the seller serves a larger fraction of the consumers when using a limited-availability scheme. The following proposition characterizes the seller's profit-maximizing strategy.

Proposition 5 *For any $\eta > 0$ and $\lambda > 1$ the seller maximizes profits by supplying two different versions of item 1: the bargain version is priced at $p_1^{\min}(\widehat{v}_1)$, with degree of availability $1-\overline{q}$ and the rip-off version is priced at $p_1^*(\widehat{v}_1)$, with degree of availability $\overline{q}$. The marginal type $\widehat{v}_1$ is implicitly defined by $\frac{1-F(\widehat{v}_1)}{f(\widehat{v}_1)} + \frac{c_1}{\Phi} = \widehat{v}_1$. Furthermore, consumers whose type is in $[\widehat{v}_1, v_1^s)$, where $v_1^s = \overline{q}p_1^*(\widehat{v}_1)[1+(1-\overline{q})\eta(\lambda-1)] + (1-\overline{q})p_1^{\min}(\widehat{v}_1)[1-\overline{q}\eta(\lambda-1)]$, get negative expected utility.*

Notice that in this case the overall welfare effect of limited availability is ambiguous, since with a limited-availability scheme the seller is serving a larger measure of consumers compared to the case of perfect availability. Nevertheless, some consumers, who would get a utility level of zero with perfect availability, are unambiguously worse off with this strategy.

The result in Proposition 5 can easily be extended to the case in which the seller's products are not perfect substitutes and have different marginal costs. Suppose the seller cannot create a perfect substitute for item 1, but she can supply item 2 at a constant marginal cost $c_2 = c_1 - k \geq 0$. Let v_2 denote consumers' taste for item 2 and assume it is distributed according to F with support $\left[v_1^l - h, v_1^h - h\right]$. To see the intuition, suppose $h = k$ so that with perfect availability the seller would be exactly indifferent between whether to supply item 1 or 2 and the marginal types would be $\widehat{p}_1$ and $\widehat{p}_2 = \widehat{p}_1 - k$, respectively.

With limited availability, we know from Lemma 7 that if $v_1 - c_1 = v_2 - c_2$ the seller maximizes profits by using item 1 as the bargain item. Therefore, she supplies $\underline{q}$ units of item 1 at price $p_1^{\min}(\widehat{v}_1)$ and $1-\underline{q}$ units of item 2 at price $p_2^*(\widehat{v}_2)$, where $\widehat{v}_1 = \widehat{v}_2 + k$ and achieves higher profits than with perfect availability. Furthermore, $\widehat{v}_1 < \widehat{p}_1$ so that, also in this case, limited availability implies less exclusion than perfect availability.

Finally, let's consider a case with both horizontally and vertically differentiated tastes. Suppose each individual consumer is characterized by a pair of valuation (v_1, v_2) uniformly distributed on the square $[\underline{v}, \overline{v}]^2 \subset \mathbb{R}_2^+$. This distribution is common knowledge, whereas a consumer's individual valuations are his own private information. The goods are substitutes and consumers demand at most one unit of one good. Assume, for simplicity, that $c_1 = c_2 = c$. With perfect availability, the seller solves the following program:

$$\max_{p_1, p_2} \pi^{**} = (p_1 - c)\int_{p_1}^{\overline{v}}\int_{\underline{v}}^{v_1 - p_1 + p_2}\left(\frac{1}{(\overline{v}-\underline{v})^2}\right)dv_2 dv_1 + (p_2 - c)\int_{p_2}^{\overline{v}}\int_{\underline{v}}^{v_2 - p_2 + p_1}\left(\frac{1}{(\overline{v}-\underline{v})^2}\right)dv_1 dv_2.$$

Since the environment is symmetric, there is no loss of generality in restricting attention to a symmetric solution with $p_1 = p_2 = p$; the seller's problem then simplifies to

$$\max_p \frac{(p-c)(\overline{v}-p)(\overline{v}-2\underline{v}+p)}{(\overline{v}-\underline{v})^2}.$$

Taking FOC and re-arranging yields:

$$p^{**} = \frac{2\underline{v} + c + \sqrt{3\overline{v}^2 - 6\overline{v}\underline{v} + 4\underline{v}^2 - 2\underline{v}c + c^2}}{3}. \tag{12}$$

Next, I turn to the analysis with limited availability. Suppose the seller employs the scheme that makes a (v, v)-type consumer exactly indifferent between planning to buy only the bargain and planning to always buy.[36] Let $p_1^{\min}(v) = \frac{1+\eta}{1+\eta\lambda} v$ and $\underline{q}(\eta, \lambda, v, v, c, c)$ be the price of the bargain and its degree of availability and $p_2^*(v)$ be the price of the rip-off.[37] Notice also that although the marginal type values the two items the same, the prices are not symmetric since $p_2^*(v) > v > p_1^{\min}(v)$. The lemma below describes the PPEs for all consumers' types.

Lemma 9 *Let $\eta \leq 1$ and suppose the seller plays the limited-availability strategy that makes a type-(v, v) consumer indifferent between buying only the bargain item and always buying. Then, for types in $[v, \overline{v}]^2$ the PPE plan is to always buy; for types in $(v, \overline{v}] \times [\underline{v}, v)$ the PPE plan is to buy only the bargain if available and nothing otherwise; and for types in $[\underline{v}, v) \times (v, \overline{v}]$, there exist $a > 1$ and $b > 0$ such that consumers' PPE plan is to always buy if $v_2 \geq av - bv_1$ and to never buy otherwise. All other types plan to never buy.*

Thus, the seller's program under limited availability can be written as

$$\max_v \pi_{LA}^{**} = \left[\underline{q} p_1^{\min}(v) + \left(1 - \underline{q}\right) p_2^*(v) - c\right] \left[\left(\frac{\overline{v} - v}{\overline{v} - \underline{v}}\right)^2 + \Omega(v)\right] + \underline{q}\left[p_1^{\min}(v) - c\right] \left(\frac{\overline{v} - v}{\overline{v} - \underline{v}}\right) \left(\frac{v - \underline{v}}{\overline{v} - \underline{v}}\right)$$

where

$$\Omega(v) = \begin{cases} \frac{\left(v - \frac{av - \overline{v}}{b}\right)(\overline{v} - av + bv)}{2} & \text{if } \frac{av - \overline{v}}{b} \geq \underline{v} \\ \frac{[(\overline{v} - av + bv) + (\overline{v} - av + b\underline{v})](v - \underline{v})}{2} & \text{if } \frac{av - \overline{v}}{b} < \underline{v} \end{cases}$$

denotes the area to the left of v and above $av - bv_1$.[38]

Let (v^{**}, v^{**}) denote the profit-maximizing marginal type. It is worth noticing that, for a fixed marginal type, under limited availability the seller is serving a smaller measure of consumers compared to the perfect-availability case. Indeed, while with perfect availability every consumer who values *at least* one item more than its price will buy something for sure, with limited availability instead a positive measure of the consumers who value item 1 more than its price prefer to stay out of the market and those consumers who value item 1 much more than item 2 prefer to plan to buy item 1 if available and nothing otherwise. Indeed, the seller's profit under limited availability might be strictly decreasing in v and, as shown in the following lemma, a limited-availability scheme can be profit-maximizing only if the monopolist serves all potential customers, so that $(v^{**}, v^{**}) = (\underline{v}, \underline{v})$.

Lemma 10 *If $\eta \leq 1$ and $(v^{**}, v^{**}) > (\underline{v}, \underline{v})$ there always exists a perfect-availability strategy that provides the seller with a higher profit than what she could achieve with any limited-availability scheme.*

The intuition for the above result relies on the fact that when the marginal type is in the interior of the support of the distribution, the heterogeneity in consumers' tastes, as captured by $\overline{v} - \underline{v}$, is so large that the seller cannot profitably exploit the attachment effect that a limited-availability deal

[36] It is easy to see that the marginal type must lie on the 45-degree line. Suppose by contradiction that the marginal type had valuations $(v + \varepsilon, v)$. The seller could then increase her profits by playing the limited-availability strategy that makes a type $\left(v + \frac{\varepsilon}{2}, v + \frac{\varepsilon}{2}\right)$ indifferent between buying only the bargain and always buying. In this way, the seller would be serving a larger measure of consumers at a higher average price.

[37] Given the symmetry assumptions about the values' distribution and the items' costs, and since the marginal type views the items as perfect substitutes, the seller is actually indifferent between which item to choose as the bargain.

[38] For $\frac{av - \overline{v}}{b} \geq \underline{v}$, $\Omega(v)$ is a right triangle with sides of length equal to $v - \frac{av - \overline{v}}{b}$ and $\overline{v} - (av - bv)$. Then, for $\frac{av - \overline{v}}{b} < \underline{v}$, $\Omega(v)$ becomes a right trapezoid of height $(v - \underline{v})$ and with sides equal to $\overline{v} - (av - bv)$ and $\overline{v} - (av - b\underline{v})$.

creates for the consumers. This happens because, with multiple dimensions of heterogeneity, there exist "extreme types" who have a relatively high valuation for one of the goods but do not care much for the other. Consider consumers with valuations in $[\underline{v}, v^{**}) \times [\underline{v}, \overline{v}]$. For these consumers not buying is a credible plan since they do not value the bargain item very much; therefore they can "resist" going to the store and will plan to buy only if they expect to obtain a strictly positive surplus. On the other hand, consumers with valuations in $(v^{**}, \overline{v}] \times [\underline{v}, v^{**})$ do not value the rip-off item very much and, although they cannot avoid planning to buy with positive probability, they prefer to leave the store empty-handed if the bargain is not there. Nevertheless, a limited-availability strategy is profit-maximizing if the heterogeneity in consumers' tastes is not too large. The following proposition derives necessary and sufficient conditions for this.

Proposition 6 *Assume $\eta \leq 1$ and $\underline{v} > 0$. The seller's profit-maximizing strategy is a limited-availability one if*

$$\overline{v} \leq \underline{v} + \frac{\sqrt{2\underline{v}^2[9\Phi(3\Phi-4)+8]+2c[2\underline{v}(10-9\Phi)-c]-2(8\underline{v}+c-9\underline{v}\Phi)[\underline{v}^2\Phi(9\Phi-8)+2\underline{v}c(4-5\Phi)+c^2]^{\frac{1}{2}}}}{4} \tag{13}$$

where $\Phi \equiv \frac{4-2\eta^2+\lambda^2\eta^2+4\lambda\eta+\lambda\eta^2-2\sqrt{2(\eta+\lambda\eta+2)(\lambda\eta-\eta^2+\lambda\eta^2+1)}}{\eta(\lambda-1)(\lambda\eta+1)} > 1$. In this case, the seller uses item 1 as the bargain, with price $p_1^{\min}(\underline{v})$ and degree of availability $\underline{q}$ and item 2 as the rip-off, with price $p_2^*(\underline{v})$ and degree of availability $1-\underline{q}$, and all consumers plan to always buy, irrespective of their type. If condition (13) does not hold, the seller maximizes profits by employing a perfect-availability strategy where both items are priced at p^{**}.

Thus, when the distance between $\overline{v}$ and $\underline{v}$ is small, as defined by (13), the seller maximizes profits with a limited-availability strategy. Intuitively, if the degree of heterogeneity in consumers' tastes is small, under screening with perfect availability there is not going to be much exclusion of low types and hence, the valuation of the marginal type — which determines the price — is relatively low. In this case, then, a limited-availability scheme is superior because it extracts more than the marginal type's intrinsic valuation and it serves the entire market with no exclusion, as shown in the example below.

Example 7 ***(Two-dimension Heterogeneity)*** *Suppose $c = 0$, $\eta = 1$ and $\lambda = 3$. Then condition (13) reduces to*

$$\overline{v} \leq \underline{v} + \frac{\underline{v}}{4}\sqrt{\frac{85}{2}\sqrt{\frac{1753}{16} - \frac{303}{4}\sqrt{2}} - 27\sqrt{2}\sqrt{\frac{1753}{16} - \frac{303}{4}\sqrt{2}} - \frac{837}{2}\sqrt{2} + \frac{4763}{8}} \simeq 1.8\underline{v}.$$

Let $(v_1, v_2) \overset{U}{\sim} [20, 30]$. The profit-maximizing price with perfect availability is $p^{**} = \frac{40+10\sqrt{7}}{3}$ for a profit of $\frac{200+140\sqrt{7}}{27}$. The profit-maximizing strategy with limited-availability is a to supply item 1 at price 10, with degree of availability $\frac{\sqrt{2}-1}{2}$ and item 2 at a price of $10\left(4-\sqrt{2}\right)$ for a profit of $65 - 30\sqrt{2}$.

5.3 Optimistic Consumers

So far I have closely followed the model of Kőszegi and Rabin (2006) by assuming that consumers' beliefs must be consistent with rationality: a consumer correctly anticipates the implications of his period-0 plans, and makes the best plan she knows she will carry through. In this section I relax the assumption about rational expectations.

Suppose that when the seller announces a degree of availability q for a bargain, consumers are overly optimistic about their chance of getting a deal and when forming their purchasing plan, they think they will get the bargain with probability $\widetilde{q} = \min\{\chi q, 1\}$, where $\chi > 1$ parametrizes the degree of consumers' optimism. The seller knows χ, but cannot be held liable for the difference between perceived and actual availability; however, she cannot reduce product availability below the level q that she announces. On the other hand, after observing the seller's announcement of availability and prices, consumers still select a PPE purchasing plan, but they base their decisions and payoffs' comparison on the biased beliefs $\widetilde{q}$.

For simplicity, let's assume that the products are perfect substitutes ($v_1 = v_2 = v > 0$) and that marginal cost is zero for both of them, and as a normalization, let item 1 be the bargain item. Denote by $\widehat{q}$ the profit-maximizing degree of availability of item 1 when consumers have rational expectations ($\chi = 1$).

At first glance one could be tempted to guess that with naïve consumers, the seller would always choose a lower degree of availability for the bargain item, compared to the rational case. After all, the seller can just announce $q = \frac{\widehat{q}}{\chi}$, inducing the same attachment effect as with rational consumers but actually selling the bargain less often and hence making even higher profits. However, this intuition is incomplete. To see why, notice that for given q and p_1 that the seller announces for the bargain item, she can raise the price of the rip-off up to

$$p_2^*(q, p_1) = v + \left[\frac{2\eta(\lambda - 1)\chi q}{1 + \eta(\lambda - 1)\chi q}\right] p_1.$$

This means

$$\frac{\partial^2 p_2^*(q, p_1)}{\partial\chi\partial q} > 0 \Leftrightarrow 1 - q\chi\eta(\lambda - 1) > 0,$$

implying that if χ is small, the marginal gain from raising q is higher when consumers are optimistic.

The monopolist will then choose the degree of availability and price for item 1 that solves:

$$\max_{q, p_1} \widetilde{\pi} = qp_1 + (1 - q)\, p_2^*(q, p_1).$$

Let $q_\chi(p_1)$ be the solution to this maximization problem. The following proposition characterizes the seller's profit-maximizing strategy.

Proposition 7 *Fix any $\eta > 0$ and $\lambda > 1$. There exists a $\widetilde{\chi}$ such that the seller's profit-maximizing strategy is as follows:*

(i) *if $\chi < \widetilde{\chi}$, she announces a degree of availability for the bargain equal to $q_\chi(p_1^{\min})$, and prices $p_1^{\min}$ and $p_2^*(q_\chi(p_1^{\min}), p_1^{\min})$;*

(ii) *if $\chi \geq \widetilde{\chi}$, she announces a degree of availability for the bargain equal to $q_\chi = \frac{1}{\chi}$ and prices v and $p_2^* = v\left(1 + \frac{\eta(\lambda-1)}{1+\eta\lambda}\right)$.*

Furthermore, the seller's expected profit is strictly greater than v.

The first implication of Proposition 7 is that the monopolist profit displays a discontinuity at $\widetilde{\chi}$. The intuition is as follows. For moderate levels of consumers' optimism, the seller's profit-maximization problem is very similar to the one with rational consumers: she chooses the highest price for the bargain that makes not buying not a credible plan and the price of the rip-off is such that consumers ex-ante are (perceive to be) indifferent between planning to buy only the bargain and planning to always buy. Then, she announces a degree of availability for the bargain that trades off the gains from exploiting the attachment effect with those from selling the rip-off more often than the bargain. Hence, except for the fact that consumers believe to be more likely to make a deal than they actually are, the seller's profit-maximizing limited-availability scheme is qualitatively similar to the one derived in Section 4.

Things are different, however, when consumers are very optimistic. For $\chi = \widetilde{\chi}$ we have that:

$$\widetilde{q}\left(v - p_1^{\min}\right) - \widetilde{q}\left(1 - \widetilde{q}\right)\eta\left(\lambda - 1\right)\left(v + p_1^{\min}\right) = 0,$$

where $\widetilde{q} = q_{\widetilde{\chi}}\left(p_1^{\min}\right)$. That is, $\widetilde{\chi}$ is the lowest degree of optimism for which, when the seller plays the scheme in part *(i)* of Proposition 7, consumers perceive their expected utility to be non-negative. In this case, there is no need for the seller to offer a tempting deal on item 1 to make not buying not credible; instead, she can just announce $q_\chi = \frac{1}{\chi}$, inducing consumers to believe that they will find item 1 available for sure, and price item 1 at its intrinsic value and item 2 at the highest price consumers are willing to pay *ex-post*. So at $\chi = \widetilde{\chi}$, the degree of availability of the bargain and the prices jump up and so does the seller's profit. Notice also that the optimal level of availability for the bargain is not monotone in the degree of optimism χ, as shown in Figure 3.

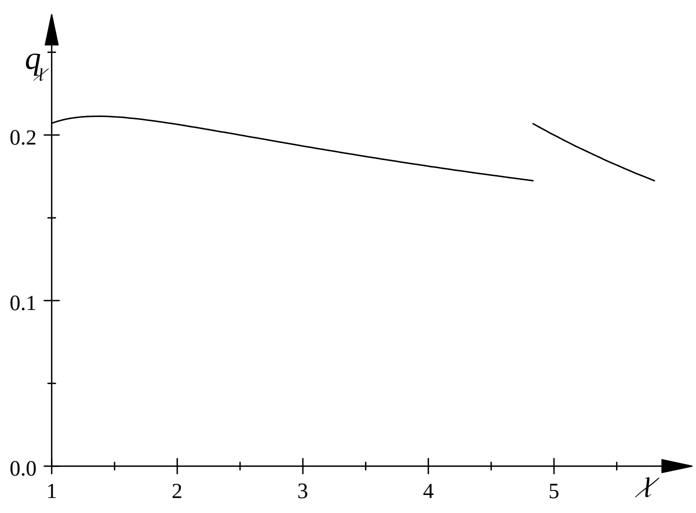

Figure 3: Level of availability with naïve consumers q_χ as a function of χ, for $\eta = 1$, $\lambda = 3$ and $v = 1$.

Clearly naïvete makes consumers worse off. However, notice that as χ tends to 1, the seller is choosing a higher degree of availability for item 1 compared to the case with rational consumers; hence, if χ is relatively small, although overly optimistic consumers on average are exploited even more than rational consumers, there is more of them that end up making a deal.

6 Related Literature

This paper belongs to a recent and growing literature on how firms respond to consumer loss aversion. Heidhues and Kőszegi (2008), Karle and Peitz (2012) and Zhou (2011) study the implications of reference-dependent preferences and loss aversion in an oligopolistic environment with differentiated goods. In a monopolistic-screening setting, Carbajal and Ely (2012), Hahn, Kim, Kim and Lee (2012) and Herweg and Mierendorff (forthcoming) analyze the implications of reference-dependent preferences and loss aversion for the design of profit-maximizing menus and tariffs. Karle (2012) studies the advertising strategy of a single–product monopolist when consumers are expectation–based loss-averse. He shows that the seller maximizes profits by releasing an advertising signal about the consumers' (unknown ex-ante) match-value for the product that, although informative, would be redundant if consumers had classical preferences; instead with loss-averse consumers this informative signal can have a persuasive effect and hence increase consumers' willingness to pay.[39]

As discussed in the Introduction, my paper is most related to Heidhues and Kőszegi (2012), which provides an explanation for why regular prices are sticky, but sales prices are variable, based on expectations-based loss aversion. In their model, a monopolist sells only one good and maximizes profits by employing a stochastic-price strategy made of low, variable sales prices and a high, sticky regular price. My results share an intuition similar to theirs: low prices work as baits to lure consumers who, once in the store, are willing to pay a price even above their intrinsic valuation for the item. However, in my model the monopolist sells two goods and uses one of them as a bait to attract the consumers and the other to exploit them. Also, in Heidhues and Kőszegi (2012) consumers face uncertainty about the price whereas in my case the uncertainty stems from the limited availability of the deal.[40] I consider my model to be an extension as well an improvement over theirs.[41] It is an extension because it shows that the intuition behind their main result holds also in the case of a multi-product monopolist and it is an improvement because I find my assumption about the seller endogenously choosing the degree of availability of a product more realistic than their assumption of the seller being able to credibly commit to an entire price distribution.[42] Moreover, by analyzing the case of a multi-product retailer, I can derive predictions about which products are more likely to be put on sale and I show that higher-value products are more likely to be used as baits.

Within the realm of industrial organization, this paper is also closely related to the literature on advertising, bait-and-switch and loss leaders. Lazear (1995) studies a duopoly with differentiated goods

[39] For other applications of the Kőszegi-Rabin model of reference-dependent preferences outside the field of IO, see Aperjis and Balestrieri (2010) on advantegeous selection in insurance markets, Crawford and Meng (2011) and Eliaz and Spiegler (2012) on labor supply, Herweg, Muller and Weinschenk (2010) and Macera (2012) on agency contracts, Lange and Ratan (2010) , Eisenhuth (2012) and Eisenhuth and Ewers (2012) on sealed-bid auctions and mechanism design, and Daido and Murooka (2012) on team incentives.

[40] If consumers value the two goods equally and the goods have the same production cost, my model coincides with a special case of theirs in which the monopolist uses a two-price distribution. However, in my model the seller can credibly announce to the consumers that she is having a sale on some selected products — as stores often do indeed — whereas in their model the seller can only announce that she might have a sale.

[41] Spiegler (2012a) proposes another simplification and extension of Heidhues and Kőszegi (2012).

[42] After entering a store that claims to use a stochastic-pricing strategy à la Heidhues and Kőszegi (2012), and faced with a high-price draw, a consumer might reasonably doubt whether he was just unlucky or whether the seller was just pretending to randomize prices. In my model, instead, if a consumer does not find a bargain available, he has less of a reason to blame the seller because other consumers might have bought all the bargain items and he might even be mad at himself for not having gone to the store earlier. I thank Kfir Eliaz for suggesting this "shifting the blame" interpretation.

in which each firm produces only one good and consumers pay a search cost to visit a firm, and derives the conditions under which bait-and-switch is a profitable strategy. Although consumers have rational expectations and understand that a firm might engage in bait-and-switch, this strategy can be profitable if the goods sold by different firms are similar and if search is costly. However, bait-and-switch is a form of false advertising in which a firm claims to sell a different good than the one it actually produces. In my model, instead, the firm is not lying to the consumers but is using a truthful version of the bait and switch strategy through endogenously reducing the availability of the goods. Furthermore, in Lazear's model prices are exogenous whereas in mine they are optimally chosen by the seller. Gerstner and Hess (1990) present a model of bait-and-switch in which retailers advertise only selected brands, low-priced advertised brands are understocked and in-store promotions are biased towards more expensive substitute brands. In their model consumers are rational and foresee stock outages. However, the authors assume that in-store promotions can create a permanent utility increase for consumers and this is the reason why in equilibrium some consumers will switch to more expensive brands.

Ellison (2005) presents a model of competitive price discrimination with horizontal and vertical taste differences across consumers in which firms advertise a base price for a product and then try to sell "add-ons" or more sophisticated versions of the product for a higher price at the point of sale. Gabaix and Laibson (2006) study a model where firms benefit from shrouding add-on prices if there myopic consumers who, mistakenly, do not consider the add-on price when forming their shopping plans. Apart from the result that the "basic" version of the product can be a loss leader, my model is different since I assume that all prices are known and that consumers correctly predict their own shopping behavior. Furthermore, the situation described in Example 2 in the Introduction, where the more sophisticated version of the product is offered at a lower price, can be rationalized by my model but not theirs. Eliaz and Spiegler (2011a) propose a model where stores compete for consumers' limited attention by expanding their product lines with "pure attention grabbers"; that is, products that have the sole purpose of attracting consumers' attention to the other products offered by the store. Once at the store, a consumer might realize that there exists another product that better suits his needs. Thus, differently from my model, a consumer might switch to another product even if the bargain item is available.

Hess and Gerstner (1987) develop a model in which multi-product firms might stock out of advertised products and offer rain checks to consumers, and Lal and Matutes (1994) consider multi-product firms competing for consumers who are initially unaware of prices. In both of these models firms might advertise loss leaders in order to increase store traffic. The profitability of this strategy, then, stems from the fact that once they arrive at the store, consumers will buy also other complementary items that are priced at a high mark-up; that is, each firm enjoys a form of monopoly on the other items once a consumer is attracted into the store by the loss leader.[43] My model is different as I consider a monopolist selling substitutable goods to consumers who demand at most one unit of one good and

[43] Related, but somewhat different explanations for the use of loss leaders are advanced by DeGraba (2006) and Chen and Rey (forthcoming). DeGraba (2006) presents a multi-product pricing model in which the loss leaders are the goods purchased mainly by more profitable consumers — consumers who are more likely to buy larger quantities of other goods as well; hence, loss-leading is a way to price discriminate between differently profitable consumers. In Chen and Rey (forthcoming), a large retailer, competing with smaller stores offering a narrower range of products, can exercise market power by pricing below costs some of the products offered also by its rivals. Thus, loss-leading emerges as an exploitative device that allows the large retailer to discriminate multi-stop shoppers from one-stop shoppers.

therefore loss-leading is not aimed at increasing store traffic in order to boost demand for complementary products. Furthermore, in these models the products with lower consumer value are the more natural candidates for loss-leading pricing; my model instead can also rationalize the use of more valuable or popular products as loss leaders.

Models of price dispersion under demand uncertainty (Dana, 1999, 2001a; Deneckere and Peck, 1995; Nocke and Peitz, 2007) and buying frenzies (De Graba 1995; Gilbert and Klemperer, 2000) also predict that rationing some consumers through voluntary stockouts can be a profit-maximizing strategy. However, these models apply mainly to new products that are launched on the market for the first time and for which either the seller or the consumers cannot predict what actual demand will turn out to be; or to industries with clear binding capacity constraints like airlines, hotels and restaurants. Yet, goods sold during bargain sales are usually not appearing on the market for the first time. Moreover, in these models, once the true demand-state is revealed, the scope for rationing disappears.

Finally, Thanassoulis (2004) studies the problem of a multi-product monopolist selling two substitute goods to risk-neutral consumers with unit demand, and derives conditions such that the optimal tariff includes lotteries.[44] In my model, when the seller endogenously reduces the availability of the goods, from the consumers' point of view this is equivalent to taking a lottery on both which good they will end up with and how much they will have to pay.[45] Nevertheless, there are several differences between his model and mine. First, my result on the optimality of limited-availability deals holds also when consumers have homogeneous tastes, whereas his result on the optimality of lotteries does not. Second, in his lotteries there is uncertainty only on the item dimension but not on the price one, whereas in my case the uncertainty is on both dimensions. Last, in his model a lottery is offered in addition to each good being offered in isolation with its own posted price; in my model instead each good is offered in isolation with its own price, but because the items are in short supply, consumers are uncertain about their consumption outcomes.

7 Conclusion

Limited-availability sales are commonly employed by retailers selling durable consumer goods such as electronics, household appliances, or clothes. However, while limited-availability sales are familiar to consumers, economists have not devoted much attention to the importance of product availability in retailing.

In this paper, I have provided an explanation, based on consumer loss aversion, for why a monopolist selling substitute goods might find it profitable to use limited-availability sales. The optimal strategy for the monopolist resembles bait-and-switch: she lures the consumers with a limited-availability tempting deal on one good and cashes in with a high price on another one. The model also predicts that more valuable or popular items are more likely to be used as baits and that the bait can be a loss leader.

[44] Pavlov (2011) solves for the optimal mechanism when selling two substitutable goods and generalizes the analysis in Thanassoulis (2004). Balestrieri and Leao (2011) extend this result to an oligopoly setting where consumers have horizonally differentiated tastes. Fay and Xie (2008) show how lotteries can provide a buffer against a seller's own demand uncertainty and capacity constraints.

[45] Thanassoulis (2004) makes also the related point that capacity constraints, actual or alleged, are an indirect way to implement lotteries.

I conclude the paper by discussing some of the model's limitations, as well as some directions for future research.

An implicit premise of my model is that consumers cannot commit not to go shopping. Although this seems sometimes to be unrealistic, there exist some real-life situations in which this assumption is not that restrictive. For example, around Christmas many consumers "have to" go shopping in order to buy gifts for their friends and relatives. Furthermore, committing not to look at ads or not to learn about sales to avoid being manipulated by firms might require some costly effort on the part of the consumers. If this is the case, then the seller could easily "bribe" the consumers into visiting the store.[46]

Another important assumption is that, from the consumers' perspective, the two products belong to the same hedonic dimension. This creates an insurance effect for the consumers: by planning to always buy a consumer can reduce the uncertainty in his consumption compared to the plan of buying only the bargain item. The monopolist then, is able to exploit this insurance effect by charging a high price for the rip-off item. If the two goods were evaluated along different hedonic dimensions, the insurance effect would disappear, making the conditions for always buying to be the PPE more restrictive.

The analysis in this paper can be extended to the case of a monopolist carrying more than two goods. If the goods are perfect substitutes, or if the seller can endogenously fine tune their degree of differentiation, then she will always use as many products as possible and price them slightly differently to mitigate the consumers' comparison effect on the price dimension, implementing *de facto* the random-price strategy described in Heidhues and Kőszegi (2012). However, if the products are not close enough in terms of substitutability, then the seller will supply only the most similar ones.

I have closely followed the model of Kőszegi and Rabin (2006) in specifying the reference point as the entire distribution of consumers' rational beliefs. However, the analysis would be the same if the reference point, for each hedonic dimension, was equal to the point expectation instead of the distribution. In fact, since all lotteries that consumers face in the model involve comparing only two possible outcomes, each realization is either a loss or a gain but not both, and the same would still be true if the reference point was a point expectation. On the other hand, the assumption that consumers assess gains and losses separately on each hedonic dimension of consumption utility is crucial for the results. If gain-loss utility were defined on the consumers' intrinsic surplus, $v - p$, then the seller could never raise p above v and the profit-maximizing scheme would be a perfect-availability one.

I have also assumed that all consumers show up at the store at the same time and are served randomly with equal probability. In reality, however, especially during popular promotions like Black Friday, consumers line up outside stores before they open. This suggests that consumers' heterogeneity in waiting costs is likely to play a role. Also, those consumers planning to go later in the day would most likely hold different believes about their chances of getting the bargain.

Since my model is one-shot, once a consumer arrives at the store and realizes there are no items left

[46] In fact, introducing a small shopping cost into the model would not significantly affect the results. To see why, suppose that consumers must incur a positive shopping cost ϕ, with $0 < \phi < p_2^{\min}$, to go to the store and let the gain-loss utility in the shopping cost be evaluated separately from the product and money dimensions. Then, there exists a $\phi^*(\eta, \lambda, v_1, v_2, c_1, c_2)$ such that for $\phi \leq \phi^*$ the seller's profit-maximizing strategy is a limited-availability scheme with the only difference that now the price of the bargain must be reduced by $\frac{\phi}{\underline{q}}$ (or $\frac{\phi}{1-\overline{q}}$, depending on which item is the bargain) in order to make never buying non-credible for the consumers and therefore induce them to visit the store (the price of the rip-off should also be adjusted accordingly).

for a discounted price anymore, he has to choose between the feeling of loss on the item dimension by returning home empty-handed or the feeling of loss by paying a higher price for a substitute. In reality, the consumer could decide to wait and return to the store some time later. More generally, sales and promotions appear to be periodic and inter-temporal price discrimination on the part of firms is a big part of the story.

It would be interesting to study which results of this model, if any, continue to hold in a (possibly imperfect) competitive environment. Indeed, one of the most striking features of popular sales like Black Friday is that all retailers use limited-availability deals at the same time. At first glance, since Heidhues and Kőszegi (2012) show that their result does not hold in an environment with two retailers selling a homogeneous product and competing à la Bertrand, one might think that also the results of this paper would not survive. However, given the multi-product framework that characterizes my model, firms would have a different strategy-space than in Heidhues and Kőszegi (2012).

The interaction between the retailer and the manufacturing sector, not modeled in this paper, could also modify the results. For example, if both goods are produced by the same upstream firm, then since the retailer is able to extract more surplus from the consumers through a limited-availability scheme, the firm could try to design a contractual agreement through which she extracts some of this extra surplus. On the other hand, if the goods are produced by two independent manufacturers, the firm producing the good used as a bargain would want to prohibit the retailer from using a limited-availability scheme, since this scheme shifts sales away from the bargain and towards the rip-off.

A Consumers' Plans

For each plan, below I first derive the necessary and sufficient conditions for it to be a PE and then I compute its expected reference-dependent utility at time 0. For any (p_1, p_2) and $(q, 1-q)$, suppose a buyer enters the store with the expectation of not buying; in this case his reference point is to consume nothing and pay nothing. Let the price of good 1 be p_1 and suppose the consumer sticks to his plan. Then, his overall utility is

$$U\left[(0,0)\,|\,\{\varnothing\}\right] = 0.$$

What if instead the consumer decides to deviate from his plan and buys item 1? In this case his overall utility equals

$$U\left[(v_1,p_1)\,|\,\{\varnothing\}\right] = v_1 - p_1 + \eta v_1 - \eta\lambda p_1$$

where $v_1 - p_1$ is his intrinsic consumption utility from buying item 1 at price p_1, the term ηv_1 captures the gain he feels from consuming item 1 when he was expecting to consume nothing and $-\eta\lambda p_1$ captures the loss he feels from paying p_1 when he was expecting to pay nothing. Thus, the consumer will not deviate in this way from the plan to never buy if

$$U\left[(0,0)\,|\,\{\varnothing\}\right] \geq U\left[(v_1,p_1)\,|\,\{\varnothing\}\right] \Leftrightarrow \; p_1 > \frac{1+\eta}{1+\eta\lambda}v_1.$$

A similar threshold can be derived for the case in which the consumer considers deviating from his original plan and buy item 2 at price p_2. Therefore, the plan to never buy is a PE if and only if $p_1 > \frac{1+\eta}{1+\eta\lambda}v_1$ and $p_2 > \frac{1+\eta}{1+\eta\lambda}v_2$. The expected reference-dependent utility associated with the plan to never buy is

$$EU\left[\{\varnothing\}\,|\,\{\varnothing\}\right] = 0$$

as the expected utility from planning to consume nothing and pay nothing and expecting to follow this plan is zero.

Now suppose a buyer enters the store planning to buy item 1 if available and nothing otherwise. In this case his reference point on the item dimension is to enjoy v_1 with probability q and to consume nothing with probability $1-q$; similarly, on the price dimension he expects to pay p_1 with probability q and to pay nothing with probability $1-q$. If the consumer follows this plan his total utility if item 1 is indeed available is

$$U\left[(v_1,p_1)\,|\,\{1,\varnothing\}\right] = v_1 - p_1 + \eta\left(1-q\right)v_1 - \eta\lambda q p_1$$

where $v_1 - p_1$ is his intrinsic consumption utility from buying item 1 at price p_1, the term $\eta\left(1-q\right)v_1$ captures the gain he feels from consuming item 1 when he was expecting to consume nothing with probability $(1-q)$ and $-\eta\lambda q p_1$ captures the loss he feels from paying p_1 when he was expecting to pay nothing with probability q. Suppose that item 1 is available but the buyer instead deviates and does not buy. In this case his overall utility equals

$$U\left[(0,0)\,|\,\{1,\varnothing\}\right] = 0 - \eta\lambda q v_1 + \eta q p_1$$

where 0 is his intrinsic consumption utility, $-\eta\lambda q v_1$ captures the loss he feels from consuming nothing when he was expecting to consume item 1 with probability q, and $\eta q p_1$ captures the gain from paying nothing instead of p_1 which he was expecting to pay with probability q. Thus, the consumer will not deviate in this way from his plan if

$$U\left[(v_1,p_1)\,|\,\{1,\varnothing\}\right] \geq U\left[(0,0)\,|\,\{1,\varnothing\}\right] \Leftrightarrow\; p_1 \leq \frac{1+\eta\,(1-q)+\eta\lambda q}{1+\eta q+\eta\lambda\,(1-q)}v_1. \tag{14}$$

On the other hand, consider the case in which item 1 is not available. If the buyer follows his plan, his overall utility is $U\left[(0,0)\,|\,\{1,\varnothing\}\right]$. If instead he deviates and buys item 2, for $p_1 \geq p_2$ his overall utility is

$$U\left[(v_2,p_2)\,|\,\{1,\varnothing\}\right] = v_2 - p_2 + \eta\,(1-q)\,v_2 - \eta\lambda q\,(v_1-v_2) + \eta q\,(p_1-p_2) - \eta\lambda\,(1-q)\,p_2$$

where $v_2 - p_2$ is his intrinsic consumption utility from from buying item 2 at price p_2, the term $\eta\,(1-q)\,v_2$ captures the gain he feels from consuming item 2 compared to the expectation of consuming nothing with probability $(1-q)$, the term $-\eta\lambda q\,(v_1-v_2)$ captures the loss he feels from consuming item 2 instead of item 1 when he was expecting to consume item 1 with probability q (recall that $v_1 \geq v_2$), the term $\eta q\,(p_1-p_2)$ captures the gain from paying p_2 instead of p_1 which he was expecting to pay with probability q and, finally, $-\eta\lambda\,(1-q)\,p_2$ captures the loss from paying p_2 when he was expecting to pay nothing with probability $1-q$. Thus, the consumer will not deviate in this way from his plan if

$$U\left[(0,0)\,|\,\{1,\varnothing\}\right] > U\left[(v_2,p_2)\,|\,\{1,\varnothing\}\right] \Leftrightarrow\; p_2 > \frac{1+\eta\,(1-q)+\eta\lambda q}{1+\eta q+\eta\lambda\,(1-q)}v_2. \tag{15}$$

Notice that conditions (14) and (15) together imply that $U\left[(v_1,p_1)\,|\,\{1,\varnothing\}\right] > U\left[(v_2,p_2)\,|\,\{1,\varnothing\}\right]$ so that there is no need to check that a consumer does not want to deviate and buy item 2 when item 1 is available. Therefore, for $p_1 \geq p_2$, $\{1,\varnothing\}$ is a PE if and only if $p_2 > \frac{1+\eta(1-q)+\eta\lambda q}{1+\eta q+\eta\lambda(1-q)}v_2$ and $p_1 \leq \frac{1+\eta(1-q)+\eta\lambda q}{1+\eta q+\eta\lambda(1-q)}v_1$. Similarly, for $p_1 < p_2$, $\{1,\varnothing\}$ is a PE if and only if $p_1 < \frac{1+\eta(1-q)+\eta\lambda q}{1+\eta q+\eta\lambda(1-q)}v_1$ and $p_2 > \frac{v_2[1+\eta(1-q)+\eta\lambda q]+q\eta(\lambda-1)p_1}{1+\eta\lambda}$. The expected reference-dependent utility associated with this plan is

$$EU\left[\{1,\varnothing\}\,|\,\{1,\varnothing\}\right] = q\,(v_1-p_1) - q\,(1-q)\,\eta\,(\lambda-1)\,(v_1+p_1)\,. \tag{16}$$

The first term in (16), $q\,(v_1-p_1)$, captures standard expected consumption utility. The second term, $-q\,(1-q)\,\eta\,(\lambda-1)\,(v_1+p_1)$, captures expected gain-loss utility and it is derived as follows. On the consumption dimension, the consumer compares the outcome in which with probability q he gets to consume the good and to enjoy v_1 with the outcome in which with probability $1-q$ he does not consume and gets 0. Similarly, on the price dimension he compares paying the price p_1 with probability q with paying 0 with probability $1-q$. Notice that the expected gain-loss component of the consumer's overall expected utility is always negative as, because of $\lambda > 1$, losses are felt more heavily than equal-size gains. Also, notice that uncertainty in the product and uncertainty in the money dimension are "added up" so that the expected gain-loss term is proportional to $v_1 + p_1$.

Similarly, suppose a buyer enters the store with the plan of buying item 2 if available and nothing otherwise. In this case his reference point on the item dimension is to enjoy v_2 with probability $1-q$ and to consume nothing with probability q; similarly, on the price dimension he expects to pay p_2 with probability $1-q$ and to pay nothing with probability q. If the consumer follows his plan his total utility if item 2 is indeed available is

$$U\left[(v_2,p_2)\,|\,\{2,\varnothing\}\right] = v_2 - p_2 + \eta q v_2 - \eta\lambda\,(1-q)\,p_2.$$

Suppose that item 2 is available but the buyer instead deviates and does not buy. In this case his overall utility equals

$$U\left[(0,0)\mid\{2,\varnothing\}\right]=0-\eta\lambda\left(1-q\right)v_2+\eta\left(1-q\right)p_2.$$

Thus, the consumer will not deviate in this way from his plan to buy only good 2 if

$$U\left[(v_2,p_2)\mid\{2,\varnothing\}\right]\geq U\left[(0,0)\mid\{2,\varnothing\}\right]\Leftrightarrow\ p_2\leq\frac{1+\eta q+\eta\lambda\left(1-q\right)}{1+\eta\left(1-q\right)+\eta\lambda q}v_2. \tag{17}$$

On the other hand, consider the case in which item 2 is not available. If the buyer follows his plan, his overall utility is $U\left[(0,0)\mid\{2,\varnothing\}\right]$. If instead he deviates and buys item 1, for $p_1\geq p_2$ his overall utility is

$$U\left[(v_1,p_1)\mid\{2,\varnothing\}\right]=v_1-p_1+\eta qv_1+\eta\left(1-q\right)\left(v_1-v_2\right)-\eta\lambda qp_1-\eta\lambda\left(1-q\right)\left(p_1-p_2\right).$$

Thus, the consumer will not deviate in this way from his plan if

$$U\left[(0,0)\mid\{2,\varnothing\}\right]>U\left[(v_1,p_1)\mid\{2,\varnothing\}\right]\Leftrightarrow\ p_1>\frac{\left(1+\eta\right)v_1+\eta\left(\lambda-1\right)\left(1-q\right)\left(v_2+p_2\right)}{1+\eta\lambda}. \tag{18}$$

Notice that conditions (17) and (18) together imply that $U\left[(v_2,p_2)\mid\{2,\varnothing\}\right]>U\left[(v_1,p_1)\mid\{2,\varnothing\}\right]$. Therefore, for $p_1\geq p_2$, $\{2,\varnothing\}$ is a PE if and only if $p_1>\frac{(1+\eta)v_1+\eta(\lambda-1)(1-q)(v_2+p_2)}{1+\eta\lambda}$ and $p_2\leq\frac{1+\eta q+\eta\lambda(1-q)}{1+\eta(1-q)+\eta\lambda q}v_2$.[47] The expected reference-dependent utility associated with this plan is

$$EU\left[\{2,\varnothing\}\mid\{2,\varnothing\}\right]=\left(1-q\right)\left(v_2-p_2\right)+\eta\left(1-\lambda\right)q\left(1-q\right)\left(v_2+p_2\right). \tag{19}$$

The above expression is the obvious analogous of (16) for the plan of buying only good 2 if available and nothing otherwise.

For the plan to buy item 1 if available and item 2 otherwise, a consumer's reference point in the item dimension is to consume good 1 and enjoy v_1 with probability q and to consume good 2 and enjoy v_2 with probability $1-q$; similarly, in the price dimension, a buyer expects to pay p_1 with probability q and p_2 with probability $1-q$. Let $p_1\geq p_2$ and suppose that when the consumer arrives at the store item 1 is indeed available. Then, if he follows his plan and buys item 1 his overall utility is

$$U\left[(v_1,p_1)\mid\{1,2\}\right]=v_1-p_1+\eta\left(1-q\right)\left(v_1-v_2\right)-\eta\lambda\left(1-q\right)\left(p_1-p_2\right).$$

If instead he deviates and buys item 2, his overall utility equals

$$U\left[(v_2,p_2)\mid\{1,2\}\right]=v_2-p_2-\eta\lambda q\left(v_1-v_2\right)+\eta q\left(p_1-p_2\right).$$

Thus, the consumer will not deviate in this way from his plan if

$$U\left[(v_1,p_1)\mid\{1,2\}\right]\geq U\left[(v_2,p_2)\mid\{1,2\}\right]\Leftrightarrow\ p_1\leq p_2+\frac{1+\eta\left(1-q\right)+\eta\lambda q}{1+\eta q+\eta\lambda\left(1-q\right)}\left(v_1-v_2\right). \tag{20}$$

Now, instead, suppose that once the buyer arrives at the store, item 2 is everything that is left. If he follows his plan and buys item 2 his overall utility is $U\left[(v_2,p_2)\mid\{1,2\}\right]$. If instead he deviates and

[47] It is easy to verify that $\{2,\varnothing\}$ cannot be a PE for $p_2>p_1$ as a consumer would always deviate and buy item 1 if this is available.

does not buy his utility is

$$U[(0,0)\,|\,\{1,2\}] = 0 - \eta\lambda q v_1 - \eta\lambda(1-q)\,v_2 + \eta q p_1 + \eta(1-q)\,p_2.$$

Thus, the consumer will not deviate in this way from his plan if

$$U[(v_2,p_2)\,|\,\{1,2\}] \geq U[(0,0)\,|\,\{1,2\}] \Leftrightarrow\; p_2 \leq \frac{1+\eta\lambda}{1+\eta}v_2. \tag{21}$$

Notice that conditions (20) and (21) together imply that $U[(v_1,p_1)\,|\,\{1,2\}] > U[(0,0)\,|\,\{1,2\}]$. Therefore, for $p_1 \geq p_2$, $\{1,2\}$ is a PE if and only if $p_1 \leq p_2 + \frac{1+\eta(1-q)+\eta\lambda q}{1+\eta q+\eta\lambda(1-q)}(v_1-v_2)$ and $p_2 \leq \frac{1+\eta\lambda}{1+\eta}v_2$. Similarly, for $p_1 < p_2$, $\{1,2\}$ is a PE if and only if $p_2 \leq \frac{(1+\eta\lambda)v_2+q\eta(\lambda-1)p_1}{1+\eta(1-q)+\eta\lambda q}$ and $p_1 \leq \frac{1+\eta(1-q)+\eta\lambda q}{1+\eta}v_1 + \frac{\eta(\lambda-1)(1-q)}{1+\eta}v_2$. The expected reference-dependent utility associated with this plan is

$$\begin{aligned} EU[\{1,2\}\,|\,\{1,2\}] &= q(v_1-p_1) + (1-q)(v_2-p_2) \\ &\quad + q(1-q)\,\eta(1-\lambda)(v_1-v_2) \\ &\quad + q(1-q)\,\eta(1-\lambda)(\max\{p_1,p_2\} - \min\{p_1,p_2\}). \end{aligned} \tag{22}$$

The first and second terms in (22), $q(v_1-p_1) + (1-q)(v_2-p_2)$ are the standard expected consumption utility terms. The third term, $q(1-q)\,\eta(1-\lambda)(v_1-v_2)$, captures expected gain-loss utility in the consumption dimension: with probability q the consumer expects to buy good 1 and with probability $1-q$ he expects to buy good 2. Notice again that this term is negative, but now it is proportional to (v_1-v_2). This is because with this plan, the consumer is "guaranteeing" himself to enjoy at least the item he values v_2 and the expected gain-loss utility is therefore related only to by how much more he would prefer to consume the other good (or, the degree of substitutability between the two goods). The fourth term, $q(1-q)\,\eta(1-\lambda)(\max\{p_1,p_2\} - \min\{p_1,p_2\})$, captures expected gain-loss utility in the money dimension and can be explained in a similar fashion.

Finally, let's consider the plan to buy item 2 if available and item 1 otherwise. The reference point for this plan is the same as for the previous one. Let $p_1 \geq p_2$ and suppose that when the consumer arrives at the store item 2 is indeed available. Then, if he follows his plan and buys item 1 his overall utility is

$$U[(v_2,p_2)\,|\,\{2,1\}] = v_2 - p_2 - \eta\lambda q(v_1-v_2) + \eta q(p_1-p_2).$$

If instead he deviates and buys item 1, his overall utility equals

$$U[(v_1,p_1)\,|\,\{2,1\}] = v_1 - p_1 + \eta(1-q)(v_1-v_2) - \eta\lambda(1-q)(p_1-p_2).$$

Thus, the consumer will not deviate in this way from his plan if

$$U[(v_2,p_2)\,|\,\{2,1\}] \geq U[(v_1,p_1)\,|\,\{2,1\}] \Leftrightarrow\; p_2 \leq p_1 - \frac{1+\eta(1-q)+\eta\lambda q}{1+\eta q+\eta\lambda(1-q)}(v_1-v_2). \tag{23}$$

Now, instead, suppose that once the buyer arrives at the store, item 1 is everything that is left. If he follows his plan and buys item 1 his overall utility is $U[(v_1,p_1)\,|\,\{2,1\}]$. If instead he deviates and does not buy his utility is

$$U[(0,0)\,|\,\{2,1\}] = 0 - \eta\lambda q v_1 - \eta\lambda(1-q)\,v_2 + \eta q p_1 + \eta(1-q)\,p_2.$$

Thus, the consumer will not deviate in this way from his plan if

$$U\left[(v_1, p_1) \mid \{2,1\}\right] \geq U\left[(0,0) \mid \{2,1\}\right]$$

$$\Leftrightarrow \; p_1 \leq \frac{1+\eta(1-q)+\eta\lambda q}{1+\eta q+\eta\lambda(1-q)} v_1 + \frac{\eta(\lambda-1)(1-q)}{1+\eta q+\eta\lambda(1-q)}(v_2+p_2). \tag{24}$$

Notice that conditions (23) and (24) together imply that $U\left[(v_2, p_2) \mid \{2,1\}\right] > U\left[(v_1, p_1) \mid \{2,1\}\right]$. Therefore, for $p_1 \geq p_2$, $\{2,1\}$ is a PE if and only if $p_1 \leq \frac{1+\eta(1-q)+\eta\lambda q}{1+\eta q+\eta\lambda(1-q)} v_1 + \frac{\eta(\lambda-1)(1-q)}{1+\eta q+\eta\lambda(1-q)}(v_2+p_2)$ and $p_2 \leq p_1 - \frac{1+\eta(1-q)+\eta\lambda q}{1+\eta q+\eta\lambda(1-q)}(v_1-v_2)$. And the expression of the expected reference-dependent utility associated with this plan is the same as in (22).[48]

B Proofs

Proof of Lemma 1: As shown in Kőszegi and Rabin (2006), the plan of buying good $i=1,2$ is a PE if and only if $p_i \leq \frac{1+\eta\lambda}{1+\eta} v_i \equiv p_i^{\max}$ and the plan of not buying good i is a PE if and only if $p_i > \frac{1+\eta}{1+\eta\lambda} v_i \equiv p_i^{\min}$. Therefore, for $p_i \in \left(p_i^{\min}, p_i^{\max}\right]$ both plans are consistent. However, the plan of buying good i at p_i is the PPE if and only if

$$EU\left[\{i\} \mid \{i\}\right] \geq EU\left[\{\varnothing\} \mid \{\varnothing\}\right]$$

$$\Leftrightarrow \quad v_i - p_i \geq 0$$

and this proves the statement. ■

Proof of Lemma 2: The result holds trivially for the case of perfect availability. Then, let $q_1 > 0$, $q_2 > 0$ with $q_1 + q_2 < 1$ and suppose the seller charges p_1 for item 1 and p_2 for item 2, with $p_2 \geq p_1$. The highest price the seller can charge for item 2 is the one that makes the following inequality bind:

$$EU\left[\{1,2\} \mid \{1,2\}\right] \geq EU\left[\{\varnothing\} \mid \{\varnothing\}\right]. \tag{25}$$

Substituting and re-arranging yields

$$p_2 \leq \frac{v_2\left[1+\eta(\lambda-1)q_1 - \eta(\lambda-1)(1-q_1-q_2)\right] - 2\eta(\lambda-1)q_1}{1+\eta(\lambda-1)(1-q_2)}.$$

It is easy to see that the right-hand-side of the above inequality is increasing in q_2. Therefore, the seller can raise q_2 up to $1-q_1$ and increase her profits without violating condition (25). A similar analysis applies if $p_2 < p_1$. ■

Proof of Lemma 3: I prove the result by contradiction. Suppose that $q \in (0,1)$ and $p_i = v_i$ for $i = 1,2$ and that $v_1 > 2v_2$; then we have that

$$\begin{aligned} EU\left[\{\varnothing\} \mid \{\varnothing\}\right] &= 0 \\ &> -2\eta(\lambda-1)q(1-q)v_2 = EU\left[\{2,\varnothing\} \mid \{2,\varnothing\}\right] \\ &> -2\eta(\lambda-1)q(1-q)(v_1-v_2) = EU\left[\{1,2\} \mid \{1,2\}\right] \\ &> -2\eta(\lambda-1)q(1-q)v_1 = EU\left[\{1,\varnothing\} \mid \{1,\varnothing\}\right]. \end{aligned}$$

[48] It is easy to verify that $\{2,1\}$ cannot be a PE for $p_2 > p_1$ as a consumer would always deviate and buy item 1 if this is available.

Furthermore, we know that not buying is a PE when $p_i = v_i$. Therefore, for this quantity vector and this price vector the buyers would strictly prefer the plan of not buying . The seller would then do better by setting $p_i = p_i^{\min}$ for at least one good and thus force the consumers to buy it. The same argument applies to the case in which $v_1 \leq 2v_2$ (just switch the first and second inequalities). ■

Proof of Lemma 4: I prove the result by contradiction. Suppose that $q \in (0,1)$ and $p_i > p_i^{\min}$ for $i = 1, 2$ and that $v_1 - c_1 \geq v_2 - c_2$. By producing a strictly positive quantity of both goods, the seller wants the buyers to choose the plan to always buy; however, for this plan to be the PPE it must be that

$$EU[\{1,2\} \mid \{1,2\}] \geq EU[\{\varnothing\} \mid \{\varnothing\}]$$

$$\Rightarrow \quad q(v_1 - p_1) + (1-q)(v_2 - p_2) > 0$$

$$\Leftrightarrow \quad qp_1 + (1-q)p_2 < qv_1 + (1-q)v_2$$

$$\Rightarrow \quad q(p_1 - c_1) + (1-q)(p_2 - c_2) < q(v_1 - c_1) + (1-q)(v_2 - c_2) \leq v_1 - c_1.$$

But then the seller would prefer to set $q = 1$ and $p_1 = v_1$ and this contradicts the assumption that seller produces a strictly positive quantity of both goods. The same argument applies to the case in which $v_1 - c_1 < v_2 - c_2$. ■

Proof of Lemma 5: Let $q \in (0,1)$. From Lemma 4 we know that $p_i = p_i^{\min}$ for at least one good; let this be good 2. I now show that it is not profitable for the seller to choose p_1 such that the plan to always buy is the unique credible plan for the consumers. First, we have that, for $p_2 = p_2^{\min}$, the highest price the seller can use, in order to make the plan to buy only good 2 not credible, is

$$p_1 \leq \frac{(1+\eta)v_1 + \eta(\lambda-1)(1-q)v_2\left(1 + \frac{1+\eta}{1+\eta\lambda}\right)}{1+\eta\lambda} \equiv \widetilde{p}_1(q)$$

Then, we have that, for $p_2 = p_2^{\min}$, the plan to always buy is a PE if and only if

$$p_1 \leq \frac{[1+\eta(1-q)+\eta\lambda q]v_1 + \eta(\lambda-1)(1-q)v_2\left(1 + \frac{1+\eta}{1+\eta\lambda}\right)}{1+\eta q+\eta\lambda(1-q)} \equiv \overline{p}_1(q).$$

It is readily verified that $\overline{p}_1(q) > \widetilde{p}_1(q) \Leftrightarrow q > 0$. However, for $\overline{p}_1(q) \geq p_1 > \widetilde{p}_1(q)$ both the plan to always buy and the plan to buy only item 2 are personal equilibria; but the plan of always buying is the PPE if and only if

$$p_1 \leq v_1 + \frac{2(1-q)\eta(\lambda-1)[v_2(2+\eta+\eta\lambda) - v_1(1+\eta\lambda)]}{(1+\eta\lambda)[1+\eta(\lambda-1)(1-q)]} \equiv \widehat{p}_1(q).$$

It is easy to see that $\widehat{p}_1(q) > \widetilde{p}_1(q)$. Therefore, the highest price p_1^* at which a buyer prefers the plan to always buy is given by

$$p_1^* = \min\{\overline{p}_1(q), \widehat{p}_1(q)\}$$

and this proves that it is not profit-maximizing for the seller to make always buying the unique consistent plan.

Then, in order to prove that $p_1^* = \widehat{p}_1(q)$, notice that

$$\overline{p}_1(q) < \widehat{p}_1(q)$$

$$\Leftrightarrow\ q < \frac{v_2\left(1+2\eta\lambda\right)\left(2+\eta+\eta\lambda\right)-\eta v_1\left(1+\lambda\right)\left(1+\eta\lambda\right)-\sqrt{A^2v_1^2-2Bv_1v_2+C^2v_2^2}}{2v_2\eta\left(\lambda-1\right)\left(2+\eta+\eta\lambda\right)}$$

where $A \equiv \eta\left(1+\eta\lambda\right)\left(1+\lambda\right)$, $B \equiv \eta\left(1+\eta\lambda\right)\left(2+\eta+\eta\lambda\right)\left[3+2\eta+2\eta\left(\lambda-1\right)\right]$ and $C \equiv \left(1+2\eta\right)\left(2+\eta+\eta\lambda\right)$.

It is also easy to verify that $\frac{v_2(1+2\eta\lambda)(2+\eta+\eta\lambda)-\eta v_1(1+\lambda)(1+\eta\lambda)-\sqrt{A^2v_1^2-2Bv_1v_2+C^2v_2^2}}{2v_2\eta(\lambda-1)(2+\eta+\eta\lambda)} < 1$.
However, it is in the seller's interest to select the p_1^* that maximizes qp_1^* and since

$$\frac{\partial\left[q\widehat{p}_1\left(q\right)\right]}{\partial q} = \frac{\left(2+\eta+\eta\lambda\right)v_2-\left(1+\eta\lambda\right)v_1}{1+\eta\lambda} > 0$$

it follows that $p_1^* = \widehat{p}_1\left(q\right)$.

Finally, I show that $p_1^* > v_1$. Suppose, by contradiction, that $p_1^* \leq v_1$. The seller's profit is

$$q\left(p_1^*-c_1\right)+\left(1-q\right)\left(p_2^{\min}-c_2\right).$$

We have that

$$p_1^* \leq v_1 \Rightarrow q\left(p_1^*-c_1\right)+\left(1-q\right)\left(p_2^{\min}-c_2\right) < q\left(v_1-c_1\right)+\left(1-q\right)\left(v_2-c_2\right) < \max\left\{v_1-c_1, v_2-c_2\right\}.$$

But then the seller would prefer to choose either $q=1$ or $q=0$, contradicting the hypothesis that she is producing a strictly positive quantity of both goods. Notice also that

$$p_1^* > v_1 \Leftrightarrow v_2\left(2+\eta+\eta\lambda\right)-v_1\left(1+\eta\lambda\right) > 0$$

$$\Leftrightarrow v_2+p_2^{\min} > v_1.$$

The same argument applies if the seller uses item 1 as the bargain (i.e., $p_1 = p_1^{\min}$). ■

Proof of Lemma 6: Suppose the seller uses item 2 as the bargain and thus prices it at $p_2^{\min}$. Then, by Lemma 5 we know that the optimal price for item 1 is

$$p_1^* = v_1 + \frac{2\left(1-q\right)\eta\left(\lambda-1\right)\left[v_2\left(2+\eta+\eta\lambda\right)-v_1\left(1+\eta\lambda\right)\right]}{\left(1+\eta\lambda\right)\left[1+\eta\left(\lambda-1\right)\left(1-q\right)\right]}.$$

This pair of prices provides the seller with profits equal to

$$q\left\{\frac{v_1\left[1-\eta\left(\lambda-1\right)\left(1-q\right)\right]+2\eta\left(\lambda-1\right)\left(1-q\right)\left(1+\frac{1+\eta}{1+\eta\lambda}\right)v_2}{1+\eta\left(\lambda-1\right)\left(1-q\right)}-c_1\right\}$$
$$+\left(1-q\right)\left(p_2^{\min}-c_2\right).$$

The above expression is maximized at

$$\begin{aligned} q &= \frac{1+\eta\lambda-\eta}{\eta\left(\lambda-1\right)}-\frac{\sqrt{2}}{\eta\left(\lambda-1\right)}\frac{\sqrt{\left(1+\eta\lambda-\eta\right)\left(-v_1+2v_2+\eta v_2-\lambda\eta v_1+\lambda\eta v_2\right)}}{\sqrt{\left(-c_1+c_2-v_1\right)\left(1+\eta\lambda\right)+v_2\left(3+\eta+2\eta\lambda\right)}} \\ &\equiv \overline{q}\left(\eta,\lambda,v_1,v_2,c_1,c_2\right). \end{aligned}$$

Notice that for the above expression to be well-defined, it must be that

$$\left(-c_1+c_2-v_1\right)\left(1+\eta\lambda\right)+v_2\left(3+\eta+2\eta\lambda\right) > 0$$

since we know that $(2+\eta+\eta\lambda)v_2 > (1+\eta\lambda)v_1$ for p_1^* to be greater than v_1. It is easy to see that $\overline{q} > 0$. Furthermore, we have that

$$\begin{aligned}\overline{q} &< 1 \Leftrightarrow v_1\left[1+3\eta\lambda-2\eta+2\eta^2\lambda(\lambda-1)\right] < (c_1-c_2)(1+\eta\lambda)+ \\ & v_2\left[1+4\eta\lambda-3\eta+2\eta^2(\lambda-1)(\lambda+1)\right].\end{aligned}$$

Notice that

$$\overline{q}(\eta,\lambda,v_1,v_2,c_1,c_2) > \frac{1}{2}$$

since

$$\overline{q}(\eta,\lambda,v_1,v_2,c_1,c_2) > \overline{q}(\eta,\lambda,v,v,c,c)$$

$$\begin{aligned}\Leftrightarrow \quad & \frac{1+\eta\lambda-\eta}{\eta(\lambda-1)} - \frac{\sqrt{2}}{\eta(\lambda-1)}\frac{\sqrt{(1+\eta\lambda-\eta)(-v_1+2v_2+\eta v_2-\eta\lambda v_1+\eta\lambda v_2)}}{\sqrt{(-c_1+c_2-v_1)(1+\eta\lambda)+v_2(3+\eta+2\eta\lambda)}} > \\ & \frac{1+\eta\lambda-\eta}{\eta(\lambda-1)} - \frac{\sqrt{2}}{\eta(\lambda-1)}\sqrt{\frac{(1+\eta)(\eta\lambda-\eta+1)}{(\eta+\eta\lambda+2)}}\end{aligned}$$

$$\Leftrightarrow (1+\eta\lambda)(1+\eta\lambda-\eta)\left[(c_1-c_2)(1+\eta)+(v_2-v_1)(1+\eta\lambda)\right] < 0$$

which is true for any $\eta > 0$ and $\lambda > 1$ provided that $v_1 - c_1 > v_2 - c_2$ (which, as shown below, is a necessary condition for the seller to use item 2 as the bargain); and

$$\overline{q}(\eta,\lambda,v,v,c,c) > \frac{1}{2}$$

$$\begin{aligned}\Leftrightarrow \quad & \frac{1}{\eta(\lambda-1)}(\eta\lambda-\eta+1) - \frac{1}{\eta(\lambda-1)}\sqrt{\frac{2(1+\eta)(\eta\lambda-\eta+1)}{\eta+\eta\lambda+2}} > \frac{1}{2} \\ \Leftrightarrow \quad & \frac{\eta(\lambda-1)\left(\eta^2\lambda^2+6\eta\lambda-\eta^2-6\eta+4\right)}{\eta+\eta\lambda+2} > 0\end{aligned}$$

which is true for any $\eta > 0$ and $\lambda > 1$.

If instead the seller uses item 1 as the bargain, then by Lemma 5 we know that the optimal price for item 2 is

$$p_2^* = v_2 + \frac{2qv_1\eta(\lambda-1)(1+\eta)}{(1+\eta\lambda)\left[1+\eta(\lambda-1)q\right]}.$$

This pair of prices provides the seller with profits equal to

$$q\left(p_1^{\min}-c_1\right)+(1-q)\left\{\frac{v_2\left[1+\eta(\lambda-1)q\right]+2\eta(\lambda-1)q\left(\frac{1+\eta}{1+\eta\lambda}\right)v_1}{1+\eta(\lambda-1)q} - c_2\right\}.$$

The above expression is maximized at

$$\begin{aligned}q &= \frac{\sqrt{2}}{\eta(\lambda-1)}\frac{\sqrt{v_1(1+\eta)(1+\eta\lambda-\eta)}}{\sqrt{(c_1-c_2+v_2)(1+\eta\lambda)+v_1(1+\eta)}} - \frac{1}{\eta(\lambda-1)} \\ &\equiv \underline{q}(\eta,\lambda,v_1,v_2,c_1,c_2).\end{aligned}$$

We have that

$$\underline{q} < 1 \Leftrightarrow v_1 (1+\eta)(1+\eta-\eta\lambda) < (1-\eta+\eta\lambda)(v_2-c_2+c_1)(1+\eta\lambda).$$

Similarly, we also have

$$\underline{q} > 0 \Leftrightarrow v_1 (1+\eta)(1+2\eta\lambda-2\eta) > (v_2-c_2+c_1)(1+\eta\lambda).$$

Notice that

$$\begin{aligned} & \underline{q}(\eta,\lambda,v_1,v_2,c_1,c_2) < \frac{1}{2} \\ \Leftrightarrow \quad & 2\sqrt{\frac{2v_1(1+\eta)(1+\eta\lambda-\eta)}{(c_1-c_2+v_1+v_2+\eta v_1+\lambda\eta c_1-\lambda\eta c_2+\lambda\eta v_2)}} < \eta(\lambda-1)+2 \\ \Leftrightarrow \quad & \left\{8(1+\eta\lambda-\eta)-[\eta(\lambda-1)+2]^2\right\}(1+\eta)v_1 < (v_2-c_2+c_1)(1+\eta\lambda)[\eta(\lambda-1)+2]^2. \end{aligned} \tag{26}$$

Condition (26) is trivially satisfied for any $\eta>0$ and $\lambda>1$ if $v_2-c_2 \geq v_1-c_1$ since $[\eta(\lambda-1)+2]^2 - \left\{8(1+\eta\lambda-\eta)-[\eta(\lambda-1)+2]^2\right\} = 2\eta^2(\lambda-1)^2 > 0$. Condition (26) holds also for $v_2-c_2 < v_1-c_1$ if $\eta \leq 1$ since, as shown below in the proof of proposition 1, if the seller prefers to use item 1 as the bargain when this is the item with the larger social surplus then $v_1 < (v_2-c_2+c_1)\left(\frac{1+\eta\lambda}{1+\eta}\right)$.

Finally, we have that

$$\begin{aligned} & \overline{q} > 1-\underline{q} \\ \Leftrightarrow \quad & \frac{v_2(2+\eta+\eta\lambda)-v_1(1+\eta\lambda)}{(-c_1+c_2-v_1)(1+\eta\lambda)+v_2(3+\eta+2\eta\lambda)} < \frac{v_1(1+\eta)}{(c_1-c_2+v_2)(1+\eta\lambda)+v_1(1+\eta)} \\ \Leftrightarrow \quad & v_2(2+\eta\lambda+\eta)(v_2-v_1+c_1-c_2)-\eta(\lambda-1)v_1(c_1-c_2) < 0 \end{aligned}$$

which holds for any $\eta>0$ and $\lambda>1$ given we know that $v_2(2+\eta\lambda+\eta) > v_1(1+\eta\lambda)$ from Lemma 5 and provided that $v_1-c_1 > v_2-c_2$ which, as shown below, is the only case in which $\overline{q}$ and $\underline{q}$ are comparable. ■

Proof of Lemma 7: Define $\pi_1 \equiv \pi\left(p_1^*, p_2^{\min}, \overline{q}; c_1, c_2\right)$ and $\pi_2 \equiv \pi\left(p_1^{\min}, p_2^*, \underline{q}; c_1, c_2\right)$ and recall that $\overline{q} = \arg\max_q \pi\left(p_1^*, p_2^{\min}, q; c_1, c_2\right)$ and $\underline{q} = \arg\max_q \pi\left(p_1^{\min}, p_2^*, q; c_1, c_2\right)$.

First, consider the special case with $v_1 = v_2$ and $c_1 = c_2$. It is easy to see that in this case $p_1^{\min} = p_2^{\min}$, $p_1^* = p_2^*$, $\overline{q} = 1-\underline{q}$ so that $\pi_1 = \pi_2$. Therefore the seller is indifferent between which item to use as the bargain. Furthermore, by the envelope theorem we have that $\frac{d\pi_1}{dc_1} = -\overline{q}$, $\frac{d\pi_1}{dc_2} = -(1-\overline{q})$, $\frac{d\pi_2}{dc_1} = -\left(1-\underline{q}\right)$ and $\frac{d\pi_2}{dc_2} = -\underline{q}$. By lemma 6 we know that $\overline{q} > 1-\underline{q}$ and therefore it follows that when the two goods are perfect substitutes, the seller maximizes profits by using the more expensive one as the bargain.

Next, suppose to change v_1 by dv_1 and c_1 by dc_1 with $dv_1 = dc_1 = \delta > 0$ so that $v_1 > v_2$ but $v_1 - v_2 = c_1 - c_2$.

By the envelope theorem the effect of these changes on profits are

$$d\pi_1 \simeq \frac{\partial \pi_1}{\partial v_1}dv_1 + \frac{\partial \pi_1}{\partial c_1}dc_1 = \left(\overline{q}\frac{\partial p_1^*}{\partial v_1} - \overline{q}\right)\delta$$

and

$$d\pi_2 \simeq \frac{\partial \pi_2}{\partial v_1}dv_1 + \frac{\partial \pi_2}{\partial c_1}dc_1 = \left[\underline{q}\frac{1+\eta}{1+\eta\lambda} + \left(1-\underline{q}\right)\frac{\partial p_2^*}{\partial v_1} - \underline{q}\right]\delta.$$

By substituting and re-arranging, it follows that $d\pi_2 > d\pi_1$ if and only if

$$\underline{q}\frac{\eta(\lambda-1)}{1+\eta\lambda}\left[\frac{2\left(1-\underline{q}\right)(1+\eta)-\underline{q}\eta(\lambda-1)-1}{\underline{q}\eta(\lambda-1)+1}\right] > \overline{q}\left[\frac{1-\eta(\lambda-1)(1-\overline{q})}{1+\eta(\lambda-1)(1-\overline{q})}-1\right]. \tag{27}$$

As the expression on the right-hand-side of (27) is negative, it suffices to show that

$$2\left(1-\underline{q}\right)(1+\eta)-\underline{q}\eta(\lambda-1)-1>0 \Leftrightarrow \frac{1+2\eta}{2+\eta+\eta\lambda} > \underline{q}.$$

Substituting $v_1 = v_2$ and $c_1 = c_2$ into the expression for $\underline{q}$ yields

$$\frac{2+\eta+\eta\lambda+\eta(\lambda-1)(1+2\eta)}{2+\eta+\eta\lambda} > \sqrt{\frac{2+\eta+\eta\lambda+\eta(\lambda-1)(1+2\eta)}{2+\eta+\eta\lambda}}$$

which is of course true for any $\eta > 0$ and $\lambda > 1$. Thus, the seller maximizes profits by using item 1 as the bargain if $v_1 > v_2$ and $v_1 - v_2 = c_1 - c_2$. Furthermore, it is easy to see that the same result holds also if $dc_1 > dv_1 > 0$ so that $v_1 - c_1 < v_2 - c_2$. Therefore, we have that $\pi_2 \geq \pi_1$ for $v_1 - c_1 \leq v_2 - c_2$.

Finally, consider the case in which $v_1 - c_1 > v_2 - c_2$. Again, let's start with $v_1 = v_2$ and $c_1 = c_2$ so that $\pi_1 = \pi_2$ and suppose to change v_1 by dv_1 and c_1 by dc_1 with either $dv_1 > dc_1 \geq 0$ or $dv_1 \geq 0 > dc_1$. By the envelope theorem the effect of these changes on profits are

$$d\pi_1 \simeq \frac{\partial \pi_1}{\partial v_1}dv_1 + \frac{\partial \pi_1}{\partial c_1}dc_1 = \overline{q}\frac{\partial p_1^*}{\partial v_1}dv_1 - \overline{q}dc_1$$

and

$$d\pi_2 \simeq \frac{\partial \pi_2}{\partial v_1}dv_1 + \frac{\partial \pi_2}{\partial c_1}dc_1 = \left[\underline{q}\frac{1+\eta}{1+\eta\lambda} + \left(1-\underline{q}\right)\frac{\partial p_2^*}{\partial v_1}\right]dv_1 - \underline{q}dc_1.$$

By substituting and re-arranging, it follows that $d\pi_1 \geq d\pi_2$ if and only if

$$\left[\frac{1-\eta(\lambda-1)(1-\overline{q})}{1+\eta(\lambda-1)(1-\overline{q})}\overline{q} - \frac{1+\eta}{1+\eta\lambda}\underline{q} - \left(1-\underline{q}\right)\frac{1+\eta}{1+\eta\lambda}\frac{2\underline{q}\eta(\lambda-1)}{1+\underline{q}\eta(\lambda-1)}\right]dv_1 \geq \left(\overline{q}-\underline{q}\right)dc_1. \tag{28}$$

We know that for $dv_1 = dc_1 > 0$ condition (28) is violated; but for either $dv_1 > dc_1 \geq 0$ or $dv_1 \geq 0 > dc_1$ it can hold (for example, it is readily satisfied for $dv_1 = 0$ and $dc_1 < 0$). Then, let $\widetilde{v}_1$ be the value of v_1 for which (28) binds; if such a value exists then it is unique because the term on the left-hand-side of (28) is continuous and increasing in dv_1. Notice also that $\widetilde{v}_1$ increases with $c_1 - c_2$.

However, from lemma 6 we know that

$$\overline{q} < 1 \Leftrightarrow v_1 < \frac{(c_1-c_2)(1+\eta\lambda) + v_2(1+4\eta\lambda-3\eta) + 2\eta^2 v_2(\lambda-1)(\lambda+1)}{1+3\eta\lambda-2\eta+2\eta^2\lambda(\lambda-1)}.$$

Therefore, a necessary condition for the seller to use item 2 as the bargain when $v_1 - c_1 > v_2 - c_2$ is that

$$\frac{(c_1-c_2)(1+\eta\lambda) + v_2(1+4\eta\lambda-3\eta) + 2\eta^2 v_2(\lambda-1)(\lambda+1)}{1+3\eta\lambda-2\eta+2\eta^2\lambda(\lambda-1)} > v_2 - c_2 + c_1$$

$$\Leftrightarrow \ \eta(\lambda-1)\left[2(1+\eta\lambda)(c_2-c_1) + v_2(1+2\eta)\right] > 0$$

$$\Leftrightarrow \ 2(1+\eta\lambda)(c_2-c_1) + v_2(1+2\eta) > 0$$

$$\Leftrightarrow \; v_2 > \frac{2\left(1+\eta\lambda\right)\left(c_1-c_2\right)}{1+2\eta}.$$

However, the above condition is not sufficient as it could still be that

$$\widetilde{v}_1 > \frac{\left(c_1-c_2\right)\left(1+\eta\lambda\right)+v_2\left(1+4\eta\lambda-3\eta\right)+2\eta^2 v_2\left(\lambda-1\right)\left(\lambda+1\right)}{1+3\eta\lambda-2\eta+2\eta^2\lambda\left(\lambda-1\right)}. \blacksquare$$

Proof of Proposition 1: For an arbitrary price-pair (p_1, p_2) and an arbitrary quantity-pair $(q, 1-q)$ the monopolist's profit is

$$\pi\left(p_1, p_2, q; c_1, c_2\right) = q\left(p_1-c_1\right)+\left(1-q\right)\left(p_2-c_2\right).$$

By Lemma 1 we know that if the seller produces only one good, then she will price it at its intrinsic value.

By Lemma 3 and Lemma 4 we know that if the seller produces a strictly positive quantity of both goods then one of them, say good i, must be priced at the discounted price $p_i^{\min}$. By Lemma 5 we also know that in this case the seller will price good j at p_j^*. Therefore, the seller has three options:

i) Set $p_2 = p_2^{\min}$, $p_1 = p_1^*$ and $q = \overline{q}$. In this case the seller's profit is

$$\overline{q}\left(p_1^*-c_1\right)+\left(1-\overline{q}\right)\left(p_2^{\min}-c_2\right) \equiv \pi_1.$$

ii) Set $p_1 = p_1^{\min}$, $p_2 = p_2^*$ and $q = \underline{q}$. In this case the seller's profit is

$$\underline{q}\left(p_1^{\min}-c_1\right)+\left(1-\underline{q}\right)\left(p_2^*-c_2\right) \equiv \pi_2.$$

iii) Set $p_i = v_i$ for $i = 1, 2$. This pair of prices provides the seller with profits equal to

$$q\left(v_1-c_1\right)+\left(1-q\right)\left(v_2-c_2\right).$$

The above expression is maximized at $q = 1$ (resp. $q = 0$) if $v_1-c_1 > v_2-c_2$ (resp. if $v_1-c_1 \leq v_2-c_2$).

Depending on the degree of substitutability between the two goods, their marginal costs and the degree of loss aversion, the seller will choose the option that will give her the highest profit. Suppose first that $v_1 - c_1 \leq v_2 - c_2$. By Lemma 7 we know that if she were to produce both goods, the seller would prefer to use item 1 as the bargain Then,

$$\pi\left(p_1^{\min}, p_2^*, \underline{q}; c_1, c_2\right) \geq v_2 - c_2$$

$$\Leftrightarrow \quad v_1 \geq \frac{v_2-c_2+c_1}{1+2\eta\left(\lambda-1\right)}\frac{1+\eta\lambda}{1+\eta} \equiv \alpha\left(v_2, c_1, c_2, \eta, \lambda\right).$$

Now suppose that $\widetilde{v}_1 > v_1 > v_2 - c_2 + c_1$. By Lemma 7 we know that if she were to produce both goods, the seller would again prefer to use item 1 as the bargain. Therefore,

$$\pi\left(p_1^{\min}, p_2^*, \underline{q}; c_1, c_2\right) \geq v_1 - c_1$$

$$\Leftrightarrow \quad v_1 \leq \left(v_2-c_2+c_1\right)\left(\frac{1+\eta\lambda}{1+\eta}\right)\Xi\left(\eta,\lambda\right) \equiv \beta\left(v_2, c_1, c_2, \eta, \lambda\right).$$

where

$$\Xi(\eta,\lambda) \equiv [1+\eta(\lambda-1)] \times \left[\frac{3\eta+4\eta^2+2\eta^3+\eta^2\lambda^2(1+\eta)-\eta\lambda(1+3\eta^2+4\eta)-2\eta(\lambda-1)\sqrt{2(1+\eta)^3}+1}{4\eta(1+\eta^3)+\eta^4\lambda^4-2\eta^3\lambda^3(1+3\eta)+\eta^2\lambda^2(13\eta^2+2\eta-5)-2\eta\lambda(6\eta^3-3\eta+1)+1}\right].$$

Furthermore, since $\Xi(\eta,\lambda) < 1$ for $\eta \leq 1$, we have that

$$\eta \leq 1 \Rightarrow \beta(v_2, c_1, c_2, \eta, \lambda) < (v_2 - c_2 + c_1)\left(\frac{1+\eta\lambda}{1+\eta}\right).$$

Finally, if $v_1 \geq \tilde{v}_1$ then by Lemma 7 the seller prefers to use item 2 as the bargain and we have

$$\pi\left(p_1^*, p_2^{\min}, \overline{q}; c_1, c_2\right) \geq v_1 - c_1 \Leftrightarrow v_2 \geq \frac{v_1 - c_1 + c_2 + 2\eta(\lambda-1)v_1}{1+\eta(\lambda-1)\left(\frac{3+2\eta\lambda+2\eta}{1+\eta\lambda}\right)} \equiv \gamma(v_2, c_1, c_2, \eta, \lambda).$$

To conclude the proof, notice that the seller's profits, if she chooses to produce only one good, are equal to $\max\{v_1 - c_1, v_2 - c_2\}$. Since she would choose a different option only if this provides her with at least as much, it thus follows that $\pi \geq \max\{v_1 - c_1, v_2 - c_2\}$, and the inequality is strict when either option i) or ii) is profit-maximizing. ■

Proof of Proposition 2: Suppose the seller uses item 2 as the bargain. We have:

$$\begin{aligned} \overline{q}(p_1^* - c_1) + (1-\overline{q})\left(p_2^{\min} - c_2\right) &> \max\{v_1 - c_1, v_2 - c_2\} \\ &\geq \overline{q}(v_1 - c_1) + (1-\overline{q})(v_2 - c_2) \end{aligned}$$

$$\Rightarrow \overline{q}p_1^* + (1-\overline{q})p_2^{\min} > \overline{q}v_1 + (1-\overline{q})v_2.$$

In this case, therefore, a consumer expects to buy with probability one at an expected price strictly greater than his expected valuation. Hence, his consumption utility is negative. Furthermore, in any PE expected gain-loss utility is non-positive. If instead he could commit to the plan of never buying, both his consumption utility and his gain-loss utility would be zero. The same argument applies for the case in which the seller uses item 1 as the bargain. ■

Proof of Proposition 3: First, consider the seller's profits when item 1 is used as the bargain. We have:

$$\pi_2 = \pi\left(p_1^{\min}, p_2^*, \underline{q}; c_1, c_2\right) = \underline{q}\left(p_1^{\min} - c_1\right) + \left(1-\underline{q}\right)(p_2^* - c_2).$$

By the envelope theorem, we have that:

$$\begin{aligned} \frac{d\pi_2}{dv_1} &= \underline{q}\frac{\partial p_1^{\min}}{\partial v_1} + \left(1-\underline{q}\right)\frac{\partial p_2^*}{\partial v_1} \\ &= \frac{1+\eta}{1+\eta\lambda}\underline{q} + \left(1-\underline{q}\right)\frac{1+\eta}{1+\eta\lambda}\frac{2\eta(\lambda-1)\underline{q}}{1+\eta(\lambda-1)\underline{q}} \\ &= \frac{1+\eta}{1+\eta\lambda}\underline{q}\left[1 + \frac{2\eta(\lambda-1)\left(1-\underline{q}\right)}{1+\eta(\lambda-1)\underline{q}}\right] \\ &> \underline{q} \\ &= \left|\frac{d\pi_2}{dc_1}\right| \end{aligned}$$

where the inequality follows from

$$1+\frac{2\eta(\lambda-1)\left(1-\underline{q}\right)}{1+\eta(\lambda-1)\underline{q}}>\frac{1+\eta\lambda}{1+\eta}$$

$$\begin{aligned}
&\Leftrightarrow\quad 2\left(1-\underline{q}\right)(1+\eta)>1+\eta(\lambda-1)\underline{q}\\
&\Leftrightarrow\quad \frac{1+2\eta}{2+\eta\lambda+\eta}>\underline{q}\\
&\Leftrightarrow\quad v_1<\frac{2(\eta+1)\left(\eta^2\lambda^2-\eta^2\lambda+2\eta\lambda-\eta+1\right)(c_1-c_2+v_2)}{2\eta+3\eta^2+2\eta^3+\eta^2\lambda^2+2\eta\lambda-2\eta^2\lambda-2\eta^3\lambda+2}
\end{aligned}$$

and

$$\frac{2(\eta+1)\left(\eta^2\lambda^2-\eta^2\lambda+2\eta\lambda-\eta+1\right)(c_1-c_2+v_2)}{2\eta+3\eta^2+2\eta^3+\eta^2\lambda^2+2\eta\lambda-2\eta^2\lambda-2\eta^3\lambda+2}>\beta(v_2,c_1,c_2,\eta,\lambda)$$

for $\eta\leq 1$.

Similarly, we also have that

$$\begin{aligned}
\frac{d\pi_2}{dv_2} &= \left(1-\underline{q}\right)\frac{\partial p_2^*}{\partial v_1}\\
&= \left(1-\underline{q}\right)\\
&= \left|\frac{d\pi_2}{dc_2}\right|.
\end{aligned}$$

Next, consider the seller's profits when item 2 is used as the bargain. We have:

$$\pi_1=\pi\left(p_1^*,p_2^{\min},\overline{q};c_1,c_2\right)=\overline{q}(p_1^*-c_1)+(1-\overline{q})\left(p_2^{\min}-c_2\right).$$

Then, we have that

$$\begin{aligned}
\frac{d\pi_1}{dv_1} &= \overline{q}\frac{\partial p_1^*}{\partial v_1}\\
&= \overline{q}\left[\frac{1-\eta(\lambda-1)(1-\overline{q})}{1+\eta(\lambda-1)(1-\overline{q})}\right]\\
&< \overline{q}\\
&= \left|\frac{d\pi_1}{dc_1}\right|.
\end{aligned}$$

Similarly,

$$\begin{aligned}
\frac{d\pi_1}{dv_2} &= \overline{q}\frac{\partial p_1^*}{\partial v_2} + (1-\overline{q})\frac{\partial p_2^{\min}}{\partial v_2} \\
&= \frac{2\eta(\lambda-1)(2+\eta\lambda+\eta)\overline{q}(1-\overline{q})}{(1+\eta\lambda)[1+\eta(\lambda-1)(1-\overline{q})]} + (1-\overline{q})\frac{1+\eta}{1+\eta\lambda} \\
&= (1-\overline{q})\left[\frac{2\eta(\lambda-1)(2+\eta\lambda+\eta)\overline{q}}{(1+\eta\lambda)[1+\eta(\lambda-1)(1-\overline{q})]} + \frac{1+\eta}{1+\eta\lambda}\right] \\
&> (1-\overline{q}) \\
&= \left|\frac{d\pi_1}{dc_2}\right|
\end{aligned}$$

where the inequality follows from

$$\begin{aligned}
\frac{2\eta(\lambda-1)(2+\eta\lambda+\eta)\overline{q}}{(1+\eta\lambda)[1+\eta(\lambda-1)(1-\overline{q})]} + \frac{1+\eta}{1+\eta\lambda} &> 1 \\
&\Leftrightarrow \quad 2(2+\eta\lambda+\eta)\overline{q} > 1+\eta(\lambda-1)(1-\overline{q}) \\
&\Leftrightarrow \quad (4+3\eta\lambda+\eta)\overline{q} > 1+\eta(\lambda-1) \\
&\Leftarrow \quad \overline{q} > \frac{1}{2}
\end{aligned}$$

and this concludes the proof. ■

Proof of Proposition 4: First, we prove that if the seller can create artificial substitutes, a combination limited availability, bargains and rip-offs always yields higher profits than perfect availability. Let $v_1 - c_1 > v_2 - c_2$ so that the maximum level of profits the seller can achieve with perfect availability is $v_1 - c_1$. If the seller can create perfect substitutes for item 1, then her profits are equal to

$$\widehat{q}\left(p_{1,1}^* - c_1\right) + (1-\widehat{q})\left(p_1^{\min} - c_1\right)$$

where $\widehat{q} = \overline{q}(\eta, \lambda, v, v, c, c)$. Then, it suffices to show that

$$\widehat{q}\left(1 + \frac{2(1-\widehat{q})\eta(\lambda-1)}{1+(1-\widehat{q})\eta(\lambda-1)}\frac{1+\eta}{1+\eta\lambda}\right) + \frac{1+\eta}{1+\eta\lambda}(1-\widehat{q}) > 1$$

$$\Leftrightarrow \ \widehat{q} > \frac{1+\eta(\lambda-1)}{\eta+\lambda\eta+2}.$$

Substituting for $\widehat{q}$ yields

$$\frac{2+2\lambda\eta-2\eta^2+2\lambda\eta^2-\sqrt{2(\eta+1)(\lambda\eta-\eta+1)}\sqrt{\eta+\lambda\eta+2}}{\eta(\lambda-1)(\eta+\lambda\eta+2)} > 0$$

$$\Leftrightarrow \ 2\eta(\lambda-1)(\lambda\eta-\eta+1)\left(2\eta^2+3\eta+1\right) > 0$$

which is of course true for any $\eta > 0$ and $\lambda > 1$. A similar argument applies if $v_1 - c_1 \leq v_2 - c_2$.

Next, we prove the first part of the proposition. Define $\pi_{1,2} \equiv \pi\left(p_{1,2}^*, p_2^{\min}, \overline{q}; c_1, c_2\right)$, $\pi_{2,1} \equiv \pi\left(p_1^{\min}, p_{2,1}^*, \underline{q}; c_1, c_2\right)$, $\pi_{1,1} \equiv \pi\left(p_{1,1}^*, p_1^{\min}, \widehat{q}; c_1, c_1\right)$ and $\pi_{2,2} \equiv \pi\left(p_{2,2}^*, p_2^{\min}, \widehat{q}; c_2, c_2\right)$. Recall that $\overline{q} = \arg\max_q \pi\left(p_1^*, p_2^{\min}, q; c_1, c_2\right)$, $\underline{q} = \arg\max_q \pi\left(p_1^{\min}, p_2^*, q; c_1, c_2\right)$ and let $\widehat{q} = \arg\max_q \pi\left(p_i^{\min}, p_{i,i}^*, q; c_i, c_i\right)$, for $i \in \{1, 2\}$. If $v_1 = v_2$ and $c_1 = c_2$, then $p_1^{\min} = p_2^{\min}$, $p_{1,2}^* = p_{1,1}^* = p_{2,2}^* = p_{2,1}^*$ and $\overline{q} = 1 - \underline{q} = \widehat{q}$ so that $\pi_{1,1} = \pi_{1,2} = \pi_{2,1} = \pi_{2,2}$.

Suppose to change v_1 by dv_1 and c_1 by dc_1 with either $dv_1 > dc_1 \geq 0$ or $dv_1 \geq 0 > dc_1$. By the envelope theorem the effect of these changes on profits are

$$d\pi_{1,2} \simeq \frac{\partial \pi_{1,2}}{\partial v_1} dv_1 + \frac{\partial \pi_{1,2}}{\partial c_1} dc_1 = \overline{q} \frac{\partial p_{1,2}^*}{\partial v_1} dv_1 - \overline{q} dc_1$$

$$d\pi_{2,1} \simeq \frac{\partial \pi_{2,1}}{\partial v_1} dv_1 + \frac{\partial \pi_{2,1}}{\partial c_1} dc_1 = \left[\underline{q} \frac{1+\eta}{1+\eta\lambda} + \left(1 - \underline{q}\right) \frac{\partial p_{2,1}^*}{\partial v_1} \right] dv_1 - \underline{q} dc_1$$

$$d\pi_{1,1} \simeq \frac{\partial \pi_{1,1}}{\partial v_1} dv_1 + \frac{\partial \pi_{1,1}}{\partial c_1} dc_1 = \widehat{q} \left[\frac{\partial p_{1,1}^*}{\partial v_1} dv_1 - dc_1 \right] + (1 - \widehat{q}) \left[\frac{1+\eta}{1+\eta\lambda} dv_1 - dc_1 \right]$$

and

$$d\pi_{2,2} = 0.$$

By substituting and re-arranging, we have that $d\pi_{1,1} > d\pi_{2,1}$ since

$$\left[\widehat{q} + \frac{2(1-\widehat{q})\eta(\lambda-1)\widehat{q}}{1+\eta(\lambda-1)(1-\widehat{q})} \frac{1+\eta}{1+\eta\lambda} + \left(1 - \widehat{q} - \underline{q}\right) \frac{1+\eta}{1+\eta\lambda} - \frac{2\underline{q}\eta(\lambda-1)\left(1-\underline{q}\right)}{1+\eta(\lambda-1)\underline{q}} \frac{1+\eta}{1+\eta\lambda} \right] dv_1 > \left(1 - \underline{q}\right) dc_1$$

$$\Leftrightarrow \ dv_1 > dc_1$$

where the last inequality follows from $1 - \underline{q} = \widehat{q}$. Similarly, $d\pi_{1,1} > d\pi_{1,2}$ since

$$\left[\widehat{q} + \frac{2(1-\widehat{q})\eta(\lambda-1)\widehat{q}}{1+\eta(\lambda-1)(1-\widehat{q})} \frac{1+\eta}{1+\eta\lambda} + (1-\widehat{q}) \frac{1+\eta}{1+\eta\lambda} - \overline{q} + \frac{2(1-\overline{q})\eta(\lambda-1)\overline{q}}{1+\eta(\lambda-1)(1-\overline{q})} \right] dv_1 > (1-\overline{q}) dc_1$$

$$\Leftrightarrow \quad \left[\frac{2\eta(\lambda-1)\overline{q}}{1+\eta(\lambda-1)(1-\overline{q})} \left(\frac{2+\eta\lambda+\eta}{1+\eta\lambda} \right) + \frac{1+\eta}{1+\eta\lambda} \right] dv_1 > dc_1$$

where the last inequality follows from $\overline{q} = \widehat{q} > \frac{1}{2}$ and $dv_1 > dc_1$.

Finally, consider the case in which $v_1 - c_1 \leq v_2 - c_2$. Again, let's start with $v_1 = v_2$ and $c_1 = c_2$ and suppose to change v_2 by dv_2 and c_2 by dc_2 with $dc_2 \leq dv_2 < 0$ so that $v_1 - c_1 \leq v_2 - c_2$. By the envelope theorem the effect of these changes on profits are

$$d\pi_{1,2} \simeq \frac{\partial \pi_{1,2}}{\partial v_2} dv_2 + \frac{\partial \pi_{1,2}}{\partial c_2} dc_2 = \left[\overline{q} \frac{\partial p_{1,2}^*}{\partial v_2} + (1-\overline{q}) \frac{1+\eta}{1+\eta\lambda} \right] dv_2 - (1-\overline{q}) dc_2$$

$$d\pi_{2,1} \simeq \frac{\partial \pi_{2,1}}{\partial v_2} dv_2 + \frac{\partial \pi_{2,1}}{\partial c_2} dc_2 = \left(1 - \underline{q}\right) \frac{\partial p_{2,1}^*}{\partial v_2} dv_2 - \left(1 - \underline{q}\right) dc_2$$

$$d\pi_{1,1} = 0$$

and

$$d\pi_{2,2} \simeq \frac{\partial \pi_{2,2}}{\partial v_2} dv_2 + \frac{\partial \pi_{2,2}}{\partial c_2} dc_2 = \left[\widehat{q} \frac{\partial p_{2,2}^*}{\partial v_2} + (1-\widehat{q}) \frac{1+\eta}{1+\eta\lambda} \right] dv_2 - dc_2.$$

By substituting and re-arranging, we have that $d\pi_{2,1} \geq d\pi_{1,2}$ since

$$\left(1-\underline{q}\right)(dv_2 - dc_2) \geq (1-\overline{q})\left\{\left[\frac{2\eta(\lambda-1)\overline{q}}{1+\eta(\lambda-1)(1-\overline{q})}\frac{2+\eta\lambda+\eta}{1+\eta\lambda}+\frac{1+\eta}{1+\eta\lambda}\right]dv_2 - dc_2\right\}$$

$$\Leftarrow \frac{2\eta(\lambda-1)\overline{q}}{1+\eta(\lambda-1)(1-\overline{q})}\frac{2+\eta\lambda+\eta}{1+\eta\lambda}+\frac{1+\eta}{1+\eta\lambda} \geq 1$$

where the last inequality follows from $\overline{q} = 1-\underline{q} > \frac{1}{2}$ and $0 > dv_2 \geq dc_2$.

Finally, we have that $d\pi_{2,1} \geq d\pi_{2,2}$ if and only if

$$\left(1-\underline{q}\right)(dv_2 - dc_2) \geq \widehat{q}\left[1+\frac{2(1-\widehat{q})\eta(\lambda-1)}{1+\eta(\lambda-1)(1-\widehat{q})}\frac{1+\eta}{1+\eta\lambda}\right]dv_2 + (1-\widehat{q})\frac{1+\eta}{1+\eta\lambda}dv_2 - dc_2$$

$$\Leftrightarrow\ 0 \geq \frac{1+\eta}{1+\eta\lambda}\left[\frac{2\eta(\lambda-1)\widehat{q}}{1+\eta(\lambda-1)(1-\widehat{q})}+1\right]dv_2 - dc_2 \tag{29}$$

where the last inequality follows from $\widehat{q} = 1-\underline{q}$. Notice that, although $dv_2 - dc_2 > 0$, condition (29) might hold. Therefore, let $\widetilde{v}_2$ be the value of v_2 for which condition (29) binds. This completes the proof of the proposition. ■

Proof of Lemma 8: We already know that if a consumer of type v is indifferent between the plan of buying only the bargain and the plan of always buying, then his equilibrium expected utility must be negative since he is paying a price above his valuation and, moreover, he is facing uncertainty over the price. Next, consider the equilibrium expected utility for a consumer with type $\overline{v} \in \left(v, v_1^h\right]$. If he plans to buy only the bargain item, his expected utility in equilibrium equals

$$(1-\overline{q})\left[\overline{v} - p_1^{\min}(v)\right] - \eta(\lambda-1)\overline{q}(1-\overline{q})\left(\overline{v} + p_1^{\min}(v)\right). \tag{30}$$

Differentiating (30) with respect to $\overline{v}$ yields $(1-\overline{q})[1-\eta(\lambda-1)\overline{q}]$. On the other hand, if he plans to always buy, his expected utility in equilibrium is

$$\overline{v} - (1-\overline{q})p_1^{\min}(v) - \overline{q}p_1^*(v) - \eta(\lambda-1)\overline{q}(1-\overline{q})\left[p_1^*(v) - p_1^{\min}(v)\right]. \tag{31}$$

Differentiating (31) with respect to $\overline{v}$ yields 1. Therefore, all consumers with type $\overline{v} \in \left(v, v_1^h\right]$ prefer the plan to always buy to the plan to buy only the bargain item.

Next, consider the plan of buying only the rip-off item and nothing otherwise. In this case the consumers' equilibrium expected-utility is

$$\overline{q}\left[\overline{v} - p_1^*(v)\right] - \eta(\lambda-1)\overline{q}(1-\overline{q})\left(\left[\overline{v} + p_1^*(v)\right]\right). \tag{32}$$

It is easy to see that (31) is always larger than (32) since

$$\overline{v}\left[1+\eta(\lambda-1)\overline{q}\right] > p_1^{\min}(v)\left[1-\eta(\lambda-1)\overline{q}\right]$$

and therefore we have proved that all consumers with type $\overline{v} \in \left(v, v_1^h\right]$ prefer to always buy.

Last, consider the consumers with type $\underline{v} \in \left[v_1^l, v\right)$. For these types, not buying is a credible plan since $p_1^{\min}(v) > p_1^{\min}(\underline{v})$. Therefore, they are going to plan to buy with positive probability only if they can make (weakly) positive utility in expectation. From (30) we have that a consumer's expected utility when planning to buy the bargain item and nothing otherwise is non-decreasing in his own type if and only if $1-\eta(\lambda-1)\overline{q}\geq 0$. If this condition holds, then since a type-v consumer gets strictly negative utility in equilibrium so would a a type-$\underline{v}$ if he were to plan to buy; therefore, the latter would prefer planning not to buy. This argument does not work when $1-\eta(\lambda-1)\overline{q}<0$ because in this case a consumer's expected utility is decreasing with his type when he plans to buy only the bargain. However, the utility of a type-$\underline{v}$ consumer when planning to buy only the bargain is equal to

$$\begin{aligned} & (1-\overline{q})\left[\underline{v}-p_1^{\min}(v)\right]-\eta(\lambda-1)\overline{q}(1-\overline{q})\left[\underline{v}+p_1^{\min}(v)\right] \\ = \; & (1-\overline{q})\left\{\underline{v}-p_1^{\min}(v)-\eta(\lambda-1)\overline{q}\left[\underline{v}+p_1^{\min}(v)\right]\right\} \end{aligned}$$

which is negative for $1-\eta(\lambda-1)\overline{q}<0$. Therefore, also in this case consumers prefer not to buy. By the same argument, it is easy to see that these consumers would never want to plan to buy only the rip-off either and this concludes the proof of the lemma. ■

Proof of Proposition 5: From Lemma 8 we know that for a given marginal type v types above v plan to always buy and types below v plan to never buy. Then, the problem reduces to a standard linear-monopoly pricing program where the seller charges an expected price of

$$\overline{q}p_1^*(v)+(1-\overline{q})p_1^{\min}(v)=\Phi v$$

where $\Phi\equiv\frac{4-2\eta^2+\eta^2\lambda^2+4\lambda\eta+\eta^2\lambda-2\sqrt{2(2+\eta+\eta\lambda)(1+\eta)(1+\eta\lambda-\eta)}}{\eta(\lambda-1)(1+\eta\lambda)}>1$. Let $\widehat{v}_1$ be the profit-maximizing marginal type. In equilibrium a consumer of type-v attains a positive expected utility if and only if

$$v-(1-\overline{q})p_1^{\min}(\widehat{v}_1)-\overline{q}p_1^*(\widehat{v}_1)-\eta(\lambda-1)\overline{q}(1-\overline{q})\left[p_1^*(\widehat{v}_1)-p_1^{\min}(\widehat{v}_1)\right]\geq 0$$

$$\Leftrightarrow \quad v\geq(1-\overline{q})p_1^{\min}(\widehat{v}_1)+\overline{q}p_1^*(\widehat{v}_1)+\eta(\lambda-1)\overline{q}(1-\overline{q})\left[p_1^*(\widehat{v}_1)-p_1^{\min}(\widehat{v}_1)\right]\equiv v_1^s$$

and this concludes the proof. ■

Proof of Lemma 9: Suppose the seller plays the limited-availability strategy that makes a (v,v)-type consumer indifferent between planning to buy only the bargain item and planning to always buy; that is:

$$\begin{aligned} & \underline{q}\left[v-p_1^{\min}(v)\right]-\underline{q}\left(1-\underline{q}\right)\eta(\lambda-1)\left[v+p_1^{\min}(v)\right]= \\ & \underline{q}\left[v-p_1^{\min}(v)\right]+\left(1-\underline{q}\right)\left[v-p_2^*(v)\right]-\underline{q}\left(1-\underline{q}\right)\eta(\lambda-1)\left[p_2^*(v)-p_1^{\min}(v)\right] \end{aligned}$$

$$\Leftrightarrow \; v-p_2^*(v)=\underline{q}\eta(\lambda-1)\left[p_2^*(v)-v-2p_1^{\min}(v)\right]. \tag{33}$$

It is easy to see that consumers whose values lie in $\left[\underline{v},p_1^{\min}(v)\right]\times\left[\underline{v},p_2^*(v)\right]$ will plan to never buy and this plan is consistent for them. For consumers in $\left[p_1^{\min}(v),v\right)\times\left[\underline{v},v\right)$ not buying is consistent as well; hence, they would choose a different plan only if it provides them with non-negative expected utility. Planning to buy item 2 yields negative expected utility; similarly planning to buy item 1 if available and item 2 otherwise, also yields negative expected utility as $\underline{q}p_1^{\min}(v)+\left(1-\underline{q}\right)p_2^*(v)>v$. Thus, we only need to check whether the consumers would prefer to plan to buy item 1 if available:

$$\underline{q}\left(v_1-p_1^{\min}(v)\right)-\underline{q}\left(1-\underline{q}\right)\eta(\lambda-1)\left(v_1+p_1^{\min}(v)\right)>0$$

$$\Leftrightarrow \; v_1 > \frac{1+\eta(\lambda-1)\left(1-\underline{q}\right)}{1-\eta(\lambda-1)\left(1-\underline{q}\right)}\frac{1+\eta}{1+\eta\lambda}v$$

where the second inequality follows from the fact that $p_1^{\min}(v) = \frac{1+\eta}{1+\eta\lambda}v$. However, we have

$$\frac{1+\eta(\lambda-1)\left(1-\underline{q}\right)}{1-\eta(\lambda-1)\left(1-\underline{q}\right)}\frac{1+\eta}{1+\eta\lambda} > 1$$

$$\Leftrightarrow \; \eta\lambda\left(1-\underline{q}\right)+1 > \eta\left(1-\underline{q}\right)+2\underline{q}.$$

Substituting for $\underline{q}$ yields

$$\eta(\lambda-1)(1+2\eta) > 0.$$

Therefore,

$$v \geq v_1 > \frac{1+\eta(\lambda-1)\left(1-\underline{q}\right)}{1-\eta(\lambda-1)\left(1-\underline{q}\right)}\frac{1+\eta}{1+\eta\lambda}v > v$$

yielding a contradiction.

For consumers with values in $[v,\overline{v}] \times [\underline{v},v]$, neither planning of never buying nor planning to buy item 2 is credible. They prefer the plan to buy item 1 if available to the plan of always buying if and only if

$$v_2 - p_2^*(v) \leq \underline{q}\eta(\lambda-1)\left[p_2^*(v) - v_2 - 2p_1^{\min}(v)\right].$$

Since $v_2 \leq v$, the result follows.

Next, consider those consumers with values in $[v,\overline{v}] \times [v,\overline{v}]$. For these consumers planning to never buy is not a credible option. Suppose first that $v_1 > v_2$. In this case planning to buy item 2 is not a PE, as if a consumer were to find item 1 available, he would prefer to deviate and buy it. Then, a consumer with this type prefer the plan of always buying to the plan of buying item 1 if available if and only if

$$v_2 - p_2^*(v) \geq \underline{q}\eta(\lambda-1)\left[p_2^*(v) - v_2 - 2p_1^{\min}(v)\right].$$

Since $v_2 \geq v$, the result follows. Now, suppose instead that $v_1 \leq v_2$. Consumers prefer always buying to buying only item 2 if and only if

$$v_1 - p_1^{\min}(v) \geq \left(1-\underline{q}\right)\underline{q}\eta(\lambda-1)\left[-v_1 - p_1^{\min}(v)\right]$$

which is of course true; and they prefer always buying to buying only item 1 if and only if

$$v_2 - p_2^*(v) \geq \underline{q}\eta(\lambda-1)\left[v_2 + p_2^*(v) - 2v_1 - 2p_1^{\min}(v)\right]$$

$$\Leftrightarrow \; v_2\left[1-\underline{q}\eta(\lambda-1)\right] \geq p_2^*(v)\left[1+\underline{q}\eta(\lambda-1)\right] - 2\underline{q}\eta(\lambda-1)\left[v_1 + p_1^{\min}(v)\right]$$

$$\Leftrightarrow \; v_2\left[1-\underline{q}\eta(\lambda-1)\right] \geq v\left[1+\underline{q}\eta(\lambda-1)\right] - 2\underline{q}\eta(\lambda-1)v_1$$

where the last inequality follows from (33). If $1-\underline{q}\eta(\lambda-1) > 0$, then the result follows since $v < v_1 < v_2$. Notice that

$$1-\underline{q}\eta(\lambda-1) > 0 \Leftrightarrow 3+2\eta+\eta\lambda+\eta^2-\eta^2\lambda > 0$$

and because we assumed $\eta \leq 1$, the result follows.

Finally, consider those consumers whose type is in $[\underline{v}, v] \times [v, \overline{v}]$, with $v_i > p_i$ for $i = 1, 2$. Again, for these consumers, planning not to buy is credible and yields zero. Planning to buy item 1 yields negative utility (the proof is the same as for consumers in $\left[p_1^{\min}(v), v\right] \times [\underline{v}, v]$) and planning to always buy is preferred to planning to buy only item 2 (the proof is the same as for those consumers with values in $[v, \overline{v}] \times [v, \overline{v}]$ and $v_2 \geq v_1$). Finally, planning to always buy is preferred to never buying if and only if

$$\underline{q}\left[v_1 - p_1^{\min}(v)\right] + \left(1 - \underline{q}\right)\left[v_2 - p_2^*(v)\right] - \underline{q}\left(1 - \underline{q}\right)\eta(\lambda - 1)\left[v_2 - v_1 + p_2^*(v) - p_1^{\min}(v)\right] \geq 0$$

$$\begin{aligned} \Leftrightarrow \quad & v_2\left[1 - \underline{q}\eta(\lambda - 1)\right] \geq p_2^*(v)\left[1 + \underline{q}\eta(\lambda - 1)\right] + \\ & \frac{\underline{q}}{1 - \underline{q}}\left\{p_1^{\min}(v)\left[1 - \left(1 - \underline{q}\right)\eta(\lambda - 1)\right] - v_1\left[1 + \left(1 - \underline{q}\right)\eta(\lambda - 1)\right]\right\}. \end{aligned} \tag{34}$$

Since $\eta \leq 1$ implies that $1 - \underline{q}\eta(\lambda - 1) > 0$, condition (34) can be re-written as

$$v_2 \geq av - bv_1$$

where

$$\begin{aligned} a &= \frac{\underline{q}}{1 - \underline{q}} \frac{1 + \eta}{1 + \eta\lambda} \frac{1 - \left(1 - \underline{q}\right)\eta(\lambda - 1)}{1 - \underline{q}\eta(\lambda - 1)} + \\ & \frac{1 + \underline{q}\eta(\lambda - 1)}{1 - \underline{q}\eta(\lambda - 1)} \frac{\sqrt{2(\eta + 1)(\lambda\eta - \eta + 1)}(2\eta + \lambda\eta + 3) - 2(\eta + 1)\sqrt{\eta + \lambda\eta + 2}}{(\lambda\eta + 1)\sqrt{2(\eta + 1)(-\eta + \lambda\eta + 1)}} \\ &> 1 \end{aligned}$$

and

$$b = \frac{\underline{q}}{1 - \underline{q}} \frac{1 + \left(1 - \underline{q}\right)\eta(\lambda - 1)}{1 - \underline{q}\eta(\lambda - 1)} > 0.$$

This concludes the proof. ■

Proof of Lemma 10: Under limited availability the seller solves the following program:

$$\max_v \ \pi_{LA}^{**} = [\Phi v - c]\left[\left(\frac{\overline{v} - v}{\overline{v} - \underline{v}}\right)^2 + \Omega(v)\right] + \left[\Psi v - \underline{q}c\right]\left(\frac{\overline{v} - v}{\overline{v} - \underline{v}}\right)\left(\frac{v - \underline{v}}{\overline{v} - \underline{v}}\right) \tag{35}$$

where $\Phi \equiv \frac{4 - 2\eta^2 + \eta^2\lambda^2 + 4\lambda\eta + \eta^2\lambda - 2\sqrt{2(2 + \eta + \eta\lambda)(1 + \eta)(1 + \eta\lambda - \eta)}}{\eta(\lambda - 1)(1 + \eta\lambda)} > 1$ and $\Psi \equiv \frac{1 + \eta}{1 + \eta\lambda} \frac{\sqrt{2\left(\frac{\eta + 1}{\eta + \lambda\eta + 2}\right)(\lambda\eta - \eta + 1)} - 1}{\eta(\lambda - 1)} < 1$. Since $\Omega(v) \leq \left(\frac{\overline{v} - v}{\overline{v} - \underline{v}}\right)\left(\frac{v - \underline{v}}{\overline{v} - \underline{v}}\right)$, the value of expression (35) is bounded above by

$$\max_v \ \tilde{\pi}_{LA}^{**} = [\Phi v - c]\left[\left(\frac{\overline{v} - v}{\overline{v} - \underline{v}}\right)^2 + \left(\frac{\overline{v} - v}{\overline{v} - \underline{v}}\right)\left(\frac{v - \underline{v}}{\overline{v} - \underline{v}}\right)\right] + \left[\Psi v - \underline{q}c\right]\left(\frac{\overline{v} - v}{\overline{v} - \underline{v}}\right)\left(\frac{v - \underline{v}}{\overline{v} - \underline{v}}\right) \tag{36}$$

Now I show that if the maximization problem (36) has an interior solution at some value $v^{**} \in (\underline{v}, \overline{v})$, then the seller could achieve higher profits than in (36) with a perfect-availability strategy.

Let $c = 0$ (since $\Psi < \underline{q}$ if the result holds for $c = 0$, then it must hold *a fortiori* for $c > 0$). We have an interior solution for the program in (36) if and only if $\frac{\partial \widetilde{\pi}_{LA}^{**}}{\partial v}$, evaluated at $v = \underline{v}$ is positive:

$$\Phi\left(\overline{v} - \underline{v}\right) - 2\Phi\underline{v} + \left(\Phi + \Psi\right)\underline{v} > 0$$

$$\Leftrightarrow \overline{v} > \frac{2\Phi - \Psi}{\Phi}\underline{v}. \tag{37}$$

Since any uniform distribution can be translated, with appropriate re-normalization, into the $[0, 1]$ interval, for simplicity let $\overline{v} = 1$ and $\underline{v} = 0$ (notice that condition (37) is trivially satisfied in this case). Hence, the maximization program in (36) can be re-written as:

$$\max_{v} \ \widetilde{\pi}_{LA}^{**} = \Phi v\left(1 - \overline{v}\right)^2 + \left(\Phi + \Psi\right) v\left(1 - v\right) v.$$

Taking FOC and re-arranging yields:

$$v^{**} = \frac{1}{3\Psi}\left(\Psi - \Phi + \sqrt{\Psi^2 + \Psi\Phi + \Phi^2}\right)$$

for a profit of

$$\frac{\left(\Psi - \Phi\right)\left(2\Psi + \Phi\right)\left(\Psi + 2\Phi\right) + \left(2\Psi^2 + 2\Phi^2 + 2\Psi\Phi\right)\sqrt{\Psi^2 + \Psi\Phi + \Phi^2}}{27\Psi^2}.$$

Recall that with perfect availability the seller maximizes profits by selling both items at price $p^{**} = \frac{1}{\sqrt{3}}$ and obtains profits equal to $\frac{2}{9}\sqrt{3}$. Therefore,

$$\frac{2}{9}\sqrt{3} > \frac{\left(\Psi - \Phi\right)\left(2\Psi + \Phi\right)\left(\Psi + 2\Phi\right) + \left(2\Psi^2 + 2\Phi^2 + 2\Psi\Phi\right)\sqrt{\Psi^2 + \Psi\Phi + \Phi^2}}{27\Psi^2}$$

$$\Leftrightarrow \quad -3\Psi^2\left(8\sqrt{3}\Psi^3 + 9\Psi^2\Phi^2 + 12\sqrt{3}\Psi^2\Phi - 36\Psi^2 + 18\Psi\Phi^3 - 12\sqrt{3}\Psi\Phi^2 + 9\Phi^4 - 8\sqrt{3}\Phi^3\right) > 0$$

$$\Leftrightarrow \frac{\frac{8}{9}\sqrt{3}\Psi^3 + \Phi^4 + \Psi^2\Phi^2 + 2\Psi\Phi^3 + \frac{4}{3}\sqrt{3}\Psi^2\Phi}{\frac{2}{3}\sqrt{3}\Phi^3 + 3\Psi^2 + \sqrt{3}\Psi\Phi^2} < \frac{4}{3}. \tag{38}$$

Notice first that Φ is increasing in η since

$$\begin{aligned}\frac{\partial\Phi\left(\eta,\lambda\right)}{\partial\eta} \quad &> \quad 0 \Leftrightarrow \sqrt{2}\left(\eta^3 + 3\lambda\eta^2 - 4\lambda\eta^3 - \lambda\eta^4 + 9\lambda^2\eta^2 + 5\lambda^2\eta^3 + 2\lambda^3\eta^3 + \lambda^3\eta^4 + 11\lambda\eta + \eta + 4\right) > \\ &\sqrt{\left(\eta + 1\right)\left(\eta + \lambda\eta + 2\right)\left(\lambda\eta - \eta + 1\right)}\left(3\lambda^2\eta^2 - \lambda\eta^2 + 8\lambda\eta + 2\eta^2 + 4\right)\end{aligned}$$

$$\Leftrightarrow \lambda^4\eta^4\left(\eta - 1\right) + \lambda^3\eta^3\left(11\eta + 2\eta^2 - 3\right) + \lambda^2\eta^3\left(10\eta + \eta^2 + 35\right) +$$

$$4\lambda\eta\left(9\eta + 2\eta^2 + 1\right) + 2\eta\left(2\eta + 2\eta^2 + \eta^3 + 6\right) + 2 > 0.$$

Then, we have that

$$\Phi\left(1,\lambda\right) = \frac{5\lambda + \lambda^2 - 4\sqrt{\lambda\left(\lambda + 3\right)} + 2}{\lambda^2 - 1}.$$

The above function reaches its maximum for $\lambda^* \approx 2.88$, and $\Phi\left(1,\lambda^*\right) \approx 1.13$.[49] This bound implies

[49] The relative FOC cannot be solved analytically, so I had to rely on numerical methods to identify the maximum.

that with limited availability the seller can extract 13% more profits than with perfect availability, at the most.

Similar, but much more tedious algebra shows that Ψ is increasing in η as well. So we have that

$$\Psi(1,\lambda)=\frac{2}{(\lambda-1)(\lambda+1)}\left(2\frac{\sqrt{\lambda}}{\sqrt{\lambda+3}}-1\right).$$

The above function is strictly decreasing in λ, and is therefore bounded above by $\lim_{\lambda\to 1}\Psi(1,\lambda)=\frac{3}{8}$. It is easy to see that $\Phi(\eta,\lambda)+\Psi(\eta,\lambda)$ is increasing in η. Moreover, $\Phi(1,\lambda)+\Psi(1,\lambda)$ is strictly decreasing in λ and bounded above by $\lim_{\lambda\to 1}(\Phi(1,\lambda)+\Psi(1,\lambda))=\frac{11}{8}$. Since in the function to be maximized in (36) Φ carries a bigger weight than Ψ, suppose $\lambda=2.88$ so that Φ is at its maximum, 1.13 and Ψ is therefore equal to 0.11. Arithmetic shows that for these values condition (38) holds. Since the expression on the left-hand-side of (38) is increasing in both Φ and Ψ, then the condition is always satisfied. Finally, since the function to be maximized in (36) is an upper bound for the one in (35), the result easily follows. ■

Proof of Proposition 6: We know by Lemma 10 that if the profit-maximizing marginal type were in the interior of the valuations' support, the seller would never use a limited-availability scheme. Hence, her profits must be strictly decreasing in v. A necessary condition for this is $\overline{v}<3\underline{v}$.

If the seller employs the limited-availability strategy that makes a type-$(\underline{v},\underline{v})$ consumer exactly indifferent between buying only the bargain and always buying, then for all consumers not buying is not a PE. It is easy to see that for all types on the 45-degree line, with $v_1=v_2=v$, the PPE is to always buy since for these types, like for the marginal one, expected gain-loss utility in the item is zero and

$$\frac{dEU[\{1,2\}\mid\{1,2\}]}{dv}=1.$$

To see that all consumers prefer to always buy, irrespective of their type, it suffices to show that types at the corners $(\overline{v},\underline{v})$ and $(\underline{v},\overline{v})$ prefer to always buy. Consider type $(\overline{v},\underline{v})$ first. Planning to buy item 2 if available and nothing otherwise is not a PE because $v_1>v_2$ and $p_1<p_2$. Furthermore, this consumer is indifferent between planning to buy only item 1 and planning to always buy since, for $v_1>v_2$:

$$\frac{dEU[\{1,\varnothing\}\mid\{1,\varnothing\}]}{dv_1}=\underline{q}-\underline{q}\left(1-\underline{q}\right)\eta(\lambda-1)=\frac{dEU[\{1,2\}\mid\{1,2\}]}{dv_1}.$$

Next, consider type $(\underline{v},\overline{v})$. If he plans to buy only item 1, this consumer's expected utility level is the same as that of the marginal type $(\underline{v},\underline{v})$, since $EU[\{1,\varnothing\}\mid\{1,\varnothing\}]$ does not depend on v_2. Furthermore, this consumer is indifferent between planning to buy only item 2 and planning to always buy since, for $v_1<v_2$:

$$\frac{dEU[\{2,\varnothing\}\mid\{2,\varnothing\}]}{dv_2}=\left(1-\underline{q}\right)-\underline{q}\left(1-\underline{q}\right)\eta(\lambda-1)=\frac{dEU[\{1,2\}\mid\{1,2\}]}{dv_2}.$$

Notice that

$$\left(1-\underline{q}\right)-\underline{q}\left(1-\underline{q}\right)\eta(\lambda-1)>0$$
$$\Leftrightarrow\ 1-\underline{q}\eta(\lambda-1)>0$$

which is always true for $\eta\le 1$. Therefore, this consumer's PPE is to always buy.

Since the PPE plan for types $(\overline{v},\underline{v})$ and $(\underline{v},\overline{v})$ is to always buy and these are the types with the most asymmetric preferences, then it follows that all consumers with $(v_1,v_2)\in(\underline{v},\underline{v})\times(\overline{v},\overline{v})$ will also prefer to always buy.

For this limited-availability scheme, the seller's profit is equal to $\pi^{**}_{LA}(\underline{v}) = \Phi\underline{v} - c$. This scheme is profit-maximizing if and only if

$$\pi^{**}_{LA}(\underline{v}) > \pi^{**}(v^{**})$$

$$\Leftrightarrow \quad \Phi\underline{v} - c > \frac{\left(3\overline{v} - 2\underline{v} - c - \sqrt{-6\overline{v}\underline{v} - 2\underline{v}c + 3\overline{v}^2 + 4\underline{v}^2 + c^2}\right)}{27\left(\overline{v} - \underline{v}\right)^2} \times \tag{39}$$
$$\left(2\underline{v} - 2c + \sqrt{-6\overline{v}\underline{v} - 2\underline{v}c + 3\overline{v}^2 + 4\underline{v}^2 + c^2}\right)\left(3\overline{v} - 4\underline{v} + c + \sqrt{-6\overline{v}\underline{v} - 2\underline{v}c + 3\overline{v}^2 + 4\underline{v}^2 + c^2}\right).$$

Solving (39) for $\overline{v}$, re-arranging and simplifying yield the desired result. ■

Proof of Proposition 7: Suppose $p_1 = p_1^{\min} \equiv \frac{1+\eta}{1+\eta\lambda}v$. Then, not buying is not a credible plan for the consumers and for a given q their perceived expected utility when planning to buy item 1 if available and nothing otherwise is

$$EU\left[\{1,\varnothing\} \mid \{1,\varnothing\}\right] = \widetilde{q}\left(v - p_1^{\min}\right) - \widetilde{q}\left(1 - \widetilde{q}\right)\eta\left(\lambda - 1\right)\left(v + p_1^{\min}\right) \tag{40}$$

where $\widetilde{q} = \chi q > q$. Consumers will be indifferent between the above plan and the plan to always if and only if

$$p_2 \leq v + \left[\frac{2\eta\left(\lambda - 1\right)\widetilde{q}}{1 + \eta\left(\lambda - 1\right)\widetilde{q}}\right] p_1^{\min} \equiv p_2^*.$$

This pair of prices provides the seller with profits equal to

$$qp_1^{\min} + (1 - q)\, p_2^*.$$

The above expression is maximized at

$$q_\chi = \frac{\sqrt{2\left(\eta + 1\right)\left(\eta + \lambda\eta + 2\right)\left(-\chi\eta + \lambda\chi\eta + 1\right)}}{\chi\eta\left(\lambda - 1\right)\left(\eta + \lambda\eta + 2\right)} - \frac{1}{\chi\eta\left(\lambda - 1\right)}.$$

Next, notice that expression (40) is a continuous function of $\widetilde{q}$, and its value is 0 for $\widetilde{q} = 0$ and $v - p_1^{\min} > 0$ for $\widetilde{q} = 1$. Furthermore, its derivative evaluated at $\widetilde{q} = 0$ is equal to

$$\begin{aligned} & v - p_1^{\min} - \eta\left(\lambda - 1\right)\left(v + p_1^{\min}\right) \\ = \quad & -\frac{\eta\left(\lambda - 1\right)}{1 + \eta\lambda}\left(1 + \eta + \eta\lambda\right) v < 0 \end{aligned}$$

and therefore, it must have another zero. It follows that

$$\begin{aligned} 0 \quad = \quad & \widetilde{q}\left(v - p_1^{\min}\right) - \widetilde{q}\left(1 - \widetilde{q}\right)\eta\left(\lambda - 1\right)\left(v + p_1^{\min}\right) \\ \Leftrightarrow \quad & \widetilde{q} = \frac{1 + \eta + \eta\lambda}{2 + \eta + \eta\lambda} \\ \Leftrightarrow \quad & \chi q_\chi = \frac{1 + \eta + \eta\lambda}{2 + \eta + \eta\lambda} \\ \Leftrightarrow \quad & \chi = \frac{\lambda^3\eta^3 + \lambda^2\eta^3 + 4\lambda^2\eta^2 - \lambda\eta^3 + 4\lambda\eta^2 + 8\lambda\eta - \eta^3 + 6\eta + 6}{2\left(\eta + 1\right)\left(\eta + \lambda\eta + 2\right)} \equiv \widetilde{\chi}. \end{aligned}$$

Therefore, for $\chi \geq \widetilde{\chi}$ consumers' perceived expected utility is zero. But then,the seller can set $q = \frac{1}{\chi}$ and $p_1 = v$, and still leave consumers' perceived expected utility at zero. In this case, since consumers believe they will consume item 1 at price v for sure, the highest price they are willing to pay for item 2 if they do not find item 1 available is

$$p_2 = v\left(1 + \frac{\eta(\lambda - 1)}{1 + \eta\lambda}\right)$$

and it is easy to see that this scheme provides the seller with higher profits since consumers' realized consumption utility is at most zero in any contingency. ■

C Partial Commitment

While retailers frequently advertise their good deals, it is rather uncommon to see a store publicizing its high prices. Therefore, consistently with this observation about stores' advertising patters, in this section I assume that in period 0 the seller commits only to the price of the bargain $p_i^{\min}$, $i = 1, 2$, and its degree of availability. In this case, consumers form rational expectations about the price of the item that is not publicly advertised.

Suppose that the seller uses item 1 as a bargain by announcing that she has q units of it available for sale at price $p_1^{\min}$. Once at the store, a buyer who had planned to buy item 1 if available and item 2 otherwise will follow his plan and buy item 2 when this is the only item left in the store if

$$U[(v_2, p_2) \mid \{1,2\}] \geq U[(0,0) \mid \{1,2\}]$$

$$\Leftrightarrow \quad p_2 \leq \frac{(1+\eta\lambda)\, v_2 + \eta(\lambda-1)\, q\left(\frac{1+\eta}{1+\eta\lambda}\right) v_1}{1+\eta\lambda q + \eta(1-q)}. \tag{41}$$

Notice that this price is higher than the one we found under full commitment because now the price of the rip-off is the highest price consumers are willing to pay *ex-post.* However, for the consumers to be willing to make the plan of always buying to begin with, the seller's announced degree of availability for the bargain must be such that

$$EU[\{1,2\} \mid \{1,2\}] \geq EU[\{2,\varnothing\} \mid \{2,\varnothing\}]. \tag{42}$$

To have an optimum for the seller both conditions (41) and (42) have to bind, defining a system of two non-linear equations in q and p_2. The relevant solution is

$$\begin{aligned} p_2^* &= \frac{v_1(1+\eta)(1+2\eta) + v_2(1+\eta\lambda)(1+\eta+\eta\lambda) - \sqrt{Y}}{2\eta(1+\eta\lambda)} \\ \underline{q} &= \frac{v_2\lambda(1+\eta\lambda) - \frac{1+\eta\lambda}{2\eta(1+\eta\lambda)}\left[v_1(1+\eta)(1+2\eta) + v_2(1+\eta\lambda)(1+\eta+\eta\lambda) - \sqrt{Y}\right]}{v_1(1+\eta)(\lambda-1) + v_2(1+\eta\lambda)(\lambda-1)} \end{aligned}$$

where

$$Y \equiv v_1^2(1+\eta)^2(2\eta+1)^2 + v_2^2(1-\eta+\eta\lambda)^2(1+\eta\lambda)^2 - 2v_1v_2(1+\eta)(1+\eta\lambda)\left(-\eta - 2\eta^2 - \lambda\eta + 2\eta^2\lambda - 1\right).$$

Similarly, if the seller uses item 2 as a bargain, degree of availability of item 1 and its price are

$$
\begin{aligned}
p_1^* &= \frac{v_1\eta\left(\lambda-1\right)\left(1+\eta\lambda\right)+v_2\left(1+2\eta\right)\left(2+\eta+\lambda\eta\right)-\sqrt{Z}}{2\eta\left(1+\eta\lambda\right)} \\
\overline{q} &= \frac{v_2\left(2\lambda-\eta-2\eta^2-\eta\lambda-2\eta^2\lambda+\eta\lambda^2-2\right)-v_1\lambda\left(\eta-\eta^2+\eta\lambda+\eta^2\lambda+1\right)}{v_2\left(\lambda-1\right)\left(\eta+\lambda\eta+2\right)} \\
&+ \frac{\frac{1+\eta\lambda}{2\eta(1+\eta\lambda)}\left[v_1\eta\left(\lambda-1\right)\left(1+\eta\lambda\right)+v_2\left(1+2\eta\right)\left(2+\eta+\eta\lambda\right)-\sqrt{Z}\right]}{v_2\left(\lambda-1\right)\left(\eta+\eta\lambda+2\right)}
\end{aligned}
$$

where

$$
Z \equiv v_1^2\eta^2\left(1+\lambda\right)^2\left(1+\eta\lambda\right)^2+v_2^2\left(1+2\eta\right)^2\left(2+\eta+\lambda\eta\right)^2-2\eta v_1v_2\left(-\lambda+2\eta+2\eta\lambda+3\right)\left(1+\eta\lambda\right)\left(2+\eta+\eta\lambda\right).
$$

Compared to the situation where she is able to commit in advance to both prices, now the price of the rip-off is higher but the degree of availability of the bargain is higher as well. Intuitively, since the seller is charging a higher price for the rip-off, and the consumers anticipate this, she must compensate them with a higher ex-ante chance of making a deal otherwise they would not plan to always buy. Thus, given both prices, the seller is not choosing the degree of availability that maximizes her profits. This is because by not committing in advance to the price of the rip-off, the seller must use the degree of availability of the bargain to induce the consumers to select the to plan to always buy. Furthermore, the optimal degree of availability with full commitment takes into account also the difference in the marginal costs of the two items, whereas with partial commitment it does not. Therefore, the seller's profits are lower when she cannot commit to both prices.

Unfortunately, in this case it is hard to obtain a full characterization, like the one in proposition 1, for when the seller would find it profitable to use a limited-availability strategy made of bargains and rip-offs. Nevertheless, a combination of bargains and rip-offs might be profit-maximizing as the following example shows.

Example 8 *Let $v_1 = 250$, $v_2 = 230$, $c_1 = 20$ and $c_2 = 10$. If the seller produces only one good, then she would produce item 1 and price it at $p_1 = 250$, obtaining a profit of 230. Let $\eta = 1$ and $\lambda = 2$ and suppose the seller uses item 1 as a bargain by pricing it at $p_1^{\min} = \frac{500}{3}$. In this case the seller will also commit to sell $\underline{q} = \frac{2}{119}\sqrt{3459} - \frac{75}{119}$ units of item 1 and will price item 2 at $p_2^* = 710 - \frac{20}{3}\sqrt{3459}$, obtaining a profit of 250.15.*

Moreover, example 8 shows that also in this case of partial commitment the seller might prefer to use the superior item as the bargain, exactly for the same reason as in the analysis with full commitment.[50].

[50] For the parameters in example 8, if the seller were to use item 2 as the bait by pricing it at $p_2^{\min} = \frac{460}{3}$ then the optimal degree of availability of the bait would be $1-\overline{q} = \frac{1}{23}\sqrt{489} - \frac{12}{23}$ and the price of item 1 would be $p_1^* = 700 - \frac{50}{3}\sqrt{489}$ for a total profit of 237.52. Less than what the seller can obtain by using item 1 as the bait, but still better than what she would make by selling only item 1.

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