HOUSING AND FINANCIAL FRICTIONS IN A SMALL OPEN ECONOMY

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The role of the housing sector in the recent US crisis has served to emphasise the importance of understanding how shocks originating in that sector can influence broader economic activity. The crisis has also highlighted the importance of credit market linkages in amplifying real shocks. The comparatively high exposure of Australian households to housing-secured debt suggests that housing sector and credit market developments are potentially important drivers of fluctuations in the Australian economy. In this spirit we adapt a medium scale Dynamic Stochastic General Equilibrium (DSGE) model with a housing sector and an embedded financial accelerator mechanism following Iacoviello and Neri (2010), to an open economy setting and estimate using Australian data. We find that for some parameterisations shocks to the housing sector have a substantial impact on the broader economy and that the inclusion of the financial accelerator mechanism leads to consumption dynamics that are broadly consistent with wealth effects from housing.

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Tim Robinson and Michael Robson

1. Introduction

At the most basic level housing is a durable good that provides shelter, but in a modern developed economy housing also serves as most households’ primary store of wealth. The direct effects of residential investment on activity are relatively well understood because demand for housing is strongly linked to the need for housing services that naturally emerges as a result of population growth (see Berger-Thomson and Ellis (2004) and Leamer (2007)). However, the implications of the more prominent role of housing as a store of wealth that has accompanied financial market liberalisation are not as easy to identify.

One way housing may influence activity is through wealth effects on consumption. Numerous studies have tried to quantify indirect effects of housing wealth on activity via consumption, with mixed results in Australia and abroad (see for example Fisher, Otto and Voss (2010), Williams (2010) and Case, Quigley and Shiller (2011)). Alternatively, there have been fewer attempts to investigate the potential indirect effects of developments in the housing market on activity through changes in the household balance sheet. As housing is the primary store of wealth for households it also serves as the collateral supporting most borrowing. Fluctuations in house prices, therefore, have implications for the value of households’ collateral, their access to credit and their demand for housing. This paper will assess each of these channels using a structural model that incorporates housing production and credit.

The recent crisis in the US provides an example of the central role that housing can play as a driver of the business cycle, initially as the source of the shock; and then as an amplifier of the shock via financial linkages (Bernanke (2010)). Viewing the crisis through this lens the decline in lending standards and related credit boom through the early 2000s were structural vulnerabilities that amplified the impact of the initial housing market correction. Once prices began to fall a deflationary spiral followed driven by the elevated number of low credit quality borrowers with
high loan-to-valuation ratios that were particularly vulnerable to a fall in prices. The fallout in the housing market was exacerbated by a supply overhang which emerged as new houses continued to enter the market despite the fall in demand, a result of the inherent inertia in residential investment (Ellis (2010)). In this paper we aim to investigate the importance of the housing sector as a driver of activity in the broader macroeconomy and its interaction with the financial sector, which may amplify housing sector specific shocks.

A range of existing studies have developed structural models exploring activity in the housing sector and spill-overs to the rest of the economy. Davis and Heathcote (2005) incorporate multiple production sectors (including housing) into a Real Business Cycle (RBC) model and find this feature useful in matching some of the properties of the data. Another strand of literature embeds a financial accelerator mechanism centered on housing credit into an otherwise standard DSGE model. In these models credit flows between two types of households differentiated by their willingness to borrow to fund current purchases, with housing as collateral (Iacoviello 2005). Iacoviello and Neri (2010) extend the earlier model of Iacoviello (2005) to include key aspects of housing construction from Davis and Heathcote (2005).

In this paper we build on Iacoviello and Neri (2010), adding open-economy features and estimating the model using Australian data. We then examine how the shocks from the housing sector influence developments in the rest of the economy, and whether the inclusion of a financial accelerator mechanism improves the ability of the model to match the data when compared to a model without credit. The inclusion international credit flows in the model also allows us to investigate the importance of external shocks as a driver of fluctuations in domestic credit flows, a useful feature given the importance of foreign funding for the Australian banking system.

The model also allows us to investigate the potential for housing related consumption wealth effects. In the closed-economy model of Iacoviello and Neri (2010), estimated using US data, they find positive comovement of consumption and house prices that is evidence in favour of such effects. Empirical studies using Australian data, such as Tan and Voss (2003), Dvornak and Kohler (2007), Fisher et al (2010) and Williams (2010), find varying degrees of positive consumption wealth effects from increases in house prices.¹

¹
Estimating the model using 14 observable variables we find that the model with housing is able to generate plausible results both in terms of parameter estimates and dynamics. Fluctuations in the housing market, while responsive to key shocks in the model, are largely driven by the sector-specific shocks with the housing preference shock particularly important. In contrast the housing-sector specific shocks have little influence on developments outside the housing sector for most parameterisations. The combination of the financial accelerator mechanism and a high steady-state loan-to-valuation ratio (LVR) is an exception. With this combination the model is able to replicate the comovement of consumption and house prices in response to a housing preference shock that is found in studies using US data. When the steady-state LVR is set at a low level the evidence of consumption wealth effects from housing is not present, nevertheless the inclusion of the financial accelerator mechanism does alter the model dynamics although the results from a cycle-dating exercise suggesting that the strength of the financial accelerator mechanism (governed by the steady-state LVR) has surprisingly little influence on the models ability to match the growth cycle properties of the data.

The paper is structured as follows. Section 2 provides a more detailed discussion of related models before outlining the key details of the model employed here. Section 3 outlines the data used in the process of estimation. Section 4 discusses the estimation methodology as well as the process of calibration and the choice of priors. The results from estimation are presented in Section 5 including parameter estimates, impulse response functions, forecast-error variance and historical decompositions for a selection of variables as well as some cycle-dating exercises. Section 6 concludes.

2. Model

2.1 Related Models

2.1.1 Housing and Collateral Channels in General Equilibrium models

The vein of literature that centers on adding housing to DSGE models has expanded rapidly over the past decade. Initial efforts focused on housing as

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1 It is worth noting that in general these studies also find that it is long-run (or permanent) price changes, rather than short-term fluctuations, that have a significant effect on consumption.
collateral underpinning the borrowing behaviour of households. Aoki, Proudman and Vlieghe (2004) present a model along the lines of Benanke, Gertler and Gilchrist (1999) (BGG) in which the borrowing of the entrepreneurs in the model is tied to housing rather than the firm’s net worth. Housing investment in this model involves the transformation of the intermediate good subject to an adjustment cost, and the rate at which agents can borrow in this model is tied to their leverage ratio.

Iacoviello (2005) moves away from the external finance premium as a mechanism for constraining liquidity (as in the BGG model) and instead introduces financing constraints by including two different types of households. One of group of households is impatient (they discount future consumption more than the typical household) and borrow to fund current period expenditure. Impatient households’ borrowing is constrained by the value of their holdings of housing. Movements in house prices in the initial period affect households ability to borrow in future and initiate a potentially powerful amplification process. A shock causing a reduction in demand for housing by impatient households results in a fall in the value of their collateral holdings and a subsequent reduction in available credit. Since the supply of housing in this model is fixed the user cost of housing must decline to encourage unconstrained households to purchase housing to clear the market and thus the price of housing declines further depressing collateral values in the initial period. The fall in collateral values implies a further fall in borrowing and housing demand in the next period, in an intertemporal multiplier effect that compounds the contraction in the initial period. This structure allows the model to generate comovement of real house prices and aggregate consumption in response to a shock to housing preferences as well as replicating the response of spending to an inflation shock. A drawback of this model is the fixed stock of housing precludes the consideration of residential investment, which is one of the key variables of interest in the current study.

Iacoviello and Neri (2010) deal explicitly with the supply side of housing in more detail. The paper draws heavily on earlier work by Davis and Heathcote (2005) which demonstrates that the introduction of a housing production sector in an RBC framework enables the model to match the relative volatility of residential investment (compared with business investment) and the comovement of consumption, GDP, and business and residential investment. Iacoviello and Neri (2010) incorporate this supply-side for housing into a model that is very
similar to that used in Iacoviello (2005). They remove entrepreneurs so that the credit channel only operates between the two household types, and makes housing a combination of capital, intermediate goods, labour, and land. In this framework the positive comovement of consumption and house prices in response to a housing preference shock is maintained, additionally results from estimation using US data suggest that shocks to technology and housing preferences explain most of the variation of residential investment. The model of Iacoviello and Neri (2010) forms the basis for the model employed here.

In this paper we incorporate some of the key small-open economy aspects that are important for Australia. Chistensen, Corrigan, Mendicino and Nishiyama (2009) undertake this exercise for Canada and Bao, Lim and Li (2009) adapt the model developed in Aoki et al. (2004) and estimate it using Australia data. However, both papers employ a very simple production function for housing. Rather than including specific inputs to housing production they introduce housing as a transformed intermediate good. This potentially diminishes the channels by which the housing production sector may influence the broader economy. In this paper housing specific labour, capital and domestic final goods are employed in production in an effort to strengthen linkages with the economy, which sources such as input-output tables suggest are important in the Australian housing market.

2.2 The Model

The model consists of two types of households differentiated by their discount factors. Patient households have a higher discount factor are more inclined to save, households with a lower discount factor borrow to fund current period consumption and are labelled impatient. This assumption drives credit flows between the households, with housing used as collateral in the model. This means that a shock that negatively impacts the value of impatient households holdings of housing will negatively impact available collateral, and feed back into lower demand for housing and consumption goods, both contemporaneously and in future periods, given the nature of the borrowing constraint. Housing is produced using a technology which employs specialised labour from both household types, capital, domestic final goods and land. The supply of new land is fixed in each period. Domestic firms produce differentiated goods using specialised labour and capital, the goods are sold at a mark-up to final goods bundlers or exported. Final goods are consumed, transformed into capital by patient households or used in
the production of housing. Monopolistically competitive importing firms import foreign goods and sell them to households to be bundled with domestic goods to produce final consumption goods. Monetary policy is conducted via a Taylor rule and facilitated by transfers to households.

2.2.1 Households

The two types of households in the model work, consume final consumption goods and accumulate housing. They derive utility from consumption, $C_{j,t}$, housing, $H_{j,t}$, and real money balances, $m_{j,t}$, and disutility from labour, $L_{j,t}$. They maximise lifetime utility according to

$$U(C_{j,t}, L_{j,t}, H_{j,t}, M_{j,t}) = \max_{C_{j,t}, L_{j,t}, H_{j,t}, m_{j,t}}$$

$$E_t \sum_{t=0}^{\infty} \beta^t_j \left[ \kappa_j \ln(C_{j,t} - \Omega C_{j,t-1}) + J_t \ln H_{j,t} + \ln m_{j,t} - \chi_t \frac{(L^d_{j,t})^{1+\xi} + (L^h_{j,t})^{1+\xi}}{1 + \psi} \right],$$

where $j$ is equal to 1 for patient households and 2 for impatient households. Housing is included in the utility function because of the dual role it serves for most households as a source of housing services (for example shelter) but also as a significant store of wealth. The labour share of income represents the economic size of each household type in the model. All households supply labour to intermediate good firms, $L^d_{j,t}$, and housing producers, $L^h_{j,t}$. Total labour supplied by each household is a constant elasticity of substitution (CES) aggregate of the two types of industry specific labour. When the intersectoral elasticity of labour substitution, $\xi$, is positive and non-zero the two types of labour are imperfect substitutes, and wages will vary between the sectors, with a higher value of $\xi$ implying greater sector specificity of labour. In this we follow Iacoviello and Neri (2010) and Horvath (2000). All households are subject to a common consumption preference shock, $\kappa_j$, a housing preference shock, $j_t$, which can be thought of as an exogenous shock to housing demand, and a labour supply shock, $\chi_t$. They also exhibit external habits in consumption where household preferences reflect the desire to smooth consumption across time using the
aggregate consumption of their household type in the previous period as an anchor
or reference point. The parameter $\Omega$ governs the degree of habit persistence.

Patient households maximise utility according to their real flow budget constraint

\[
C_{1,t} + \frac{P^h_t I^h_t}{P_t \eta} + \frac{P^h_t I^l_t}{P_t \eta} + q^h_t I^h_t + \frac{R_{t-1} b_{1,t-1}^h}{\pi_t} + S_t b^f_t \frac{\Pi_t}{\eta} + m_{1,t} + \frac{\Pi^m_{1,t}}{\eta} + \Pi^l_{1,t} + \frac{R^f_{t-1} S_t b^f_{t-1}}{\eta} + r^h_t \frac{K_{t-1}^h}{\eta} + P^h_t \Pi_t + \\
\eta + \frac{R^f_{t-1} S_t b^f_{t-1}}{\eta} + \Phi(a_{t-1}, e^{\theta^{b_{t-1}}}) + \frac{m_{1,t-1}}{\pi_t} + t^m_{1,t} + t^l_{1,t}.
\]

(2)

The final consumption good is a CES bundle of the domestically produced good
and imported consumption goods.\footnote{\textsuperscript{2}}

Patient households are able to invest in housing assets, $IH_{1,t}$, with their holdings
$H_{1,t}$ evolving according to a standard law of motion that accounts for the effects
of depreciation. The real price of housing is $q^h_t$ (where the aggregate consumption
good is the numeraire so that nominal house prices, are deflated by the CPI, $P_t$).
Due to their lower rate of discounting patient households also lend to impatient
households, with a positive value of $b_{1,t}$ indicating lending. Patient households
receive interest flows in each period stemming from the preceding periods lending
where $R_t$ is the nominal gross interest rate. They also have access to international
financial markets from which they can borrow, $b^f_t$, with borrowing limited by
intermediation costs similar to Benigno and Thoenissen (2008) ,$\Phi()$, which are

\[
C_t = \left( \gamma^\sigma C_t^h \frac{\sigma-1}{\sigma} + (1-\gamma) \frac{\sigma-1}{\sigma} C_t^m \frac{\sigma-1}{\sigma} \right) \frac{1}{\sigma},
\]

where $\gamma$ will influence the degree of openness of the economy. The related price index (the CPI)
is

\[
P_t = \left( (1-\gamma) P_t^m \frac{1-\sigma}{1-\sigma} + \gamma P_t^h \frac{1-\sigma}{1-\sigma} \right)^{\frac{1}{1-\sigma}}.
\]
increasing in the nominal net foreign assets to output ratio, \( a_t \), a standard method applied in ‘closing’ small-open economy models, and a shock, \( \xi_{t-1}^b \). \( R_t^f \) is the foreign nominal gross interest rate.

Patient households invest in the stock of capital employed both by domestic intermediate goods firms, \( I_t \), and the housing construction sector, \( I_{h_t}^h \). Impatient households do not invest in the capital stock. Capital for both sectors (\( K_t \) and \( K_{h_t}^h \)) is formed from domestic final goods (priced at \( P_{t}^h \)) and evolve according to a standard law of motion subject to adjustment costs which are a function of the rate of investment as in Christiano, Eichenbaum and Evans (2005) and, for capital employed by the intermediate goods producers, an investment efficiency shock, \( \varepsilon_t \). After it is formed patient households provide the capital to the relevant production firm and receive rental payments at rates \( r_t^k \) and \( r_t^h \). In each period they also receive wages from these firms in return for their labour. The wage does not, however, flow directly from the firm to the household. Instead the households sell their labour to a labour union, in return for real wages \( w_t^i \). The labour unions differentiate the labour and sell on to a costless labour bundler who rents it to firms at a mark-up over the base wage charged by households (see Smets and Wouters (2007) for a similar labour market structure). Patient households receive transfers from the government in each period. In addition to the usual money transfers, \( t_{m,t} \), that allow the conduct of monetary policy in the model, they receive a share of the revenues derived from the sale of land to housing producers, \( t_{l,t} \). Patient households also receive profits from the monopolistically competitive firms they own - the domestic intermediate good producers \( \Pi_t \), importing firms \( \Pi_{m,t}^m \), and labour unions \( \Pi_{l,t} \).

The patient household’s optimality conditions are:

---

3 We assume that in the steady state \( \Phi(\bar{a}, 1) = 1 \), that the function is twice differentiable and that \( \Phi_{\xi}^b(\bar{a}, 1) = 1 \) and \( \Phi_a(\bar{a}, 1) > 0 \).

4 There are four labour unions in the model, one for each sector-household combination.

5 Note also that \( \eta \) is the population weight of the patient households while \( \tau_{h}^h \) is the price of final goods relative to the CPI. \( \eta \) is used to scale the variables whose evolution is determined by the patient households alone and \( \tau_{h}^h \) is applied to ensure that variables such as the return on capital are consistently deflated both in the household and firms problems.
which are respectively the consumption Euler equation, the labour supply expressions for each production sector, an uncovered interest parity condition and the housing optimality condition. Note here that $p_{st}$ is the change in the nominal exchange rate, $p_t$ is CPI inflation and $p_{ht}$ is domestic final goods price inflation. The money demand condition is standard and will not be discussed further. Patient households also choose the level of investment in each sectors capital stock according to rules governing the respective Tobin’s Q variables, $q_t$ and $q_{kh}^t$, and the investment optimality conditions

\[
q_t = \beta_1 E_t \left[ \frac{\pi_{t+1}}{R_t} \left( q_{t+1} (1 - \delta) + r_{t+1}^k \right) \right] 
\]  

\[
q_{kh}^t = \beta_1 E_t \left[ \frac{\pi_{t+1}}{R_t} \left( q_{kh}^{t+1} (1 - \delta_{kh}) + \frac{r_{t+1}^h}{\pi_t^h} \right) \right]
\]
\[ 1 = q_t \varepsilon_t \left( 1 - S' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} - S \left( \frac{I_t}{I_{t-1}} \right) \right) \]
\[ + \beta_1 E_t \left[ q_{t+1} \frac{L_{1,t+1}}{L_{1,t}} \varepsilon_{t+1} \pi_{t+1}^{h} \frac{1}{\pi_{t+1}} \left( S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right) \right] \] (10)

\[ 1 = q^{kh}_t \left( 1 - S' \left( \frac{I^{kh}_t}{I^{kh}_{t-1}} \right) \frac{I^{kh}_t}{I^{kh}_{t-1}} - S \left( \frac{I^{kh}_t}{I^{kh}_{t-1}} \right) \right) \]
\[ + \beta_1 E_t \left[ q^{kh}_{t+1} \frac{L_{1,t+1}}{L_{1,t}} \varepsilon_{t+1} \pi_{t+1}^{h} \frac{1}{\pi_{t+1}} \left( S' \left( \frac{I^{kh}_{t+1}}{I^{kh}_t} \right) \left( \frac{I^{kh}_{t+1}}{I^{kh}_t} \right)^2 \right) \right] \] (11)

As previously discussed impatient households are not involved in investment decisions or in international transactions, and therefore their real budget constraint is

\[ C_{2,t} + q^h_t I H_{2,t} + \frac{R_{t-1} b_{2,t-1}}{\pi_t} + m_{2,t} = w^d_{2,t} L^d_{2,t} + w^h_{2,t} L^h_{2,t} + b_{2,t} \]
\[ + \frac{m_{2,t-1}}{\pi_t} + \Pi^l_{2,t} + t^m_{2,t} + t^l_{2,t}. \] (12)

As with the patient households impatient households work in both production sectors, they receive transfers from the government both of money and of revenues related to the sale of land. They also receive profits from labour unions. These inflows are supplemented by borrowing from patient households and are used to purchase consumption goods and housing. Since patient households have no way of knowing the credit worthiness of the borrower or forcing them to repay their loans at the end of each period they require housing as collateral, and since repossession in the event of default is costly they limit borrowing to some fraction, \( m_t \), of available collateral which can be thought of as a loan-to-valuation ratio, and which evolves according to an AR(1) process. Default and repossession are assumed to occur in the period after the loan is taken out so that the borrowing constraint is a function of the discounted value of housing assets in the next period:
Assuming that borrowing does not move too far away from its steady state the constraint will always bind (this assumption is necessary since we log-linearise the model around the steady state). The timing here is important. As previously outlined the static multiplier will act contemporaneously in the event of a negative housing demand shock, the value of housing collateral held by impatient households falls as the quantity held contracts and house prices fall. But the contraction in collateral in the initial period has implications for borrowing in future periods. A decline in available credit in the next period will result in a further reduction in housing purchases and another contraction in collateral, which will again feed into housing demand in the next period and so on. Kiyotaki and Moore (1997) label this effect the dynamic multiplier and find that it is much more important than the static multiplier in its effect on activity. The effect is harder to unambiguously identify here because where the supply of collateral (land) in Kiyotaki and Moore (1997) was fixed by assumption and the price forced to adjust to entice the patient agents to purchase it, whereas in this model a quantity adjustment is also likely to occur and manifest in a contraction in the production of housing. This is likely to attenuate the dynamic multiplier to some degree but is unlikely to shut it off entirely.

The first order conditions for impatient households are:

\[ \frac{\kappa_t}{C_{2,t} - \Omega C_{2,t-1}} = \beta_2 R_t \left[ \frac{\kappa_{t+1}}{(C_{2,t} - \Omega C_{2,t-1}) \pi_{t+1}} \right] + \Lambda_{2,2t} R_t; \]  

\[ w_{2,2t}^d = \chi_t L_{2,t}^d \frac{\xi}{(1+\xi)} \left( L_{2,t}^d 1+\xi + L_{2,t}^h 1+\xi \right) \frac{C_{2,t} - \Omega C_{2,t-1}}{\kappa_t}; \]  

\[ w_{2,2t}^h = \chi_t L_{2,t}^h \frac{\xi}{(1+\xi)} \left( L_{2,t}^d 1+\xi + L_{2,t}^h 1+\xi \right) \frac{C_{2,t} - \Omega C_{2,t-1}}{\kappa_t}; \]
\[
\frac{q_t^h \kappa_t}{C_{2,t} - \Omega C_{2,t-1}} = \frac{J_t}{H_{2,t}} + \beta_2 E_t \left[ \frac{q_{t+1}^h (1 - \delta^h) \kappa_{t+1}}{C_{2,t+1} - \Omega C_{2,t}} \right] + \Lambda_{2,2t} m_t E_t \left[ q_{t+1}^h \pi_{t+1} \right]. \tag{17}
\]

These are the consumption Euler equation (where \( \Lambda_{2,2t} \) is the Lagrange multiplier on the borrowing constraint), the labour supply conditions for each sector and the housing optimality condition, noting that the labour supply expressions are of the same form as those for the patient households and that the Euler equation and housing condition are also similar although each is augmented with a term derived from the borrowing constraint. These terms are one channel linking borrowing and the decisions about consumption and housing taken by impatient households in the model.

2.2.2 Production

**Domestic Goods**

Intermediate goods producers minimise total costs, \( t c_t \):

\[
\min_{L^d_{1,t}(i),L^d_{2,t}(i),K_{t-1}(i)} t c_t = \frac{w_{1,t}^h}{\tau_t} L^d_{1,t}(i) + \frac{w_{2,t}^h}{\tau_t} L^d_{2,t}(i) + r_t^k K_{t-1}(i), \tag{18}
\]

subject to their production technology

\[
Y_t(i) = Z_t K_{t-1} \alpha (L^d_{1,t}(i) \mu L^d_{2,t}(i) - \mu)^{1-\mu} \mu^{1-\alpha}, \tag{19}
\]

which is a Cobb-Douglas constant returns to scale production function with capital and labour from each of the household types as inputs and an AR(1) technology shock, \( Z_t \), and where \( i \) in this instance is a firm index.

The goods are differentiated and the market monopolistically competitive. Calvo pricing is assumed, with \( P_t^h \), the price of the domestic final good, a mark-up, \( \delta_t^h \), over marginal cost. Firms that are unable to reset prices to the optimal level in any period index to the previous period’s CPI inflation. This results in the standard
New Keynesian Phillips Curve augmented with a backward looking inflation term. A cost-push shock is also added to the Phillips curve by employing a time varying elasticity of substitution between differentiated goods, $\lambda_t$. Some of the goods are sold to a perfectly competitive final goods producer who costlessly bundles to produce domestic final goods, $D_t$, according to an Armington aggregator

$$D_t = \left[ \int_0^1 D_t(i) \frac{\lambda_{i-1}}{\lambda_i} \, di \right]^{\lambda_i},$$  \hspace{1cm} (20)$$

the rest are exported. Aggregating across individual intermediate goods producers:

$$Y_t = D_t + X_t.$$  \hspace{1cm} (21)$$

The final goods producers sell to households who bundle the goods with imports to form final consumption goods, and in the case of patient households transform some into capital for use in future production. The domestic final good is also purchased by housing producers. The market clearing condition for domestic final goods is

$$D_t = C_t^h + I_t + I_t^h + Y_t^h.$$  \hspace{1cm} (22)$$

**Housing**

Housing producers maximise profits

$$\pi_t^h = q_t^h IH_t - w_{1,t}^h L_{1,t}^h - w_{2,t}^h L_{2,t}^h - r_t^h K_{t-1}^h - \tau_t^l L_t^* - \tau_t^h Y_t^h$$  \hspace{1cm} (23)$$

subject to their production technology

$$IH_t = Z_t Z_t^h K_{t-1}^h L_t^* Y_t^h (L_{1,t}^h L_{2,t}^h)^{1-\theta^h-\theta^l-\theta^y}.$$  \hspace{1cm} (24)$$
which is a constant returns to scale Cobb-Douglas production function with capital, $K_{t-1}^h$, domestic final goods, $Y_t^h$, specialised labour from both households, $L_{1,t}^h$ and $L_{1,t}^d$, and land, $L^*$. As well as the economy wide technology shock, $Z_t$, there is an AR(1) sector specific technology shock, $Z_t^h$. A set amount of land, normalised to 1, is released in each period (this can be thought of as a government land release). The limited supply of land in each period has a similar effect to adding adjustment costs to production, so that housing production cannot completely adjust to changes in demand and will lead to increased house price volatility. This feature is included in an attempt to match the sluggish response of residential investment to changes in demand for housing that is a well known feature of the housing market (Ellis (2010)). The inclusion of domestic final goods as an input to housing production is an attempt to better capture the linkage between the housing sector and the rest of the economy and is supported by analysis of input-output tables for Australia which suggest a strong multiplier effects from residential construction activity.\footnote{One obvious limitation of this model is that the housing production sector only produces output equivalent to new housing. In reality a significant portion of residential investment in any period is likely to be alterations and additions to established dwellings. Incorporating this activity into the model framework is a potentially interesting avenue for further research.}

2.2.3 Importers

Importers purchase homogeneous foreign consumption goods at price, $P_{m1}^t = S_t P_f^t$, where $S_t$ is the nominal exchange rate and $P_f^t$ is the foreign price level. Importers differentiate goods before selling them at a markup over marginal cost, $X_{t}^{m}$, to a good bundler who then sells to the households who combine the final imported good with the domestic final good to produce the final consumption good. The monopolistically competitive importers are assumed to employ Calvo pricing and index to the previous periods import price inflation when they are unable to optimally reset their prices. The result is a Phillips Curve of the same form as that employed by the domestic intermediate goods producers, and as with the domestic Phillips Curve a cost-push shock is incorporated. In the current model it is assumed that all imports are consumed.\footnote{An interesting avenue for further research would be to allow some proportion of imports to flow into the formation of capital.}
2.2.4 Foreign Sector

We model export demand using a typical demand function following Kollmann (2002)

\[ X_t = \left( \frac{P^h_t}{S_t P^f_t} \right)^{-\lambda^f} Y_t^{\lambda^f}. \]  

(25)

Combining the household budget constraints, expressions for profits, and market clearing conditions we are able to derive an expression for the trade balance

\[ \frac{X_t}{Y_t} - \frac{P^m_t C^m_t}{P^h_t Y_t} = a_t - R_{t-1}^f \Phi(a_t, e^x_t) \pi^s_t \frac{1}{\pi^h_t} \Delta Y_t a_{t-1}. \]

The remainder of the foreign sector is modelled as a VAR(2) including foreign inflation, output and interest rates.

2.2.5 Value-added

We develop an expression for the combined value added of the two productive sectors. A simple summation of output from the sectors is not possible because some of the final goods serve as inputs to the production of housing. Our definition of real value added is then

\[ VA_t = (Y_t - Y_t^h) \tau_t^h + q_t^h IH_t, \]  

(26)

where we deflate by the CPI.

2.2.6 Aggregators

Finally the aggregators for total consumption and housing are the population weighted sums of each households respective holdings are
\( C_t = \eta C_{1,t} + (1 - \eta)C_{2,t}, \) \hspace{1cm} (27)

and

\( H_t = \eta H_{1,t} + (1 - \eta)H_{2,t}. \) \hspace{1cm} (28)

3. Data

Fourteen observable variables are employed in estimating the model, 11 in the core model and 3 in the foreign VAR(2). The sample period is 1993:1-2011:1. In the model GDP, residential investment, business investment, consumption, house prices, total hours worked and household credit (all real) are detrended using a Hodrick-Prescott filter; the remainder, the domestic policy rate, domestic prices, unit labour costs and import prices are log-differenced and demeaned. The data entering the VAR is transformed in a similar way with major trading partner GDP detrended and inflation and G7 policy rates log-differenced and demeaned. One drawback of detrending the data is that we are unable to address the impact of population growth which may be expected to be an interesting avenue of investigation in the housing market. The inclusion of trends in the model is one obvious extension.

In this model, as in most macroeconomic models, the match between the available data and the equivalent model concept is not exact. While we do not anticipate that the extent of measurement error in this model is especially great (relative to similar studies), it is worth discussing the mismatch between the data and model concepts of household credit. In our estimation we map total household credit to \( b_2 \) which is the credit extended to impatient households (Iacoviello and Neri (2010) do not employ credit as an observable variable in their estimation). In a closed economy this would be entirely appropriate and a very close conceptual match, but because we allow the patient households to borrow from the external sector and use the borrowed funds for the purchase of consumption goods and housing (rather than all of these funds being on-lent to impatient households or used for

---

8 The key activity variables are all private non-government.
the purchase of investment goods), the household credit series we employ as an observable is likely to include credit extended to both household types. The data underlying our credit observable do not allow us to construct a separate impatient household credit series. In light of this we persist with the aggregate measure, but this limitation should be kept in mind when interpreting the results, particularly the importance of the loan-to-valuation ratio shock in explaining the credit series.

4. Estimation

4.1 Methodology

The model is linearised around the steady-state, and solved and estimated in Dynare. We informally calibrate some of the key parameters in the model in an attempt to match the steady-state values for some of the key expenditure ratios in the model to their sample means. We also set values for some parameters that are known to be difficult to identify in estimation, in such cases we employ values that are common in the literature. For the parameters to be estimated we employ generally similar prior distributions to other Australian studies or Iacoviello and Neri (2010) (see Table 3). The posterior distributions of the parameters are estimated using the Metropolis-Hastings algorithm and Markov Chain Monte Carlo (MCMC) methods, using two chains of 500 000 draws and a burn-in phase of 200 000 draws.

4.2 Calibration

For some of the parameters that are assigned values we follow Iacoviello and Neri (2010). There are two reasons for this. First, the core of the model here is obviously similar to that employed by Iacoviello and Neri (2010). Second, we have little disaggregated data available for Australia (particularly disaggregated industry level labour and wages data) that could be used to inform our own calibration or improve that used in the earlier paper. We set the shares of land, \( q^l \), capital, \( q^h \), and intermediate goods, \( q^y \) in housing construction to 0.1. The patient households discount factor, \( \beta_1 \), is set to 0.9925 to achieve a steady-state real interest rate of 3 per cent. We set the value of \( \beta_2 \) at 0.95 as used by Iacoviello (2005).\(^9\) In the full model we assume that impatient households borrow

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9 See Iacoviello (2005) for a discussion of the studies on which this value is based.
Table 1: Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibrated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>Population weight</td>
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</tr>
<tr>
<td>$\beta_1$</td>
<td>Patient household discount factor</td>
<td>0.9925</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>Impatient household discount factor</td>
<td>0.95</td>
</tr>
<tr>
<td>$\tilde{j}$</td>
<td>Housing weight</td>
<td>0.2</td>
</tr>
<tr>
<td>$\tilde{m}$</td>
<td>Steady-state loan-to-valuation ratio</td>
<td>0.6</td>
</tr>
<tr>
<td>$\bar{a}$</td>
<td>Steady-state net foreign assets ratio</td>
<td>2.1</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital weight in goods production</td>
<td>0.35</td>
</tr>
<tr>
<td>$\mu$</td>
<td>HH1 labour weight in goods production</td>
<td>0.65</td>
</tr>
<tr>
<td>$\theta_h$</td>
<td>Capital weight in housing production</td>
<td>0.1</td>
</tr>
<tr>
<td>$\theta_y$</td>
<td>Goods weight in housing production</td>
<td>0.1</td>
</tr>
<tr>
<td>$\theta_l$</td>
<td>Land weight in housing production</td>
<td>0.1</td>
</tr>
<tr>
<td>$\nu$</td>
<td>HH1 labour weight in housing production</td>
<td>0.65</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\delta_h$</td>
<td>Housing depreciation rate</td>
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</tr>
<tr>
<td>$\psi$</td>
<td>Inverse Frisch elasticity</td>
<td>1</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Intersectoral labour elasticity of</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>substitution</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Elasticity of substitution between</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>domestic and imported goods</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Domestic goods share in final consumption</td>
<td>0.6</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Interest debt sensitivity of</td>
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</tr>
<tr>
<td></td>
<td>foreign borrowing</td>
<td></td>
</tr>
<tr>
<td>$\lambda_f$</td>
<td>Foreign good elasticity of substitution</td>
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</tr>
<tr>
<td>$\lambda$</td>
<td>Domestic good elasticity of substitution</td>
<td>4</td>
</tr>
<tr>
<td>$\lambda_w$</td>
<td>Intrasectoral labour elasticity of</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>substitution</td>
<td></td>
</tr>
<tr>
<td>$\lambda_m$</td>
<td>Imported good elasticity of substitution</td>
<td>8</td>
</tr>
</tbody>
</table>

at an LVR that is closer to the average found in Australian data, 0.6, rather than a higher LVR closer to what may be expected for the average first home buyer as used by Iacoviello and Neri (2010). Finally, we use roughly the mid-point of the Iacoviello and Neri (2010) estimates of the intersectoral elasticity of labour for patient and impatient households, $\xi = 0.75$, as there is little Australian information available to inform this choice.

The calibrated parameters are set to broadly match some of the key nominal activity ratios found in data over the sample period, or, alternatively, on the basis of other data sources. In order to match the steady-state expenditure ratios we set
the steady-state weight on housing in the utility function, $\tilde{j}$, equal to 0.2 which is somewhat higher than the value used in Iacoviello and Neri (2010), $\alpha$ is set to 0.35 which broadly matches the empirical evidence on the weight of capital in domestic goods production from the Australian National Accounts. The elasticity of substitution of imported goods, $\lambda^m$, is set to 8 implying a steady-state mark-up of 14 per cent. The higher elasticity (in comparison with the elasticity of substitution for domestic goods) narrows the gap between the ratio of exports to value-added found in the data (0.19) and that implied by the model (0.18). The weight of domestic goods in final consumption, $\gamma$, is set to 0.6, which is important in matching the ratio of domestic consumption to value-added. Finally, we set $\sigma$, the elasticity of substitution of foreign and domestic goods, to 4. This value is somewhat higher than empirical estimates may suggest, however, as discussed in Adolfson, Laseen, Linde and Villani (2007) a high value of $\sigma$ may be needed to absorb the discrepancy between the high volatility of imports in the data and the relative smoothness of domestic consumption. This parameterisation also allows us to relatively closely match the investment ratios both for residential and non-residential investment which can be seen in Table 2.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Sample Mean</th>
<th>Model Implied</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>0.76</td>
<td>0.77</td>
</tr>
<tr>
<td>Non-residential</td>
<td>0.18</td>
<td>0.20</td>
</tr>
<tr>
<td>investment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residential</td>
<td>0.07</td>
<td>0.06</td>
</tr>
<tr>
<td>investment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exports</td>
<td>0.19</td>
<td>0.18</td>
</tr>
</tbody>
</table>

The parameters governing external interactions are based on values common in the small-open economy literature. The interest rate sensitivity to the ratio of foreign debt to output, $\varphi$, is set to 0.001 within the range of values used by Benigno and Thoenissen (2008). We set the elasticity of substitution for domestic goods, $\lambda$, and intrasectoral labour, $\lambda^w$, equal to 4 which implies a steady-state mark-up of 33 per cent, following Caglarini, Robinson and Tran (2011).

The quarterly depreciation rates for capital, $\delta$, and housing, $\delta^h$, are based on the implied quarterly rates from the Annual National Accounts publication of the
Australian Bureau of Statistics (ABS). The steady-state net foreign assets ratio is set to the quarterly sample average of foreign debt to GDP which is 2.1. The population weight of the patient households, $\eta$, as well as their labour weight, both in good, $\mu$, and housing production, $\nu$, are set to 0.65 based on the estimate that around 35 per cent of Australian households have outstanding mortgage debt (see Australian Bureau of Statistics (2006)).

4.3 Priors

The priors we employ are fairly standard in the small-open economy literature and are outlined in Table 3. The priors for the Calvo parameter and the parameter governing the degree of price indexation in the Phillips’s Curve are based on values that are higher than microeconomic studies would suggest. This is a common issue in DSGE based studies, the value used are those widely agreed upon as necessary to allow realistic inflation dynamics in the model (see Cagliarini et al (2011) for further discussion). The priors for the Taylor rule parameters are fairly standard with the exception of the prior for the parameter on growth in value added which is higher than that employed in other studies. We use this prior as some studies have found that the response to growth is more important than the response to the output gap (for example Justiniano and Preston (2010)). We employ fairly loose priors on the autoregressive parameters in the model similar to Iacoviello and Neri (2010). The prior means on the standard deviations of the labour disutility, investment efficiency and housing preferences shocks are set to 0.05 based on the expectation that the series that these shocks directly influence are generally more volatile. The value used for the remainder of the standard deviations is 0.01 for all except the monetary policy shock where the mean is set to 0.0025.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Density</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
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<td>$\zeta_1$</td>
<td>Beta</td>
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<td>0.1</td>
</tr>
<tr>
<td>$\zeta_\pi$</td>
<td>Normal</td>
<td>1.5</td>
<td>0.1</td>
</tr>
<tr>
<td>$\zeta_{va}$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.25</td>
</tr>
<tr>
<td>$\zeta_{\Delta va}$</td>
<td>Beta</td>
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<td>0.1</td>
</tr>
<tr>
<td>$S''$</td>
<td>Normal</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Beta</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>$\theta^d$</td>
<td>Beta</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>$\theta^m$</td>
<td>Beta</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Beta</td>
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<td>0.05</td>
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<tr>
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<td>Beta</td>
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<td>0.05</td>
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<tr>
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<td>Beta</td>
<td>0.3</td>
<td>0.05</td>
</tr>
<tr>
<td>$\rho^z$</td>
<td>Beta</td>
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<td>0.1</td>
</tr>
<tr>
<td>$\rho^zh$</td>
<td>Beta</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>$\rho^{zb}$</td>
<td>Beta</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>$\rho^j$</td>
<td>Beta</td>
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<td>0.1</td>
</tr>
<tr>
<td>$\rho^k$</td>
<td>Beta</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>$\rho^m$</td>
<td>Beta</td>
<td>0.75</td>
<td>0.1</td>
</tr>
<tr>
<td>$\rho^e$</td>
<td>Beta</td>
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<td>0.1</td>
</tr>
<tr>
<td>$\sigma^z$</td>
<td>Inv gamma</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma^{zh}$</td>
<td>Inv gamma</td>
<td>0.01</td>
<td>0.01</td>
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<tr>
<td>$\sigma^{zb}$</td>
<td>Inv gamma</td>
<td>0.01</td>
<td>0.01</td>
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<tr>
<td>$\sigma^j$</td>
<td>Inv gamma</td>
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<td>0.01</td>
</tr>
<tr>
<td>$\sigma^k$</td>
<td>Inv gamma</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma^m$</td>
<td>Inv gamma</td>
<td>0.01</td>
<td>0.01</td>
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<td>$\sigma^e$</td>
<td>Inv gamma</td>
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<td>0.01</td>
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<td>Inv gamma</td>
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<td>0.01</td>
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<td>Inv gamma</td>
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<td>0.01</td>
</tr>
<tr>
<td>$\sigma^{\lambda m}$</td>
<td>Inv gamma</td>
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<td>0.01</td>
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</table>
5. Results

5.1 Parameter Estimates

Table 4: Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter Prior</th>
<th>Mean</th>
<th>Mode</th>
<th>Posterior 90 Per Cent HPD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta^i$</td>
<td>0.5</td>
<td>0.79</td>
<td>0.75</td>
</tr>
<tr>
<td>$\zeta^\pi$</td>
<td>1.5</td>
<td>1.61</td>
<td>1.47</td>
</tr>
<tr>
<td>$\nu_{va}$</td>
<td>0.5</td>
<td>0.04</td>
<td>0.00</td>
</tr>
<tr>
<td>$\zeta_{\Delta va}$</td>
<td>0.4</td>
<td>0.35</td>
<td>0.23</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>0.3</td>
<td>0.58</td>
<td>0.46</td>
</tr>
<tr>
<td>$S'$</td>
<td>4</td>
<td>2.30</td>
<td>1.54</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.75</td>
<td>0.54</td>
<td>0.45</td>
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<td>$\theta_m$</td>
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<td>$\omega_m$</td>
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<td>0.75</td>
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<tr>
<td>$\rho^{\hat{zh}}$</td>
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<td>0.74</td>
<td>0.64</td>
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<tr>
<td>$\rho^{z_b}$</td>
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<td>0.88</td>
<td>0.84</td>
</tr>
<tr>
<td>$\rho^j$</td>
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<td>0.98</td>
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<td>$\rho^k$</td>
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<td>0.55</td>
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<tr>
<td>$\rho_m$</td>
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<td>$\sigma^{\hat{z}}$</td>
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<td>0.010</td>
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<td>$\sigma^{\hat{zh}}$</td>
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<td>0.023</td>
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<td>0.004</td>
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<td>0.001</td>
<td>0.001</td>
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<td>0.080</td>
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<td>0.011</td>
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<tr>
<td>$\sigma^{\hat{z}}$</td>
<td>0.05</td>
<td>0.025</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Notes: HPD denotes the 90 per cent highest probability density interval.
Table 4 shows some of the characteristics of the posterior distributions of the parameter estimates. The estimates of the Taylor rule parameters are relatively close to their prior means although the interest rate smoothing term is somewhat higher than expected. Aside from the fact that we find a substantial weight on growth in value-added in the Taylor rule (and a low weight on the level) our estimates are also similar to those estimated by Iacoviello and Neri (2010). The posterior mode of the curvature parameter governing the behaviour of investment adjustment costs, $S''$, is lower than expected, and substantially lower than that found in other studies for Australia (Jaaskela and Nimark 2011) although the 90 per cent highest probability density interval surrounding the estimate is fairly broad and sensitivity analysis conducted by holding all other parameters constant and altering the value of the curvature parameter suggest that the model dynamics are robust for reasonable values. We estimate that the degree of external habit persistence is close to 0.5 which is higher than our prior and in general higher than values found in other studies (Jaaskela and Nimark (2011) and Justiniano and Preston (2010)).

The estimate of the posterior mean of the Calvo parameter for imported goods suggests that prices are reset very infrequently, resulting in a very flat import price Phillips curve. In contrast, the implied frequency of adjustment in the domestic goods sector is much higher than import prices, and higher than our prior would suggest. The domestic Calvo parameter is estimated to be around 0.5, which implies that domestic goods firms will on average be able to optimally reset prices roughly every six months. This estimate while close to estimates from some studies based on micro data from the US (see for example Bils and Klenow (2004)), is lower than estimates typically found to generate plausible dynamics in DSGE studies. We found that the estimated values for the Calvo parameter on wages were closer to the prior mean than those for domestic goods or import prices, implying that the average firm optimally resets wages every 5 quarters. The value of the wage Calvo parameter seems to exert substantial influence on the dynamics of business investment in this model, particularly in terms of the short-run response to a positive monetary policy shock.

With the exception of the investment efficiency shock, which exhibits a low degree of persistence, and the housing preference shock which is highly persistent, the estimates of the autoregressive parameters governing the shocks in the model were reasonably close to the mean of our priors. The estimate of the degree of
persistence in the housing preference shock is implausibly high in the model, however, we find a trade-off between the degree of persistence in this shock and its volatility. The high degree of persistence in the housing shock seems to be important in generating comovement of consumption and house prices. The posterior mean of the monetary policy shock standard deviation implies roughly a 30 basis point increase in the annualised domestic interest rate (on impact).

5.2 Impulse Response Functions

In this section we will examine the properties of the full model that incorporates the financial accelerator (labelled FA in the charts below) via the impulse responses to some of the main shocks of interest, particularly those related to the housing sector. We will then contrast the results from the full model with those from an estimated baseline model that does not include the financial accelerator mechanism, but which maintains the separate housing production sector (the results from this model are labelled NoFA). In general we find that the financial accelerator acts as expected to amplify the impact of most shocks on real variables, there are, however, cases in which the opposite occurs. We also find that the results from the model with the financial accelerator are sensitive to the value of the steady-state LVR parameter.
Figure 1: IRFs to a monetary policy shock

A monetary policy shock increases the quarterly domestic interest rate by 8 basis points (slightly more than the standard 25 basis point increment in the annualised cash rate) and as expected results in a contraction in value-added (and all of its components) (Figure 1). Domestic prices falls along with real house prices which are down around 0.4 percentage points on impact. Residential investment falls by 1 per cent on impact. Looking at how the responses develop over time we find that total value added returns to baseline in just under 2 years. In contrast the declines in consumption, business investment and house prices are more persistent returning to the steady-state level more than 5 years after the initial shock.
The rise in residential investment in response to a positive aggregate technology shock is pronounced, increasing by more than 5 per cent on impact (Figure 2). Interest rates and inflation fall but real house prices rise as the fall in inflation swamps the impact of technology improvement in the housing sector output from the goods sector rises as do consumption business investment and exports.
Figure 3: IRFs to a housing sector specific technology shock

In response to a housing specific technology shock residential investment increases strongly resulting in an increase in total value-added (Figure 3). Real house prices fall and are persistently lower reflecting the combination of the sharp increase in the supply of new housing, which rises by almost 10 per cent on impact, and the increase in the domestic prices. Output in the goods sector falls as do consumption, business investment and exports in turn, a result of the shift in productive resources, particularly labour, from the goods sector to housing production.
Figure 4: IRFs to a housing preference shock

The housing preference shock is not as straightforward to interpret as a monetary policy shock or a technology shock. Iacoviello and Neri (2010) suggest that it could be interpreted as a change in tastes or that it may represent all of the influences on housing demand not captured by the model. They find using US data that some of the variables omitted from the model can account for part of the preference shock but not all of it so that there may be some role for the housing tastes interpretation. It seems reasonable to assume that something similar will hold for Australia, given the difficulty that empirical models have in matching and predicting movements in the housing market.

In response to a shock to housing preferences we see an increase in both residential investment and house prices (Figure 4). Domestic goods production also increases with output flowing to business investment and exports while consumption contracts. In quantitative terms we find that in response to a housing preference shock residential investment increases by more than 8 percentage points on impact while real house prices increase by 3 percentage points and take an extended
period of time to return to their baseline. The high degree of persistence in the housing preference shock in large part explains these dynamics.

These results are sensitive to the value of the steady-state LVR, $\bar{m}$, which can be thought of as a proxy for the intensity of the financial accelerator mechanism. In the full model we have set $\bar{m}$ equal to 0.6, however in the model comparison section we will investigate the impact of setting the steady-state LVR to a higher value akin to that used by Iacoviello and Neri (2010) and find that it changes the sign of the consumption response to a shock to housing preferences and generates the comovement of consumption and house prices that has been found in existing Australian empirical studies.

Figure 5: IRFs to a foreign risk premium shock
The risk premium shock is expected to result in a contraction in total output as the higher cost of borrowing from abroad reduces funds available for investment and lending to impatient households which in turn reduces the funds available for consumption and the purchase of housing. Instead we see an increase in good sector output as labour flows out of housing production driven by a fall in demand as domestic credit flows decrease sharply, compounded by falling house prices which undermine households’ collateral holdings. Domestic inflation and interest rates both rise moderately on impact while the exchange rate depreciates and exports expand, gradually returning to steady state over an extended period.

Figure 6: Credit flows and the foreign risk premium shock
Focussing more directly on the credit channels in the model Figure 6 includes impulse responses to a risk premium shock for net foreign assets, house prices and domestic credit from the full model. It shows that the ratio of foreign lending to domestic output (the net foreign asset ratio) increases on impact as domestic borrowing by the external sector becomes cheaper. Since in a net sense patient households are lending abroad there is less funding available for domestic lending and this is reflected in the decline in domestic credit. The decline in domestic lending is amplified when the steady-state LVR is set a higher level but it has little impact on foreign lending or house prices.

5.2.1 Model Comparison

In analysing the properties of the full model we find that the dynamics of residential investment are largely governed by the sector-specific shocks with the influence of shocks to the wider economy on the housing sector via labour and good market linkages more muted. This means that for this parameterisation the direct spill-overs from the housing sector to the rest of the economy are limited. However, the indirect impact of the inclusion of housing in the model can only be assessed in comparison with a baseline model with no financial accelerator mechanism. In order to investigate the importance of the financial accelerator mechanism we estimated a model that includes housing but which has a single representative consumer and no credit flows.

The effect of a domestic monetary policy shock is amplified on impact for all variables in the model and for most of the key variables it is also more persistent. Residential and real house prices both fall almost four times further in the model with credit while the response on impact of consumption is almost ten times as large. The dynamics of the responses are also substantially different for many of the key variables. In the model without the financial accelerator the variables respond on impact before tracing a smooth path back to the steady state. In the model with the financial accelerator the response is hump-shaped often overshooting the steady state level before falling back. This response pattern is sensitive to the labour market structure in the model. Experiments with a low frequency of wage adjustment resulted in dynamics more similar to the model without credit although the response on impact remained amplified.
Figure 7: IRFs to a housing preference shock with a varied LVR

The difference in the model dynamics in response to a housing preference shock is one of the most interesting results from the model. Comparing the impulse responses from the full model with the baseline we see sharp differences in value-added, business investment and exports while the decline in consumption is more pronounced in the model with the financial accelerator. However, as previously stated the dynamics are sensitive to the value of the steady-state LVR. Figure 7 illustrates the impact of a change in the steady-state LVR to the higher value used by Iacoviello and Neri (2010), 0.9, which increases the degree of leverage households can attain in the steady state. When $\bar{m}$ is set at the higher level the rise in house prices that accompanies a housing preference shock is accompanied by an increase in consumption, the opposite reaction to the lower-LVR and baseline cases. In addition many of the other variables are more volatile. The sign of the response of business investment and exports on impact changes as they fall to accommodate the increase in domestic consumption, while domestic good production, interest rates and inflation all increase sharply on impact. In the
model without credit consumption falls in response to a housing demand shock as households reduce consumption in order to fund their purchases of housing. The negative response of consumption also means that business investment and exports contract less in the initial periods in the no credit model and actually rise above their steady state level in later periods.

The response to the two technology shocks presents another interesting divergence. While the response of key variables to the aggregate technology shock is similar in the full model and the baseline, in terms of the magnitude of the response on impact and the dynamics, the presence of the financial accelerator mechanism actually attenuates the response of most of the key variables to a housing specific technology shock. Value-added rises less sharply in the model with the financial accelerator driven by a larger fall in goods output. This may be due to the increase in residential investment (which is similar in both models), which positively influences households collateral and partially counters the initial negative influence of house prices on access to credit. This diminishes the influence of the dynamic multiplier so that house prices fall less in the model with the financial accelerator. Impatient households therefore have more to spend on consumption so that in aggregate consumption is stronger in the model with the financial accelerator.

### 5.3 Forecast-Error Variance Decomposition

**Table 5: Forecast-Error Variance Decompositions for Full Model**

<table>
<thead>
<tr>
<th></th>
<th>Monetary Policy</th>
<th>Technology</th>
<th>Housing</th>
<th>Cost push</th>
<th>Foreign</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1Q 8Q</td>
<td>1Q 8Q</td>
<td>1Q 8Q</td>
<td>1Q 8Q</td>
<td>1Q 8Q</td>
</tr>
<tr>
<td>Value-added</td>
<td>5 2</td>
<td>17 38</td>
<td>1 1</td>
<td>28 23</td>
<td>25 8</td>
</tr>
<tr>
<td>Consumption</td>
<td>6 3</td>
<td>1 2</td>
<td>1 1</td>
<td>7 7</td>
<td>1 1</td>
</tr>
<tr>
<td>House prices</td>
<td>2 0</td>
<td>1 0</td>
<td>77 85</td>
<td>18 12</td>
<td>2 2</td>
</tr>
<tr>
<td>Credit</td>
<td>2 1</td>
<td>0 0</td>
<td>93 97</td>
<td>45 65</td>
<td>4 9</td>
</tr>
<tr>
<td>Residential investment</td>
<td>1 1</td>
<td>12 12</td>
<td>79 78</td>
<td>5 2</td>
<td>0 0</td>
</tr>
</tbody>
</table>

Forecast error variance decompositions are another way to examine the shocks that are driving the variables in the model. A decomposition of the full model delivers

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10 The fact that debt is nominal in the model may also play a role here, see Iacoviello (2005) for further discussion of the effect of the financial accelerator on supply versus demand shocks.
results that are consistent with the impulse response analysis in suggesting that outcomes in the housing sector are mostly driven by sector specific shocks, while at this calibration the housing specific shocks have little influence on the broader economy. Table 5 groups the shocks into four categories where the monetary policy shock and the technology shocks are simply the relevant individual shocks, housing shocks consist of the housing specific technology shock, the housing preference shock and the LVR shock and the foreign shocks are those from the foreign VAR plus the risk premium shock. The cost-push shock combines the impact of shocks to prices of domestic and imported goods. Results for one and eight quarter horizons are shown. It is worth noting that even with a fairly broad definition of foreign shocks they have a limited effect on the domestic economy a finding that is consistent with Justiniano and Preston (2010).

Table 6: Forecast-Error Variance Decompositions for Model with High LVR

<table>
<thead>
<tr>
<th></th>
<th>Monetary Policy</th>
<th>Technology</th>
<th>Housing</th>
<th>Cost push</th>
<th>Foreign</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1Q</td>
<td>8Q</td>
<td>1Q</td>
<td>8Q</td>
<td>1Q</td>
</tr>
<tr>
<td>Value-added</td>
<td>7</td>
<td>3</td>
<td>10</td>
<td>32</td>
<td>31</td>
</tr>
<tr>
<td>Consumption</td>
<td>6</td>
<td>4</td>
<td>0</td>
<td>2</td>
<td>55</td>
</tr>
<tr>
<td>House prices</td>
<td>1</td>
<td>0</td>
<td>13</td>
<td>13</td>
<td>79</td>
</tr>
<tr>
<td>Credit</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>79</td>
</tr>
<tr>
<td>Residential investment</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>95</td>
</tr>
</tbody>
</table>

Table 6 shows decompositions from a re-estimated version of the model with the steady-state LVR set at a higher level, 0.9. The key difference is the importance of the housing shocks as explanators of consumption in the model particularly at the short horizon. Results from a quarter ahead decomposition suggest that housing shocks (and the finer detail shows that it is mostly the housing preference shock) account for more than half of the variation in consumption. The strong evidence of spill-overs from the housing sector to consumption in this model is consistent with the results from the impulse response analysis which showed evidence of comovement in consumption and house prices. By the eight quarter horizon the foreign shocks are the most prominent driver of consumption. However, shocks from the housing sector remain important and continue to explain around 20 per cent of the variation in consumption.
5.4 Historical Decompositions

In general we find that the shocks that are specific to the housing sector, particularly the housing preference shock and the housing sector specific technology shock, account for most of the movement in the housing sector variables in the model. Historical shock decompositions, which describe the variation of key variables in the model over time in terms of the structural shocks, support these findings and seem to confirm that the measures employed to further integrate the housing sector into the broader economy, via the labour market and the inclusion of intermediate goods in housing production, do not seem to strengthen the influence of external shocks.

The housing preference shock in particular tends to dominate as the driver of housing market variables as can bee seen in each of the decompositions displayed in this section. In fact the housing preference shock has such a strong influence that a similarly strong shock of opposing effect is required to match the observed data.
series. In the case of residential investment over the sample period the housing preference is consistently matched by an offsetting housing specific technology shock this obscures the meaning of the technology shock (Figure 8).

Figure 9: Drivers of credit over time
Similarly, we find that the housing preference shock dominates as the explanator of credit in the model, but again the effect is balanced by contributions of opposing sign from the loan-to-valuation ratio shock. If we think of the LVR in this model as a proxy for lending standards we would expect the LVR shock to have a substantial and consistent positive contribution to credit over the period from 1997 to 2003. However, as Figure 9 illustrates, the contribution while substantial is not consistent, rather it seems to be highly negatively correlated with the contribution of the housing preference shock.

Figure 10: Drivers of housing prices over time
An interesting aspect of the historical decomposition of house prices is the prominent role that they imply for external shocks. The import cost push shock contributes substantially to real house prices in the period around the turn of the century. This is consistent with the rapid appreciation in the real exchange rate in this period which reduced the domestic price of imports and would therefore be expected to dampen inflation and drive up real house prices.

Figure 11: Drivers of inflation over time
The decompositions for the key macroeconomic indicator variables - value-added, inflation, interest rates - are largely as expected. Inflation (see Figure 11) is primarily driven by the cost push shocks to domestic good and import prices, with monetary policy also playing a role. Domestic interest rates and value added are driven by a broader range of shocks although the strongest contributors are the cost-push shocks, the aggregate technology shock and the housing preference shock.

5.5 Growth Cycle Properties

To examine how the inclusion of housing production and the financial accelerator mechanism influence the business cycles properties of the model we use the approach outlined in Pagan and Robinson (2011). Specifically we simulate data from the full model and from the baseline model without the financial accelerator and apply the Bry-Boschan Quarterly (BBQ) cycle-dating procedure due to Harding and Pagan (2002) to the simulated data. Because the data that enter the model are filtered the results describe the properties of the growth cycle and are more symmetrical than if the exercise were conducted on unfiltered levels data because the positive growth trend over the long-run has been filtered out, and it is this which creates much of the asymmetry typically evident in business cycles.

<table>
<thead>
<tr>
<th></th>
<th>Data 1993-2011</th>
<th>Full model 1960-2011</th>
<th>Baseline model</th>
<th>Low-LVR</th>
<th>High-LVR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Durations (qtrs)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expansions</td>
<td>4.0</td>
<td>9.5</td>
<td>5.2</td>
<td>5.0</td>
<td>5.4</td>
</tr>
<tr>
<td>Contractions</td>
<td>4.4</td>
<td>5.9</td>
<td>5.5</td>
<td>5.1</td>
<td>5.5</td>
</tr>
<tr>
<td><strong>Amplitude (%)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expansions</td>
<td>2.0</td>
<td>3.3</td>
<td>4.1</td>
<td>4.6</td>
<td>2.7</td>
</tr>
<tr>
<td>Contractions</td>
<td>-2.0</td>
<td>-3.6</td>
<td>-4.1</td>
<td>-4.6</td>
<td>-2.7</td>
</tr>
<tr>
<td><strong>Cumulative amplitude (%)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expansions</td>
<td>5.8</td>
<td>22.3</td>
<td>14.6</td>
<td>15.4</td>
<td>9.7</td>
</tr>
<tr>
<td>Contractions</td>
<td>-4.7</td>
<td>-12.1</td>
<td>-15.1</td>
<td>-15.3</td>
<td>-10.1</td>
</tr>
</tbody>
</table>

The results in Table 7 show that the model with the financial accelerator and the baseline model produce similar estimates for the average duration of cycles, slightly higher than the duration found in the data. The simulated data from the baseline model are a better match to the amplitude of cycles than the full
model with the amplitude implied by the full model more than twice that found in the data. These results contrast with those of a similar exercise conducted by Pagan and Robinson (2011) with the model developed in Iacoviello (2005). Pagan and Robinson (2011) find that the amplitude of cycles in data simulated from Iacoviello (2005) are significantly smaller than the amplitudes observed in the data.

When cycle-dating was employed on simulated data from a version of the model that was estimated with the steady-state LVR set at a higher value than we use in the initial estimation of the full model (and closer to the value used by Iacoviello and Neri (2010)), we found that it made little difference to the implied duration of cycles but that the implied cycle amplitudes were marginally further from the amplitudes found in the data than those from the full model with a lower LVR. This is surprising given that Pagan and Robinson (2011) find that increasing the LVR increases the absolute average amplitude of the growth cycle markedly in the Iacoviello (2005) model, although the relationship appeared to be non-linear.

The implications of the cycle-dating exercise for the models ability to match the properties of the underlying data are sensitive to the choice of sample. Thus far we have focused on the comparisons with the properties of the data from 1993-2011, the estimation sample. However, it may be argued that as this sample does not include a major business cycle contraction despite our focus on growth cycles it may be better to apply the cycle-dating procedure to a longer sample of data. Applying the procedure to data for the period 1960-2011 we find that the properties of the data lie somewhere between the model including the financial accelerator and the baseline. While the differences between the data and the cycle-dating results for the average duration of expansions and contractions are similar, the amplitude of cycles in the data is lower than that implied by the model with the financial accelerator and higher than that in the baseline model.

5.6 Robustness

The results described above are mostly robust to a plausible range of values for the parameters that we have calibrated or set. Simulating the model with a lower wage share of patient households, $\mu$, increases the sensitivity of key variables to most shocks. This is not surprising given that the financial accelerator affect will be stronger when more households are liquidity constrained. Decreasing the value
of $\gamma$, which increases the degree of openness of the model, increases the sensitivity to external shocks without altering the responses to domestic shocks substantially. Altering the Frisch elasticity of labour supply or the degree of intersectoral labour mobility both have implications for the response to a housing sector specific technology shock, with lower labour mobility and greater sensitivity to changes in the real wage increasing the magnitude of the response to the sector specific tech shock. However, because the housing sector is small and the magnitude of the differences for the range of values are also small and have a negligible effect on aggregate activity.

The key exception to the general robustness of the model is its sensitivity to the steady-state LVR parameter $\bar{m}$, which, as previously discussed, when varied can have a substantial impact on the dynamics of several of the key variables in response to a shock to housing preferences. In the initial version of the full model we set $\bar{m}$ to 0.6 a value more in line with available estimates for all loans in Australia over the sample period (Ellis 2006), than that used by Iacoviello and Neri (2010). Alternatively, when a higher value of $\bar{m}$ closer to the value used by Iacoviello and Neri (2010) is used in estimating the model we find, most notably, that the consumption and house prices both increase in response to a positive housing preference shock. Other than altering the response to a housing preference shock altering $\bar{m}$ has little influence on the results. This finding is common to models that employ a similar underlying structure incorporating the financial accelerator mechanism (see Iacoviello (2005), Iacoviello and Neri (2010) and Chistensen et al (2009)) regardless of whether the model is open or closed, or whether the supply of housing is fixed or flexible, although as previously discussed Pagan and Robinson (2011) did find that it considerably altered the growth cycle characteristics.

The inclusion of wage rigidities was found to be important in generating plausible impulse responses to a monetary policy shock. In the absence of wage rigidities residential investment increases in response to a monetary policy shock, clearly at odds with the negative correlation of residential investment activity and the cash rate that is found in the data (Berger-Thomson and Ellis 2004). In addition substantial wage rigidity (a high $\theta^d$) was required to match the contraction of business investment found in the data in response to a monetary policy shock.
6. Conclusion

Estimating a model that includes housing and credit flows we are able to generate plausible parameter estimates and model dynamics. We find that fluctuations in the housing market are mostly driven by the sector specific shocks but that the housing sector specific shocks have little influence on developments outside the housing sector. We are able to replicate the comovement of consumption and house prices in response to a housing preference shock that is found in studies using US data. However, this requires a high steady-state LVR that is not supported in the data for Australia. When the steady-state LVR is set at a lower level the evidence of consumption wealth effects from housing is not present, nevertheless the inclusion of the financial accelerator mechanism does alter the model dynamics with results from a cycle-dating exercise suggesting that the strength of the financial accelerator mechanism has surprisingly little influence on the models ability to match the growth cycle properties of the data.

There are a numbers of ways that this model could be improved and augmented. The first and most obvious is the use of model consistent detrending which would allow us to address the role of population as well as sectoral productivity differentials. The treatment of land is simple in this model, while in the Australian housing market fluctuations in land prices are likely to explain a substantial portion of the variation in house prices especially in major markets such as Sydney and Melbourne. Another inclusion that would likely help in fitting the data from the credit and housing markets is the addition of a banking sector. One possibility is to extend the model along the lines of Gerali, Neri, Sessa and Signoretti (2010) In addition to allowing us to take the role of credit provider away from the patient households helping to resolve the conceptual issues encountered in mapping credit data to the model, it would also allow us to further pursue questions about international funding of financial institutions that have only been briefly touched on in this paper. Such a framework may also be appealing in investigating the interplay of household borrowing, house prices and the financial stability of financial intermediaries.
Appendix A

The Log-Linearised Model

Households

Patient households

Labour supply

\[ \tilde{\lambda}_{1,t} = \bar{\lambda}_t + \xi \tilde{I}_{1,t}^d + (\psi - \xi)\tilde{I}_{1,t} - \tilde{w}_{1,t}^d * \] (29)

\[ \tilde{\lambda}_{1,t} = \bar{\lambda}_t + \xi \tilde{I}_{1,t}^h + (\psi - \xi)\tilde{I}_{1,t} - \tilde{w}_{1,t}^h * \] (30)

Labour market clearing

\[ \tilde{I}_{1,t} = \tilde{I}_{1,t}^d \left( \frac{\tilde{L}_{1,t}^d}{\tilde{L}_1} \right)^{1+\xi} + \tilde{I}_{1,t}^h \left( \frac{\tilde{L}_{1,t}^h}{\tilde{L}_1} \right)^{1+\xi} \] (31)

Consumption

\[ \tilde{\lambda}_{1,t} = \kappa_t - \frac{1}{1-\Omega} (\tilde{c}_{1,t} - \Omega \tilde{c}_{1,t-1}) \] (32)

Money demand

\[ \tilde{\lambda}_{1,t} = \beta_1 E_t [\tilde{\lambda}_{1,t+1} - \tilde{\pi}_{t+1}] - (1 - \beta_1) + \tilde{m}_{1,t} \] (33)

Borrowing

\[ \tilde{\lambda}_{1,t} = E_t [\tilde{\lambda}_{1,t+1} - \tilde{\pi}_{t+1}] - \tilde{R}_t \] (34)
Tobin’s Q for physical capital

\[
\tilde{q}_t = \beta_1 E_t[\tilde{q}_{t+1}(1 - \delta) + \tilde{r}_{t+1}^k] - (\tilde{R}_t - E_t[\tilde{\pi}_{t+1}^h])
\]  

(35)

Physical capital investment optimality condition

\[
\tilde{i}_t = \frac{1}{(1 + \beta_1)} \left( \frac{1}{S''(1)} \left( \tilde{q}_t + \epsilon_i \right) + \beta_1 E_t[\tilde{i}_{t+1}] + \tilde{i}_{t-1} \right)
\]  

(36)

Tobin’s Q for housing capital

\[
\tilde{q}_{t}^{kh} = (1 - \beta_1 (1 - \delta^{kh}))(\tilde{r}_{t+1}^h - \tilde{r}_t^h + \tilde{\pi}_t^h) + \beta_1 (1 - \delta^{kh})(\tilde{q}_t^h + \tilde{\pi}_t^h) - \tilde{R}_t
\]  

(37)

Housing capital investment optimality condition

\[
\tilde{i}_t^h = \frac{1}{(1 + \beta_1)} \left( \frac{1}{S''^h(1)} \tilde{q}_t^{kh} + \beta_1 E_t[\tilde{i}_{t+1}^h] + \tilde{i}_{t-1}^h \right)
\]  

(38)

Housing optimality condition

\[
\tilde{\lambda}_{1,t} = (1 - \beta_1 * (1 - \delta^h))(\tilde{j}_t - \tilde{h}_{1,t}) + \beta_1 (1 - \delta^h)(E_t[\tilde{\lambda}_{1,t+1}^h + \tilde{q}_{t+1}^h - \tilde{q}_t^h])
\]  

(39)

Housing investment law of motion

\[
\tilde{h}_{1,t} = \delta^h \tilde{h}_{1,t} + (1 - \delta^h)\tilde{h}_{1,t-1}
\]  

(40)

**Impatient households**

Labour supply
\[
\tilde{\lambda}_{2,t} = \tilde{\chi}_t + \xi \tilde{l}_{2,t}^d + (\psi - \xi) \tilde{l}_{2,t} - \tilde{w}_{2,t}^d
\]  
(41)

\[
\tilde{\lambda}_{2,t} = \tilde{\chi}_t + \xi \tilde{l}_{2,t}^h + (\psi - \xi) \tilde{l}_{2,t} - \tilde{w}_{2,t}^h
\]  
(42)

Labour market clearing

\[
\tilde{l}_{2,t} = \tilde{l}_{2,t}^d \left( \frac{L_2}{L_t} \right)^{1+\xi} + \tilde{l}_{2,t}^h \left( \frac{L_2}{L_t} \right)^{1+\xi}
\]  
(43)

Consumption

\[
\tilde{\lambda}_{2,t} = \tilde{\kappa}_t - \frac{1}{1 - \Omega^t} \left( \tilde{c}_{2,t} - \Omega \tilde{c}_{2,t-1} \right)
\]  
(44)

Money demand

\[
\tilde{\lambda}_{2,t} = \beta_2 E_t \left[ \tilde{\lambda}_{2,t+1} - \tilde{\pi}_{t+1} \right] - (1 - \beta_2) + \tilde{m}_{2,t}
\]  
(45)

Borrowing

\[
\tilde{\lambda}_{2,t} = \frac{\beta_2}{\beta_1} E_t \left[ \tilde{\lambda}_{2,t+1} - \tilde{\pi}_{t+1} \right] + \tilde{R}_t + \frac{(\beta_1 - \beta_2)}{\beta_2 \beta_1} (\tilde{\lambda}_{2,t} + \tilde{R}_t)
\]  
(46)

Housing optimality condition

\[
\tilde{\lambda}_{2,t} = (1 - \beta_2 (1 - \delta^h) + \tilde{m}(\beta_1 - \beta_2))(\tilde{j}_t - \tilde{h}_{2,t}) + \beta_2 (1 - \delta^h) E_t [\tilde{\lambda}_{2,t+1} + \tilde{q}_{t+1}^h]
\]
\[
+ \tilde{m}(\beta_1 - \beta_2)(\tilde{\lambda}_{2,2t} + \tilde{m}_t + E_t [\tilde{\pi}_{t+1} + \tilde{q}_{t+1}^h]) - \tilde{q}_t^h
\]  
(47)
Housing investment law of motion

\[ \tilde{h}_{2,t} = \delta^h \tilde{h}_{2,t} + (1 - \delta^h) \tilde{h}_{2,t-1} \]  \hspace{1cm} (48)

Net worth

\[ \tilde{b}_{2,t} \tilde{Y} = \tilde{c}_{2,t} \tilde{C} + \frac{\tilde{q}^h \tilde{H}_2}{\tilde{Y}} (\delta^h \tilde{q}^h + \tilde{h}_{2,t} - (1 - \delta^h) \tilde{h}_{2,t-1}) + \frac{1}{\beta_1} \left( \tilde{R}_{t-1} + \tilde{b}_{2,t-1} - \tilde{\pi}_t \right) - \]

\[ (\tilde{w}_{2,t}^d + \tilde{\lambda}_2^d) \frac{\tilde{w}^d L_2}{\tilde{Y}} - (\tilde{w}_{2,t}^h + \tilde{\lambda}_2^h) \frac{\tilde{w}^h L_2}{\tilde{Y}} - \tilde{\tau}_t \frac{\tilde{\pi}}{\tilde{Y}} \]  \hspace{1cm} (49)

Aggregate households

Consumption aggregator

\[ \tilde{c}_t = \eta \tilde{c}_{1,t} \frac{\tilde{C}_1}{\tilde{C}} + (1 - \eta) \tilde{c}_{2,t} \frac{\tilde{C}_2}{\tilde{C}} \]  \hspace{1cm} (50)

Housing market clearing

\[ \tilde{h}_t = \eta \tilde{h}_{1,t} \frac{\tilde{H}_1}{\tilde{H}} + (1 - \eta) \tilde{h}_{2,t} \frac{\tilde{H}_2}{\tilde{H}} \]  \hspace{1cm} (51)

Housing investment aggregator

\[ \tilde{h}_t = \delta^h \tilde{h}_t + (1 - \delta^h) \tilde{h}_{t-1} \]  \hspace{1cm} (52)

Demand for domestic consumption goods

\[ \tilde{c}_t^h = \tilde{c}_t - \sigma \tilde{\tau}_t^h \]  \hspace{1cm} (53)
Demand for imported consumption goods

\[ \hat{c}_i^m = \hat{c}_i - \sigma \hat{c}_i^m \]  

(54)

Constraints

Borrowing constraint

\[ \hat{b}_{2,t} = \hat{m}_t + \hat{h}_{2,t} + E_t[\hat{q}_t^h + \hat{\pi}_t + 1] - \hat{R}_t \]  

(55)

Physical Capital law of motion

\[ \hat{k}_t = (1 - \delta) \hat{k}_{t-1} + \delta \ln \hat{e}_i + \delta \hat{\pi}_t \]  

(56)

Housing Capital law of motion

\[ \hat{k}_t^h = (1 - \delta^{kh}) \hat{k}_{t-1}^h + \delta^{kh} \hat{\pi}_t \]  

(57)

Foreign sector

Uncovered interest parity

\[ \left( \hat{R}_t - \pi_{t+1} \right) - \left( \hat{R}_t^f - \pi_{t+1}^f \right) = E_t[\text{rer}_{t+1}] - \text{rer}_t - \tilde{\alpha}_t + \tilde{\pi}_t \]  

(58)

Current account

\[ \bar{a} \left( \tilde{\alpha}_t - \frac{1}{\beta_1} \left( \hat{R}_{t-1}^f - \tilde{\pi}_t^f + \tilde{\pi}_{t-1}^f + \tilde{\xi}_t^b - \tilde{\pi}_t^d + \hat{\pi}_t - (\hat{\pi}_t - \hat{\pi}_t) + (\text{rer}_t - \tilde{\text{rer}}_{t-1}) \right) \right) \]

\[ = (\hat{x}_t - \hat{y}_t) \frac{X}{Y} - \frac{1}{\hat{e}^h} \frac{\bar{C}_i^m}{\bar{Y}} (\hat{c}_i^f + \hat{c}_i^m) \]  

(59)
Export demand

\[ \tilde{x}_t = \lambda^f \tilde{z}^f_t + \tilde{y}_t \]  

(60)

**Domestic intermediate goods producers**

Demand for Capital

\[ \tilde{k}_{t-1} = \tilde{\Omega}_t - \tilde{r}_t^k + \tilde{y}_t \]  

(61)

Demand for labour from patient households

\[ \tilde{l}^d_{1,t} = \tilde{\Omega}_t + \tilde{y}_t + \tilde{\tau}_t^h - \tilde{w}^d_{1,t} \]  

(62)

Demand for labour from impatient households

\[ \tilde{l}^d_{2,t} = \tilde{\Omega}_t + \tilde{y}_t + \tilde{\tau}_t^h - \tilde{w}^d_{2,t} \]  

(63)

Real marginal costs

\[ \tilde{\Omega}_t = \alpha \tilde{r}_t^k + \mu (1 - \alpha) \tilde{w}^d_{1,t} + (1 - \mu) (1 - \alpha) \tilde{w}^d_{2,t} - \tilde{z}_t - (1 - \alpha) \tilde{\tau}_t^h \]  

(64)

**Housing producers**

Demand for housing capital

\[ \tilde{k}^h_{t-1} = \tilde{q}^h_t + i \tilde{h}_t - \tilde{r}_t^h \]  

(65)

Demand for patient household labour
\[ \tilde{I}^h_{1,t} = \tilde{q}_{1,t} + \tilde{i} - \tilde{w}^h_{1,t} \] (66)

Demand for impatient households labour

\[ \tilde{I}^h_{2,t} = \tilde{q}_{2,t} + \tilde{i} - \tilde{w}^h_{2,t} \] (67)

Return on land

\[ \tilde{\tau}_t = \tilde{q}_{t} + \tilde{i} \] (68)

Demand for intermediate goods

\[ \tilde{y}^h_t = \tilde{q}_{t} + \tilde{i} - \tilde{\tau}_t \] (69)

Production function

\[ \tilde{i} = \tilde{z}_t^h + \tilde{z}_t^h + \theta_h k_t^h + \theta^y y_t^h + (1 - \theta^h - \theta^l - \theta^y) (\nu \tilde{I}^h_{1,t} + (1 - \nu) \tilde{I}^h_{2,t}) \] (70)

Prices

Goods Phillips curve

\[ \tilde{\pi}_t^h = \mu_f \tilde{\pi}_{t+1}^h + \mu_b \tilde{\pi}_{t-1}^h + \mu_{mc} \tilde{\Omega}_t + \lambda_t \] (71)

Wage Phillips curves

\[ \tilde{\pi}_{1,t}^w = \mu_f^1 \tilde{\pi}_{1,t+1}^w + \mu_b^1 \tilde{\pi}_{1,t-1}^w + \mu_{mc}^1 \tilde{\omega}_{1,t}^w \] (72)
\[ \tilde{\pi}_{1,t}^w = \mu_f^h \tilde{\pi}_{1,t+1}^w + \mu_b^h \tilde{\pi}_{1,t-1}^w + \mu_1^{mc} \tilde{w}_{1,t} \] (73)

\[ \tilde{\pi}_{2,t}^d = \mu_f^d \tilde{\pi}_{2,t+1}^d + \mu_b^d \tilde{\pi}_{2,t-1}^d + \mu_2^{mc} \tilde{d}_{2,t} \] (74)

\[ \tilde{\pi}_{2,t}^w = \mu_f^h \tilde{\pi}_{1,t+1}^w + \mu_b^h \tilde{\pi}_{1,t-1}^w + \mu_2^{mc} \tilde{w}_{1,t} \] (75)

Import Phillips curve

\[ \tilde{\pi}_t^m = \mu_f^m \tilde{\pi}_{t+1}^m + \mu_b^m \tilde{\pi}_{t-1}^m + \mu_m^{mc} \tilde{\Omega}_t^m + \tilde{\lambda}_t^m \] (76)

Importers marginal costs

\[ \tilde{\Omega}_t^m = \tilde{r} \tilde{r}_t - \tilde{\tau}_t^m \] (77)

Aggregate price index (in changes)

\[ \tilde{\pi}_t = \gamma \tilde{\pi}_t^h + (1 - \gamma) \tilde{\pi}_t^m \] (78)

Relative price of aggregate domestic good to aggregate consumption good

\[ \tilde{\tau}_t^h = \tilde{\pi}_t^h - \tilde{\pi}_t + \tilde{\pi}_t^h \] (79)

Relative price of aggregate import good to aggregate consumption good

\[ \tilde{\tau}_t^m = \tilde{\pi}_t^m + \tilde{\pi}_t^m - \tilde{\pi}_t \] (80)

Relative price of domestic good faced by the foreign sector, to foreign prices
\[ \tilde{\tau}_t^f = \tilde{r}_t \tilde{r}_t - \tilde{\tau}_t^h \]  
(81)

Wage maps

\[ \tilde{w}_{1,t}^d = \tilde{w}_{1,t}^d - \tilde{X}_{1,t}^d \]  
(82)

\[ \tilde{w}_{1,t}^h = \tilde{w}_{1,t}^h - \tilde{X}_{1,t}^h \]  
(83)

\[ \tilde{w}_{2,t}^d = \tilde{w}_{2,t}^d - \tilde{X}_{2,t}^d \]  
(84)

\[ \tilde{w}_{2,t}^h = \tilde{w}_{2,t}^h - \tilde{X}_{2,t}^h \]  
(85)

**Monetary Policy**

Taylor rule

\[ \tilde{R}_t = \xi^R \tilde{R}_{t-1} + (1 - \xi^R) (\xi^\pi \tilde{\pi}_t + \xi^{\nu a} \tilde{\nu}_a_t + \xi^{gr} (\tilde{\nu}_a_t - \tilde{\nu}_a_{t-1}) + \epsilon_t) \]  
(86)

Aggregate transfers

\[ \tilde{\tau}_t = \tilde{M} \left( \tilde{c}_t - (\eta \tilde{m}_{1,t-1} + (1 - \eta) \tilde{m}_{2,t-1}) - \tilde{\kappa}_t - \tilde{R}_t \beta_1 \right) \]  
(87)

**Goods market clearing condition**

\[ \tilde{y}_t = \tilde{c}_t^h \tilde{C}_t^h + \tilde{i}_t^h \tilde{I}_t^h + \tilde{x}_t^h \tilde{X}_t^h + \tilde{y}_t^h \tilde{Y}_t^h \]  
(88)
\[ \tilde{v}a = (\tilde{v} + \tilde{v}^h) \frac{\tilde{Y} \tilde{v}^h}{VA} - (\tilde{v}^h + \tilde{v}) \frac{\tilde{Y}^h \tilde{v}^h}{VA} + (\tilde{q}^h + \tilde{h}) \frac{\tilde{q}^h IH}{VA} \]  \hspace{1cm} (89) \\

**Shock processes**

\[ \tilde{z}_t = \rho \tilde{z}_t - 1 + \varepsilon^z \]  \hspace{1cm} (90) \\

\[ \tilde{z}_t^h = \rho \tilde{z}_t^{h-1} + \varepsilon^{zh} \]  \hspace{1cm} (91) \\

\[ \tilde{\xi}_t = \rho \tilde{\xi}_t - 1 + \varepsilon^{\xi} \]  \hspace{1cm} (92) \\

\[ \tilde{r}_t^f = \rho \tilde{r}_t^f - 1 + \varepsilon^{R_f} \]  \hspace{1cm} (93) \\

\[ \tilde{\xi}_t^b = \rho \tilde{\xi}_t^{b-1} + \varepsilon^{\xi^b} \]  \hspace{1cm} (94) \\

\[ \tilde{e}_t^i = \rho \tilde{e}_t^{i-1} + \varepsilon^{e^i} \]  \hspace{1cm} (95) \\

\[ \tilde{\pi}_t^f = \rho \tilde{\pi}_t^{f-1} + \varepsilon^{\pi^f} \]  \hspace{1cm} (96) \\

\[ \tilde{y}_t^f = \rho \tilde{y}_t^{f-1} + \varepsilon^{y^f} \]  \hspace{1cm} (97) \\

\[ \tilde{j}_t = \rho \tilde{j}_t^{j-1} + \varepsilon^{j} \]  \hspace{1cm} (98)
\[ \tilde{m}_t = \rho^m \tilde{m}_{t-1} + \epsilon^m \]  

(99)

\[ \tilde{\kappa}_t = \rho^\kappa \tilde{\kappa}_{t-1} + \epsilon^\kappa \]  

(100)

\[ \tilde{\lambda}_t = \epsilon^\lambda \]  

(101)

\[ \tilde{\lambda}_t^m = \epsilon^\lambda^m \]  

(102)
Appendix B

The Steady State

Constants

\[ \bar{r}^k = \frac{1}{\beta_1} - (1 - \delta) \quad (103) \]

\[ r^h = \left( \frac{1}{\beta_1} - (1 - \delta^{kh}) \right) \bar{r}^h \quad (104) \]

\[ \bar{m}c = \frac{\bar{\lambda} - 1}{\bar{\lambda}} \quad (105) \]

\[ \bar{X}^m = \frac{\bar{\lambda}^m}{\bar{\lambda}^m - 1} \quad (106) \]

\[ \bar{X}_1^{wd} = \frac{\bar{\lambda}_1^{wd}}{\bar{\lambda}_1^{wd} - 1} \quad (107) \]

\[ \bar{X}_1^{wh} = \frac{\bar{\lambda}_1^{wh}}{\bar{\lambda}_1^{wh} - 1} \quad (108) \]

\[ \bar{X}_2^{wd} = \frac{\bar{\lambda}_2^{wd}}{\bar{\lambda}_2^{wd} - 1} \quad (109) \]

\[ \bar{X}_2^{wh} = \frac{\bar{\lambda}_2^{wh}}{\bar{\lambda}_2^{wh} - 1} \quad (110) \]
\[ \bar{\tau}^h = \left[ 1 - (1 - \gamma) \bar{\bar{X}}^m 1 - \sigma \right]^{\frac{1}{1 - \sigma}} \]  
(111)

\[ \bar{\tau}^m = \bar{\bar{X}}^m \]  
(112)

\[ \bar{\Pi}^m = (1 - \gamma)(\bar{\bar{X}}^m - 1)\bar{\bar{X}}^m - \sigma \]  
(113)

\[ \kappa_1 = \frac{\bar{f}}{1 - \beta_1(1 - \delta^h)} \]  
(114)

\[ \kappa_2 = \frac{\bar{f}}{1 - \beta_2(1 - \delta^h) - \bar{m}(\beta_1 - \beta_2)} \]  
(115)

\[ \kappa_3 = \frac{1}{1 + (1 - \beta_1)\bar{m}\kappa_2} \]  
(116)

\[ \zeta_1 = \eta(1 + \kappa_1 \delta^h (1 + \theta^h (\frac{\delta^{kh}}{\rho^h} \bar{\theta}^h - 1) - (1 - \theta^h - \theta^l - \theta^y) v - \eta \theta^l) \] 
\[ - (1 - \gamma)(\bar{\bar{X}}^m - 1)\bar{\bar{X}}^m - \sigma ) \]  
(117)

\[ \zeta_2 = (1 - \eta)(\kappa_2 \delta^h (\theta^h (\frac{\delta^{kh}}{\rho^h} \bar{\theta}^h - 1) - \bar{m}(1 - \beta_1) \delta^h - (1 - \theta^h - \theta^l - \theta^y) v - \eta \theta^l) \] 
\[ - (1 - \gamma)(\bar{\bar{X}}^m - 1)\bar{\bar{X}}^m - \sigma ) \]  
(118)

\[ \zeta_3 = \bar{\tau}^h (1 - \bar{m} c (\frac{\delta^a}{\bar{\rho}^k} + (1 - \mu)(1 - \alpha))) \]  
(119)
\[ \zeta_4 = \eta \kappa_1 \delta^h((\nu - 1)(1 - \theta^h - \theta' - \theta^\nu) - (1 - \eta)\theta') \]  
(120)

\[ \zeta_5 = (1 - \eta)(1 + \kappa_2 \delta^h(1 + \frac{\bar{m}(1 - \beta_1)}{\delta^h}) - (1 - \nu)(1 - \theta^h - \theta' - \theta^\nu) - (1 - \eta)\theta')) \]
(121)

\[ \zeta_6 = \bar{m}c(1 - \mu)(1 - \alpha)\bar{\tau}^h \]
(122)

**Steady state ratios**

\[ \bar{K} = \alpha \frac{\bar{m}c}{\bar{r}^k} \bar{Y} \]
(123)

\[ \bar{K}^h = \frac{\theta^h \delta^h}{\bar{r}^h} q^h \bar{H} \]
(124)

\[ \bar{L}^* = \frac{\theta'}{\bar{\tau}^h} q^h \bar{H} \]
(125)

\[ \bar{L}^* = 1 \]
(126)

\[ \frac{\bar{Y}^h}{\bar{Y}} = \frac{\theta^\nu \delta^h}{\bar{\tau}^h} q^h \bar{H} \]
(127)

\[ \frac{\bar{I}}{\bar{Y}} = \delta \frac{\bar{K}}{\bar{Y}} \]
(128)
\[
\frac{\tilde{T}^h}{Y} = \delta^{kh} \tilde{K}^h
\]

\[
\frac{\bar{C}}{Y} = \frac{1 - \delta \tilde{K} Y - \delta^{kh} \tilde{K}^h Y - \tilde{\Sigma}^h}{\bar{C}^h + \frac{1}{\bar{\tau}} \bar{C}^m}
\]

\[
\frac{\bar{X}}{Y} = \frac{1}{\bar{\tau}^h} \frac{\bar{C}^m}{Y}
\]

\[
\frac{\bar{C}^m}{C} = (1 - \gamma) \bar{X}^m - \sigma
\]

\[
\frac{\bar{C}^h}{C} = \gamma \bar{\tau}^h - \sigma
\]

\[
\bar{w}_1^d \bar{L}_1^d D = \mu (1 - \alpha) \bar{m} \bar{c} \bar{Y} \bar{\tau}^h
\]

\[
\bar{w}_2^d \bar{L}_2^d D = (1 - \mu) (1 - \alpha) \bar{m} \bar{c} \bar{Y} \bar{\tau}^h
\]

\[
\bar{w}_1^{h,h} \bar{L}_1^h D = \nu (1 - \theta^h - \theta^l - \theta^y) \delta^h \bar{q}^h \bar{\theta}^h
\]

\[
\bar{w}_2^{h,h} \bar{L}_2^h D = (1 - \nu) (1 - \theta^h - \theta^l - \theta^y) \delta^h \bar{q}^h \bar{\theta}^h
\]

\[
\frac{\bar{C}_2}{\bar{Y}} = \frac{\zeta_6 \zeta_1 - \zeta_4 \zeta_3}{\zeta_5 \zeta_1 - \zeta_2 \zeta_4}
\]
\[
\frac{\tilde{C}_1}{\bar{Y}} = \frac{\zeta_3 \zeta_5 - \zeta_6 \zeta_2}{\zeta_1 \zeta_5 - \zeta_4 \zeta_2} \quad (139)
\]

\[
\frac{\tilde{C}_1}{\bar{C}} = \frac{\tilde{C}_1}{\bar{Y} \bar{C}} \quad (140)
\]

\[
\frac{\tilde{C}_2}{\bar{C}} = \frac{\tilde{C}_2}{\bar{Y} \bar{C}} \quad (141)
\]

\[
\frac{\bar{q}^h \bar{H}_1}{\bar{Y}} = \kappa_1 \frac{\bar{C}_1}{\bar{Y}} \quad (142)
\]

\[
\frac{\bar{q}^h \bar{H}_2}{\bar{Y}} = \kappa_2 \frac{\bar{C}_2}{\bar{Y}} \quad (143)
\]

\[
\frac{\bar{q}^h \bar{H}}{\bar{Y}} = \eta \frac{\bar{q}^h \bar{H}_1}{\bar{Y}} + (1 - \eta) \frac{\bar{q}^h \bar{H}_2}{\bar{Y}} \quad (144)
\]

\[
\frac{\bar{H}_1}{\bar{H}} = \frac{\bar{q}^h \bar{H}_1}{\bar{Y} \bar{q}^h \bar{H}} \quad (145)
\]

\[
\frac{\bar{H}_2}{\bar{H}} = \frac{\bar{q}^h \bar{H}_2}{\bar{Y} \bar{q}^h \bar{H}} \quad (146)
\]

\[
\frac{\bar{b}_2}{\bar{Y}} = \beta_1 \bar{m} \frac{\bar{q}^h \bar{H}_2}{\bar{Y}} \quad (147)
\]
\[ \bar{L}_1^d = \left( \frac{\bar{w}_1^d \bar{L}_1^d}{\bar{C}_1 \bar{X}_1^d} \right) \frac{1}{1+\psi} \] (148)

\[ \bar{L}_1^h = \left( \frac{\bar{w}_1^h \bar{L}_1^h \bar{X}_1^h}{\bar{C}_1} \right) \frac{1}{1+\psi} \] (149)

\[ \bar{L}_2^d = \left( \frac{\bar{w}_2^d \bar{L}_2^d}{\bar{C}_2 \bar{X}_2^d} \right) \frac{1}{1+\psi} \] (150)

\[ \bar{L}_2^h = \left( \frac{\bar{w}_2^h \bar{L}_2^h \bar{X}_2^h}{\bar{C}_2 \bar{X}_2^h} \right) \frac{1}{1+\psi} \] (151)

\[ \bar{L}_1 = \left( \bar{L}_1^d 1+\xi + \bar{L}_1^h 1+\xi \right) \frac{1}{1+\xi} \] (152)

\[ \bar{L}_2 = \left( \bar{L}_2^d 1+\xi + \bar{L}_2^h 1+\xi \right) \frac{1}{1+\xi} \] (153)

\[ \bar{Y} = \frac{1}{\bar{C}^h + \frac{\delta^h q^h H}{\bar{Y}}} \] (154)
\[
\frac{\bar{q}^h \bar{H}}{V_A} = 1 - \frac{\bar{Y}}{V_A} \bar{\tau}^h
\]  
\hspace{1cm} (155)

\[t^l = \bar{t}^l\]  
\hspace{1cm} (156)

\[
\frac{\bar{M}}{\bar{Y}} = \eta \frac{1}{1 - \beta_1} \frac{\bar{C}_1}{\bar{Y}} + (1 - \eta) \frac{1}{1 - \beta_2} \frac{\bar{C}_2}{\bar{Y}}
\]  
\hspace{1cm} (157)

\[
\bar{Y} = \left(\frac{\bar{K}}{\bar{Y}}\right)^{\frac{\alpha}{1 - \alpha}} \bar{L}_1^\mu \bar{L}_2^{1 - \mu}
\]  
\hspace{1cm} (158)
References


Bao Y, G Lim and S Li (2009), ‘A Small Open Economy Model with a Housing Sector’, *Australian Conference of Economists*.


