Network Effects, Market Structure and Industry Performance*

Rabah Amir† and Natalia Lazzati‡

July 13, 2011

Abstract

This paper analyzes oligopolistic markets with network externalities. Exploiting a minimal complementarity structure on the model primitives that allows for pure network goods, we prove existence of nontrivial fulfilled-expectations equilibrium. We formalize the concept of industry viability, investigate its determinants, and show it improves with more firms in the market and/or by technological progress. These results enlighten some conclusions from case studies in the strategy literature. We also characterize the effects of market structure on industry performance, which depart substantially from ordinary markets. The approach relies on lattice-theoretic methods, supplemented with basic insights from nonsmooth analysis.

JEL codes: C72, D43, L13, L14.

Key words and phrases: Network effects, demand-side externalities, industry viability, industry start up, supermodularity.

*We gratefully acknowledge financial support provided by the NET Institute (www.NETinst.org). We thank Roberto Burguet, Federico Echenique, Laurent Linnemer, Amilcar Menichini, Stanley Reynolds, Steve Salant, Mark Stegeman, Xavier Vives, Mark Walker, John Wooders, and seminar participants at ASU, Purdue, CORE, Paris I, LEI, Paris II, the Intertic conference in Milan (May 2007), the IESE workshop on Industrial Organization (Barcelona, June 2006), the IIOC 2008, the Midwest Theory meetings (May 2008), the SWET conference (March 2009), WZB-Berlin, UABarcelona, U Luxembourg, U Michigan , U Notre Dame, Indiana U, U Carlos III, and U Zurich.

†Rabah Amir, Department of Economics, University of Arizona, Tucson, AZ 85721 (e-mail: ramir@eller.arizona.edu).

‡Natalia Lazzati, Department of Economics, University of Michigan, Ann Arbor, MI 48109-1220 (e-mail: nlazzati@umich.edu).

1 Introduction

It has often been observed that the nature of competition is qualitatively different in network industries. The presence of interlinkages in consumers' purchasing decisions induces demand-side economies of scale that may strongly affect market behavior and performance. When such effects prevail, be they of the snob or bandwagon type, purchase decisions are influenced by buyers' expectations, leading to behavior not encompassed by traditional demand theory (Veblen [37]; Leibenstein [22]). From an industrial organization perspective, these distinctive features raise new questions and impose some methodological challenges. In their pioneering work on markets with network effects, Katz and Shapiro [19] proposed the concept of fulfilled expectations Cournot equilibrium (FECE), which was adopted by some of the early literature. This has led to a number of results that distinguish network markets from ordinary ones.[1]

The purpose of the present paper is to provide a thorough theoretical investigation of markets with homogeneous goods and network externalities. We consider oligopolistic competition amongst firms in a market characterized by positive (direct) network effects when the products of the firms are perfectly compatible, so that the relevant network is industry-wide. This is motivated by both positive and normative considerations. In terms of the former, several important industries fit the perfect compatibility framework, in particular those in the telecommunications sector, such as fax, telephone, the Internet, but also many classical industries such as compact discs, fashion and entertainment.[2] More important are the normative grounds, which stem mainly from the critical problem of industry take-off that new network goods are confronted with. A single (industry-wide) network is a crucial element in surmounting the take-off hurdle, or at least in avoiding potentially long delays before achieving success (Shapiro and Varian [31]). Indeed, the business strategy literature has concluded, through a number of detailed case studies dealing with the emergence of particular industries in the last thirty years, that interconnection amongst all the firms in a network industry (i.e., a single network) is probably the most important ingredient for success in launching a new

[1] See Economides and Himmelberg [14], Economides [13], Shy [33] and Kwon [21]. In contrast, the earlier literature in management science relied on dynamic models with no expectations, e.g., Oren and Smith [27] and Dhebar and Oren [11]. See also Bensaid and Lesne [6] and Chen et. al. [10], among others.

[2] In some industries, each customer may have in mind his own social network only, not the overall network, when making a purchase decision, but we follow the literature in industrial organization in ignoring this distinction.

network product (Rohlfs [29]). Thus a good understanding of the single network case will shed quite some light on the incentives for compatibility faced by firms and consumers in the case of firm-specific networks. We shall return to this key point several times below.

In contrast to the extant literature, this paper considers general demand functions with non-separable network effects, a critical feature if one wishes to capture pure network goods (those with no stand-alone value, such as most telecommunication products), and the so-called feature of demand-side increasing returns (see Assumption $A5$). With pure network goods, the trivial outcome of zero output is always a self-fulfilling equilibrium, since there will be no actual demand if the market expectation is that there will be no eventual sales (in other words, nobody wishes to be the only person around owning a phone, say). In view of this, the industry will fail to take off at all if this is the only equilibrium, but might also end up coordinating on this worst possible outcome when other equilibria are present. In a nutshell, this is the so-called industry viability problem, a general treatment of which is the central concern of this paper. To this end, an important pre-requisite is a good understanding of the issues of existence and multiplicity of FECE, which can clearly discriminate between the trivial FECE and the non-trivial ones, in terms of meaningful conditions imposed on primitives of the oligopoly model. Another aim of the paper is to provide an extensive inquiry into the effects of market structure (or exogenous entry) on market performance. Throughout, the paper takes a comparative perspective in that results are contrasted with their Cournot counterparts, in an attempt to shed light on the distinctive features of network industries.

The underlying approach is to impart minimal complementarity structure to the model at hand, which achieves the twin goals of ensuring the existence of a fulfilled expectations Cournot equilibrium while at the same time allowing clear-cut predictions on the comparative statics of market performance with respect to the number of firms. The critical structure is imposed in the form of two economically meaningful complementarity conditions on the primitives that guarantee the key properties that, along a given firm's best response, industry output increases in rivals' total output as well as in the expected network size. In terms of methodology, the existence and comparative statics parts rely on lattice-theoretic techniques, but these need to be supplemented by basic novel insights from nonsmooth analysis, in particular for the viability analysis.[3]

[3]Relevant work includes Topkis [35], Vives [38], Milgrom and Roberts [25], Milgrom and Shannon [26], Echenique

We next provide an overview of our findings, coupled with a literature review. While existence of FECE follows from the monotonicity structure via a double application of Tarski's fixed point theorem, this is of limited interest, as the underlying equilibrium may a priori be the trivial one. To complete the analysis, we derive two sets of conditions, each of which ensures the existence of a non-trivial equilibrium. These conditions have clear economic interpretations; they amount to requiring relatively strong network effects near the origin or away from the origin.

Although the model is static in nature, we construct an explicit learning dynamics, mapping consumers' expectation of the network size to the corresponding Cournot equilibrium industry outputs. This tatonnement-type dynamics shall serve a dual purpose. It provides a natural theoretical foundation for an equilibrium concept that might be viewed as too demanding in its implicit simultaneous determination of both firms' behavior and the correct size of the market. The dynamics also serves as a convenient tool to analyze the viability of the industry. In fact, it has tacitly been the basis of earlier informal discussions of the viability issue in the literature. Studies of telecommunications markets, such as Rohlfs [28] and Economides and Himmelberg [14], often suggest that network industries typically have three equilibria. Under this natural dynamics, the two extreme equilibria are stable in expectations and the middle equilibrium (usually called critical mass) is unstable. The argument behind this structure is quite simple for pure network goods: If consumers' initial expectation is below the critical mass, so that few buyers are expected to acquire the good, then the good will be of little value to consumers and few of them will end up buying it. These low sales in turn further depress consumers' expectations through the above dynamics, and the market unravels towards the trivial (or no-trade) equilibrium, giving rise to a failure to take off for the industry. However, if expectations are higher to start with and network effects are relatively strong, higher equilibria will also be possible. This argument is often used to explain the start-up problem in network industries, or the difficulties faced by incumbent firms in attempting to generate enough expectations to achieve critical mass. In this setting, due to increasing returns on the demand side and to the need for expectations, multiple equilibria and path dependence (the notion that early events can have significant long run effects) are the norm, rather than the exception.

An important aim of the present paper is to shed light on the role of market structure as a

[12], Amir [1], [3], Amir and Lambson [4], and Kwon [21].

determinant of the viability of a network industry, a novel and fundamental issue that, surprisingly, has not been addressed in the theoretical literature. We find that the presence of more firms in the market always enhances industry viability, by lowering the critical mass needed to avoid the trivial equilibrium. The same conclusion holds for exogenous technological progress. These two effects provide a plausible explanation of several recorded failures and successes in attempts to launch new network goods, as reported in some detail in other sections, in particular with regard to the history of the fax industry. Indeed, Rohlfs [29] forcefully argues that interconnection between suppliers of a network good is a critical feature that is at the heart of past successful new industry launches, sometimes in conjunction with technological improvements.[4] Rohlfs' detailed case studies provide strong evidence for the policy relevance of our theoretical results on viability.

Regarding market performance, the basic structure leads to an industry output that increases in the number of firms, n, as in standard Cournot competition. As this also implies an increase in the equilibrium network size, market price need not decrease with more competition, i.e., quasi-competitiveness need not hold here. The most drastic departure from standard oligopoly lies in the effects of entry on per-firm profits. Whenever per-firm output and market price increase (decrease) with n, per-firm profits increase (decrease) in n as well (see Economides [13]). The conclusion that competition may increase each firm's profit is quite provocative and leads to several important implications, both from theoretical and policy-oriented perspectives. The effects of entry on social welfare and consumer surplus also display some distinctive features relative to standard Cournot competition. Demand-side economies of scale broaden the conditions under which social welfare increases with more entry, but they may have the opposite effect on consumer surplus whenever the marginal increase of price due to a higher network size increases with output. Our results build on the perception already prevalent in the literature that standard results on the workings of competition can easily be reversed in network industries. Since, for each dimension of market performance, the conventional intuitive outcome and its opposite can both hold in robust ways, it is highly desirable to arrive at a clear understanding of the respective specific market characteristics

[4] This conclusion does not apply to network industries that do not lend themselves to interconnection, due to a variety of reasons, which may be connected to technological, industry-specific, geographic, linguistic, or other factors. Examples include bank deposits (Matutes and Vives [24]), local clubs, national associations, etc.

under which these two outcomes prevail.[5]

As a consequence, a number of policy issues will need revisiting in network industries, whenever market characteristics are such that unconventional outcomes prevail. There is more scope for pro-competitive cooperation or coordination by firms in network markets. There will be a pronounced tendency towards less entry deterrence activities; a higher propensity for licensing, probably coupled with lower royalty rates or licensing fees; less patenting or a relatively more permissive attitude towards patent infringement by a firm's rivals; and more joint ventures for research and development towards common standards, improved product performance and lower production costs. Proper reaction to these new incentives for coordinated action by market competitors might well require a significant overhaul of existing antitrust policy (Shapiro [30]).

Another noteworthy aspect of this paper is that it offers three explicit examples with easy closed-form solutions to illustrate in a simple way some of the key conclusions. In particular, Example 1 captures with closed-form solutions most of the relevant features often associated with the telecommunications industry in the literature, as well as our new results on viability.

The paper is organized as follows. Section 2 presents the model, the equilibrium concept and the assumptions. Section 3 deals with existence of equilibrium. Section 4 formalizes the concept of industry viability and its determinants. Section 5 analyzes market performance as a function of the number of firms. Section 6 concludes, and section 7 contains all the proofs.

2 The Model

This section presents the standard oligopoly model with network effects along with the commonly used equilibrium concept due to Katz and Shapiro [19]. We consider a static model to analyze oligopolistic competition in industries with positive network effects, wherein consumers' willingness to pay for the good is increasing in the number of agents acquiring the same good. The firms' products are homogeneous and perfectly compatible with each other, so there is a single network comprising the outputs of all firms in the industry.

[5] Boone [9] provides interesting insights into the difficulties of deriving meaningful measures of competition in regular industries. Our results will suggest that this task will be far more daunting in network industries.

2.1 The Model and the Solution Concept

The market consists of n identical firms, with cost function $C(\cdot)$, facing the same inverse demand function $P(Z,S)$, where Z denotes the aggregate output in the market and S represents the expected size of the network. Postulating that each consumer buys at most one unit of the good, S also stands for the expected number of people acquiring the good.

For a given S, a firm's profit function is $\pi(x,y,S) = xP(x+y,S) - C(x)$, where x is the firm's output level, and y is the joint output of the other $(n-1)$ firms. Its reaction correspondence is

$$x(y,S) = \arg\max\{\pi(x,y,S) : x \geq 0\}. \tag{1}$$

Each firm chooses its output level to maximize its profits under the assumptions that (i) consumers' expectations about the size of the network, S, is given; and (ii) the output level of the other firms, y, is fixed. Alternatively, we may think of the firm as choosing total output $Z = x + y$, given the other firms' cumulative output, y, and the expected size of the network, S, in which case, with $\widetilde{\pi}(Z,y,S) = (Z-y)P(Z,S) - C(Z-y)$, its reaction correspondence is

$$Z(y,S) = \arg\max\{\widetilde{\pi}(Z,y,S) : Z \geq y\}. \tag{2}$$

Consistency requires $Z(y,S) = x(y,S) + y$.

At equilibrium, all relevant quantities x, y, Z and π will be indexed by the underlying number of firms n, e.g., we shall denote by Z_n the equilibrium industry output corresponding to n firms in the market, and x_{in} the equilibrium output of firm i. When clear from the context, we will avoid the subindex i in the latter variable.

An equilibrium in this game is a vector $(x_{1n}, x_{2n}, ..., x_{nn})$ that satisfies the following conditions

1. $x_{in} \in \arg\max\{xP(x + \sum_{j\neq i} x_{jn}, S) - C(x) : x \geq 0\}$; and
2. $\sum_i x_{in} = S$.

Katz and Shapiro [19] called this concept a "Fulfilled Expectations Cournot Equilibrium (or FECE)". It requires that both consumers and firms correctly predict the market outcome, so that their beliefs are confirmed in equilibrium. While strategic in their choice of outputs in the usual Cournot sense, firms are "network-size taking" in their perceived inability to directly influence

customers' expectations of market size. One plausible justification for this is that firms are unable to credibly commit to output levels that customers could observe and reliably use in formulating expectations about network size (Katz and Shapiro [19]). Naturally, the plausibility of the FECE concept increases with the number of firms present in the market.

Viewing S as an inverse demand shift variable, the first line (or Condition 1 above) just describes the equilibrium in standard Cournot competition with exogenously fixed S. Let the corresponding industry output Cournot equilibrium be denoted $Q_n(S)$, a multi-valued mapping in general (since no uniqueness of Cournot equilibrium will be assumed). Adding condition 2, an aggregate output $Z_n \in Q_n(S)$ constitutes a FECE industry output if it confirms the expected level of sales (or network size) that generated it, i.e., has $Z_n = S$. Thus, if we graph $Q_n(S)$ as a function of S, the FECE industry outputs coincide with the fixed points, or the points where this correspondence crosses the 45° line. This idea will play a key role in both the existence proof and the viability analysis.

An alternative, fully game-theoretic, interpretation of this equilibrium notion is in the context of a two-stage game, wherein a market maker (or a regulator) announces an expected network size S in the first stage, and firms compete in Cournot fashion facing inverse demand $P(Z, S)$ in the second stage. If the market maker's objective function is to minimize $|S - Q_n(S)|$, then to any subgame-perfect equilibrium of this game corresponds a FECE of the Cournot market with network externalities, and vice-versa. This simple conceptualization of the FECE solution also provides one natural approach for arriving at a FECE with the participation of a market maker, and in case of multiple equilibria, also for selecting a particular FECE.[6]

The FECE concept has a dual nature: It consists of the conjunction of a standard Cournot equilibrium and a rational expectations requirement. (The latter is not related in any way to uncertainty but rather to the determination of the true final demand that will prevail in the economy.) As it pins down both firms' strategic behavior in the market and the coordination of expectations as to the right market size, all in one stroke within a static model, one might feel that this solution concept is excessively ambitious.[7] In other words, it attempts to compress an intrinsically dynamic

[6] We shall briefly return to this point in the Conclusion to argue that the U.S. government-sponsored launch of the Internet fits this regulation-based description of the model.

[7] At the same time, this concept treats consumers in a reduced-form manner as being fully nonstrategic.

succession of building blocks into a static representation. Partly to address this natural critique, a theoretical foundation for FECE is provided in the form of a simple myopic learning dynamics that converges to any Cournot-stable FECE from a suitable basin of attraction.[8] This dynamics is also intended as a natural vehicle for investigating the important issue of industry viability.

This dual nature also implies that FECE is more appropriately classified not as a purely non-cooperative solution concept, but rather as one capturing co-opetition (Brandenburg and Nalebuff [8]). The familiar inter-firm rivalrous relationship inherent in Cournot competition is intertwined with an inter-firm partnership in terms of jointly creating sufficiently high expectations for the industry prospects and building up a large common network of consumers. The overall outcome is an intricate and interesting case of co-opetition, which is unambiguously confirmed by the results below on industry viability and on the effects of increased competition on firms' well-being. In fact, the congruence between the main results of this paper and some central case studies for the business strategy literature provides important real world evidence in support of FECE as a suitable static solution concept for network industries.

An alternative solution concept has been proposed for environments where firms possess the ability to make credible commitments to output levels. In such cases, standard Cournot equilibrium with inverse demand $P(Z, Z)$ would be a more appropriate concept. A direct comparison between these two concepts appears in Katz and Shapiro [19], who find that firms' market behavior is more aggressive, leading to a higher industry output than under the FECE concept, an intuitive outcome. Ultimately, the issue as to which of these concepts is more appropriate for network industries is an empirical matter, and the answer is likely to vary according to industry characteristics, in particular those relating to firms' ability to credibly commit (observability conditions, firm reputation, government participation, marketing and public awareness of the product, etc.).

2.2 The Basic Assumptions

We list the assumptions used in this paper, starting with a set of standard ones, followed by more substantive conditions. Whenever well-defined, we denote the maximal and minimal points of

[8] Some studies provide truly dynamic models of single-network competition amongst firms, which may also be viewed as foundations for the present paper (e.g., Dhebar and Oren [11] and Bensaid and Lesne [6]).

a set by an upper and a lower bar, respectively. Thus, for instance, $\overline{Z}_n$ and $\underline{Z}_n$ are the highest and lowest industry equilibrium outputs (i.e., fixed points of $Q_n(S)$) with n firms in the market.

Denote by $W(Z,S) \triangleq \int_0^Z P(t,S)\,dt - nC(Z/n)$ the Marshallian social welfare when aggregate output is Z, all firms produce the same quantity and the expected size of the network is S. Similarly, consumer surplus is $CS(Z,S) \triangleq \int_0^Z P(t,S)\,dt - ZP(Z,S)$.

The standard assumptions are

$(A1)$ $P(.,.)$ is twice continuously differentiable, $P_1(Z,S) < 0$ and $P_2(Z,S) > 0$.

$(A2)$ $C(.)$ is twice continuously differentiable and increasing, and $C(0) = 0$.

$(A3)$ $x_i \leq K$, for each firm i.

These are all commonly used assumptions, including $P_2(Z,S) > 0$, which reflects positive network effects, or the property that consumers' willingness to pay increases in the expected number of people who will buy the good. $A3$ imposes capacity constraints on the firms, a convenient way to force compact output sets in a setting where firms may otherwise wish to produce unbounded output levels. No results rely in any way on K taking on any particular set of values.

We allow for the possibility that $P(Z,0) \equiv 0$, which characterizes pure network goods, or those with no stand-alone value, such as most telecommunications devices (telephone, fax, and e-mail). We also allow for mixed network goods, or those with strictly positive stand-alone value, for which $P(Z,0) > 0$, such as various types of software, fashion goods, and entertainment goods and services.

The next two assumptions form the key complementarity structure of the model.

$(A4)$ $\Delta_1(Z,y,S) \triangleq -P_1(Z,S) + C''(Z-y) > 0$ on $\varphi_1 \triangleq \{(Z,y,S) : Z \geq y, y \geq 0, S \geq 0\}$.

$(A5)$ $\Delta_2(Z,S) \triangleq P(Z,S)\,P_{12}(Z,S) - P_1(Z,S)\,P_2(Z,S) > 0$ on $\varphi_2 \triangleq \{(Z,y,S) : Z \geq y, y \geq 0, S \geq 0\}$.

In terms of the model structure, $A4$ guarantees that the profit function $\widetilde{\pi}(Z,y,S)$ has strictly increasing differences in (Z,y), so $Z(y,S)$ increases in y, or a firm's best-response has slopes greater than -1 in other firms' quantity for fixed S.[9] Likewise, $A5$ ensures that $\log \widetilde{\pi}(Z,y,S)$ has strict increasing differences in $(Z;S)$, so that $Z(y,S)$ increases in S.[10]

[9] Also see Hoernig [18] for an extension to differentiated-goods industries.

[10] All the lattice-theoretic notions and general results used in this paper are covered in Topkis [36] or Vives [39].

In terms of economic interpretation, $A4$ allows for limited scale economies in production, and has been justified in detail in Amir and Lambson [4]. Although our analysis does incorporate this generality on the production side, we shall not stress this point when discussing our conclusions so as to keep focus on the demand side features of the model.

The novel assumption here is $A5$, which has the precise economic interpretation that the elasticity of demand increases in the expected network size S.[11] In his pioneering study of the elementary microeconomic foundations of interdependent demands, Leibenstein [22] suggested that demand is more elastic in network markets than in ordinary markets because individual reactions to price changes are followed by additional reactions, in the same direction, to each other's change in consumption. Within the present simple, static representation of demand for a good with network effects, $A5$ may be viewed as a way to formalize the cumulative outcome of these mutually reinforcing effects on aggregate demand via self-fulfilling expectations of the network size. Another plausible interpretation of $A5$ is that it provides a natural way to model the concept of demand-side scale economies that is often postulated as a characteristic of network industries: Higher expectations of ultimate market size increase consumers' willingness to pay and make demand more elastic.

$A5$ also embodies a key respect in which the present paper departs from the extant static literature, much of which deals with the case of additively separable network effects, defined by[12]

$$P(Z,S) = p(Z) + g(S) \tag{3}$$

While this specification clearly satisfies the condition in $A5$, it automatically excludes the case of pure network goods (characterized by $P(Z,0) = 0$ for all Z, which is incompatible with (3)), for which the role of expectations is often critical, making industry viability a crucial issue. Since this issue is central to the focus of the present paper, we cannot adopt the simplifying assumption of separable network effects.

On the other hand, the case of multiplicative network effects, defined by $P(Z,S) = p(Z)g(S)$, a specification that can in particular capture pure network goods (whenever g satisfies $g(0) = 0$),

[11] The price elasticity of demand is $-\left(\frac{\partial P(Z,S)}{\partial Z}\frac{Z}{P(Z,S)}\right)^{-1} = -\left(Z\frac{\partial \log P(Z,S)}{\partial Z}\right)^{-1}$, which is increasing in S if and only if $\log P(Z,S)$ has increasing differences in (Z,S) (Topkis [36], p. 66).

[12] See Katz and Shapiro [19], Economides and Himmelberg [14] and Economides [13] among others.

also satisfies $A5$ as a limit case (i.e., with equality for all Z and S).[13] This multiplicative form captures situations where a higher expected network size keeps the elasticity of demand invariant, thus generating a pure upward shift in demand. As an example of this, consider the inverse demand $P(Z,S) = S/Z^{\alpha}, \alpha > 0$.

3 Existence of Equilibrium

In this section we provide a general equilibrium existence result, exploiting the minimal monotonic structure of the model reflected in $A4$-$A5$. As the trivial (zero-output) equilibrium is often part of the equilibrium set, we derive a second result that relies on additional conditions to guarantee the existence of a non-trivial equilibrium, i.e., one with strictly positive industry output. Under these extra conditions, the market has some chance to emerge at equilibrium.

We begin with the central monotonicity result, which is a direct consequence of $A4$ and $A5$.

Lemma 1 *Every selection of the best-response correspondence $Z(y,S)$ increases in both y and S.*

This lemma leads to an abstract existence result for symmetric equilibrium, along with the fact that the same assumptions preclude the possibility of asymmetric equilibria.

Theorem 2 *For each $n \in N$, the Cournot oligopoly with network effects has (at least) one symmetric equilibrium and no asymmetric equilibria.*

The monotonicity structure behind the existence theorem will also drive other results of this paper, many of which have a comparative statics flavor. Comparing $A1$-$A5$ with the assumptions in standard Cournot oligopoly, the only extra requirement is that the price elasticity of demand increases with the network size, $A5$, taking $P_2(Z,S) > 0$ as a natural property of network markets.

Recall that $Q_n(S)$ denotes the industry output equilibrium correspondence of standard Cournot competition (with exogenous S), with n firms in the market. In Section 2 we observed that a fixed point of $Q_n(S)$ constitutes a FECE. Conditions $A1$-$A4$ guarantee that $Q_n(S)$ is non-empty using

[13] The strict inequality in $A5$ is purely for convenience, and our conclusions are all valid with a weak inequality instead. The main implication of the latter assumption is that only the extemal selections of the argmax $Z(y,S)$, as opposed to all the selections, will be increasing in S.

the results of Amir and Lambson [4]. Then the added benefit of $A5$ is that the extremal selections of $Q_n(S)$, $\overline{Q}_n(S)$ and $\underline{Q}_n(S)$, increase in S, so that the existence of FECE follows via Tarski's Theorem applied to $\overline{Q}_n(S)$ or $\underline{Q}_n(S)$. This idea also plays a key role in the proof of existence of a non-trivial equilibrium below.

It is well-known that in network markets, the trivial (zero output) outcome is often an equilibrium. This arises when the network good has little stand-alone value, i.e., $P(Z,0)$ is small. Given any such good, if end users believe no one else will acquire it, then the good will have no value, and the trivial outcome will necessarily be part of the equilibrium set. Telecommunications industries, such as fax, phone and e-mail, typically exhibit this characteristic.

In such markets, Theorem 2 is not of much interest since the underlying equilibrium may a priori be the trivial one. To complete the analysis, we give a simple characterization of the trivial equilibrium and then add extra assumptions to guarantee the existence of a non-trivial one.

Lemma 3 *The trivial outcome is an equilibrium if and only if* $xP(x,0) \leq C(x)$ *for all* $x \in [0,K]$.

This lemma simply says that the trivial outcome is part of the equilibrium set if and only if, when the common expectation (amongst firms and consumers) about the size of the network is zero, and a firm believes the other firms will produce no output, the best it can do under the required condition is to produce zero as well. Clearly, for pure network goods, this result always holds.

To provide conditions for the existence of a non-trivial equilibrium, we use a fictitious objective function that achieves its maximum at a Cournot equilibrium industry output level, for given S, as shown in Bergstrom and Varian [**?**] for standard Cournot oligopoly.[14] Define

$$\Pi(Z,S) \triangleq \frac{n-1}{n}\left[\int_0^Z P(t,S)\,dt - nC(Z/n)\right] + \frac{1}{n}\left[ZP(Z,S) - nC(Z/n)\right]. \tag{4}$$

For given S, this function is just a weighted average of welfare and industry profits, with respective weights $\frac{1}{n}$ and $\frac{n-1}{n}$, which may be viewed as a fictitious objective function for Cournot oligopoly.

Theorem 4 *A non-trivial equilibrium exists if at least one of the following is satisfied*

[14] Their simple proof just compared the first order conditions of the two problems, as their setting implied unique solutions for both. Lemma 16 in Section 7 shows that any argmax of Π is a symmetric Cournot equilibrium in our more general setting. This conversion of a game to a maximization problem is crucial to the proof of Theorem 4(iii). A similar result appeared earlier in Spence [34].

(i) $0 \notin Q_n(0)$*, i.e., zero is not an equilibrium output (or* $xP(x,0) > C(x)$ *for some* $x \in [0,K]$*);*

(ii) $0 \in Q_n(0)$*,* $P(0,0) = C'(0)$ *and* $P_1(0,0) + P_2(0,0) > [-P_1(0,0) + C''(0)]/n$*; or*

(iii) $0 \in Q_n(0)$*,* $C''(.) \geq 0$ *and* $P(Z,S) + \frac{Z}{n}P_1(Z,S) \geq C'(Z/n)$ *for some* S *and all* $Z \leq S$*.*

Theorem 2 guarantees equilibrium existence. Hence, if zero is not part of the equilibrium set, there must be an equilibrium with a strictly positive industry output, and Theorem 4(i) follows. This applies only to network goods with sufficiently high stand-alone value (cf. Lemma 3), e.g., specific computer software, some fashion goods, web sites, etc.

The extra requirements in Theorem 4(ii) guarantee that, although $Q_n(0) = 0$, $Q_n(S)$ starts above the 45° line near zero, i.e., $Q_n'(0) > 1$. The existence of a non-trivial equilibrium follows as the extremal selections of $Q_n(S)$ increase in S. Formally, this derives from applying Tarski's Theorem to any of the extremal selections of $Q_n(S)$ for $S \in [\epsilon, nK]$, and some $\epsilon > 0$ small enough. Observe that, ceteris paribus, the condition of Part (ii) is more likely to hold with a stronger network effect around the origin (as captured by $P_2(0,0)$), or with a higher number of firms in the market. In fact, by $A4$, the condition requires that $P_1(0,0) + P_2(0,0) > 0$.

The main condition in Theorem 4(iii) has a clear economic meaning: There must be some network size S such that, along a symmetric path for firms, a firm's marginal revenue exceeds marginal cost for all $Z \leq S$. As this amounts to $\Pi_1(Z,S) \geq 0$, the same interpretation using marginal revenue also holds for the planner's problem. It follows that the argmax $Q_n(S)$ of $\Pi(Z,S)$ is above the 45° line at the given S. Then a non-trivial equilibrium exists by Tarski's Theorem applied to $Q_n(.)$ mapping $[S, nK]$ to itself.[15] The economic interpretation of this condition is also that network effects must be sufficiently strong, but for some $S > 0$, and/or the number of firms in the market must be large enough. Indeed, taking the derivative of the LHS of condition (iii) with respect to S yields $P_2(Z,S) + \frac{Z}{n}P_{12}(Z,S)$. Assuming this to be strictly positive and large is related to requiring that $A5$ hold in a strong sense (i.e., with sufficient slack), which would boil down to requiring that the price elasticity of demand be sufficiently responsive to the network size.[16] This

[15] This condition is stronger than what is actually needed, which is that $\Pi(Z',S) \geq \Pi(Z,S)$ for some S, some $Z' \geq S$ and all $Z \leq S$. We prefer to use the stronger condition due to its more transparent economic interpretation.

[16] We could use the condition $P_2(Z,S) + ZP_{12}(Z,S) > 0$ in lieu of $A5$ throughout the paper. This would make $\widetilde{\pi}$ supermodular instead of having the single-crossing condition in $(Z;S)$, thus yielding the same conclusions.

is a plausible measure of the strength of network effects.

In a nutshell, the existence of a non-trivial equilibrium is guaranteed if the stand alone value of the good is high. Otherwise it depends in a crucial way on the strength of the network effect around the origin (as captured by $P_2(0,0)$) as well as away from the origin, and on the number of firms in the market.

The proof of Theorem 4 uses the following intermediate result, which also plays a key role in the viability analysis (Section 4).

Lemma 5 *If $0 \in Q_n(0)$, then $Q_n(0) = 0$, i.e., $Q_n(0)$ is single-valued. If in addition $P(0,0) = C'(0)$, then $Q_n'(0)$ exists, is also single-valued and right-continuous at 0, and*

$$Q_n'(0) = \frac{-nP_2(0,0)}{(n+1)P_1(0,0) - C''(0)}. \tag{5}$$

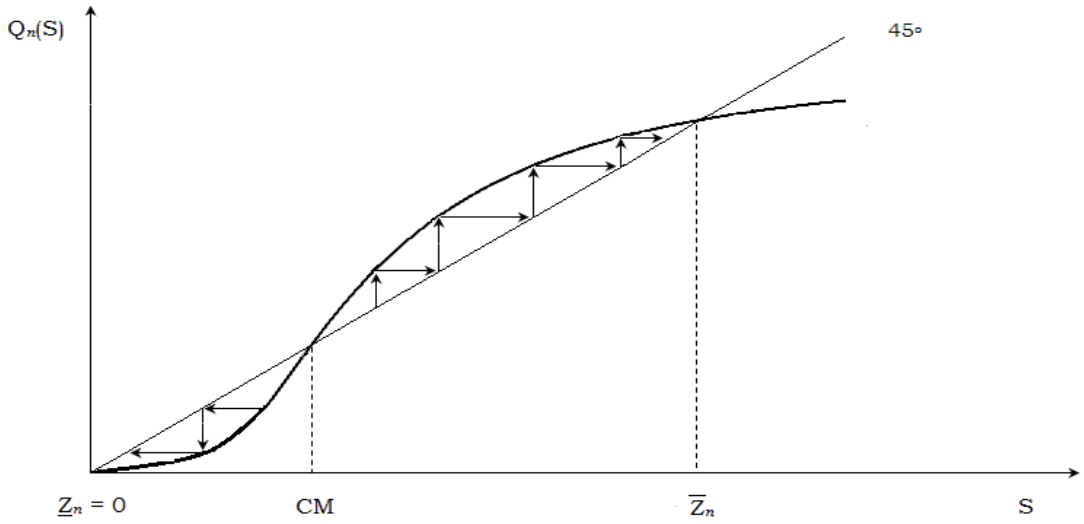
If the trivial equilibrium is not interior, i.e., if $P(0,0) < C'(0)$, then $Q_n(0) = 0$ and $Q_n'(0) = 0$.

Thus, though $Q_n(.)$ is a correspondence, when 0 is part of the equilibrium set, $Q_n(.)$ is single-valued at the origin. If in addition the trivial equilibrium satisfies the first order condition for a maximum, i.e., $P(0,0) = C'(0)$, then the slope of this correspondence at 0 is given by (5).[17]

Multiple equilibria in markets with network effects are more of a norm than an exception. They are due to the positive feedback associated with expectations: If consumers believe the good will not succeed, it will usually fail. On the contrary, if they expect it to succeed, it usually will. By assuming $P(Z,S)$ is log-concave in Z, one obtains uniqueness of Cournot equilibrium for each S, so $Q_n(.)$ is single-valued and continuous. With network effects, much stronger conditions are required for uniqueness of FECE, as a single-valued $Q_n(.)$ can cross the 45° line several times. Since our methodology easily handles multiple equilibria, there is no need to impose the overly restrictive uniqueness conditions.

[17] Lemma 5 reflects the extent to which the standard argument of (the smooth version of) the Implicit Function Theorem can be carried over to a setting in which the usual assumption of strict concavity of the objective is replaced by a supermodularity condition (so the argmax is an increasing correspondence as opposed to a differentiable function).

Figure 1: Viability and basin of attraction of the trivial equilibrium



4 Industry Viability

Building directly on the results on the existence of non-trivial equilibrium, this section provides an extensive treatment of industry viability, via a formalization of expectations dynamics and the associated stability analysis of the multiple FECE, in particular of the trivial one. This dynamics maybe viewed as a natural extension of standard Cournot dynamics, which integrates iterative expectations in the usual myopic setting. As such, it constitutes an elementary theoretical foundation for the FECE concept. We then derive insightful novel results on the effects of exogenously changing market structure and technological progress on the viability of an industry.

4.1 A Natural Dynamics for the FECE Concept

Many studies suggest that Figure 1 reflects the structure of specific telecommunications industries. The underlying game there displays three possible equilibria, the trivial equilibrium, a middle unstable equilibrium (usually called critical mass, CM), and a high stable equilibrium. The intuition behind this configuration is quite simple: If all the consumers expect that no one will acquire the good, then the good has no value and no one will end up buying it, resulting in the zero equilibrium output for the industry. However, if expectations are high enough to start with, another, non-trivial, equilibrium will prevail.

Whenever the trivial equilibrium is locally stable in expectations (as in Figure 1), the market will never emerge if the initial expected network size is too low to start with. Under such conditions, even if the industry does get going, Cournot equilibrium on the basis of small expectations cannot lead firms to produce enough output to generate prospects beyond the critical mass, and the industry will unravel through a declining process towards the trivial equilibrium. In view of the equilibrium concept adopted here, the incumbent firms are simply unable to influence these expectations to circumvent this difficulty. This argument is commonly invoked to capture the start-up problem that frequently arises in these markets, as a "chicken and egg" dilemma. Oren and Smith [27] offer an early discussion of this phenomenon in electronic communications markets.

The dynamic process underlying this analysis can be formalized through the following expectations/network size recursion, starting from any initial $S_0 \geq 0$,

$$S_k = \overline{Q}_n \left(S_{k-1} \right), \ k \geq 1 \tag{6}$$

where $\overline{Q}_n$ denotes the maximal selection of Q_n. The analysis that follows is also valid for the minimal selection $\underline{Q}_n$, and most results work for any monotonic selection of Q_n.

This process thus begins with an initial expectation S_0, then postulates that firms react by engaging in Cournot competition with demand $P(Z, S_0)$, leading to an industry output $\overline{Q}_n (S_0)$. The latter will in turn determine consumers' expectation $S_1 = \overline{Q}_n (S_0)$, and the process repeats indefinitely. This yields a sequential adjustment course in which consumers and firms behave myopically with respect to the size of the network. Taking a single-valued selection of $Q_n(.)$ amounts to selecting one particular Cournot equilibrium for each given S. Under this dynamic process, the trivial equilibrium is stable if there is a right neighborhood V of 0 such that for all S_0 in V, the orbit $S_k = \overline{Q}_n (S_{k-1}) \to 0$ as $k \to \infty$.[18]

When zero is an equilibrium, let V_n denote its basin of attraction when there are n firms in the market, i.e., the set of values of S_0 for which the trivial equilibrium is the limit of (6).

Remark 1. In view of Lemma 5, when zero is a FECE, $\overline{Q}_n(.)$ is continuously differentiable at 0. Therefore, assuming henceforth that this derivative is (generically) not equal to 1, it follows that zero is an isolated fixed-point (for a formal proof, see, e.g., Granas and Dugundji [17], pp. 326-327).

[18] The notions of stability used here are the standard ones applied to the one-dimensional dynamic system (6).

Since in addition $\overline{Q}_n(S)$ is increasing in S, V_n is an interval.

Define the critical mass (CM) as the smallest initial expectation such that for all $S_0 > CM$, the orbit given by (6) converges to a nonzero FECE.[19] In other words, $CM = \sup V_n$, so V_n can be of the form $[0, CM]$ or $[0, CM)$. The trivial equilibrium is globally stable, locally (but non-globally) stable, or unstable according as $V_n = [0, nK]$, $V_n \subsetneq [0, nK]$, or $V_n = \phi$, or equivalently according as $CM = nK$, $nK > CM > 0$, or $CM = 0$.

4.2 Industry Viability and its Determinants

We begin by formalizing the concept of viability used in this paper, illustrated in Figure 2.

Definition 1 An industry is (i) uniformly viable if zero output is either not part of the equilibrium set or it is an unstable equilibrium; (ii) conditionally viable if zero output is a locally (but non-globally) stable equilibrium; and (iii) nonviable if zero output is a globally stable equilibrium.

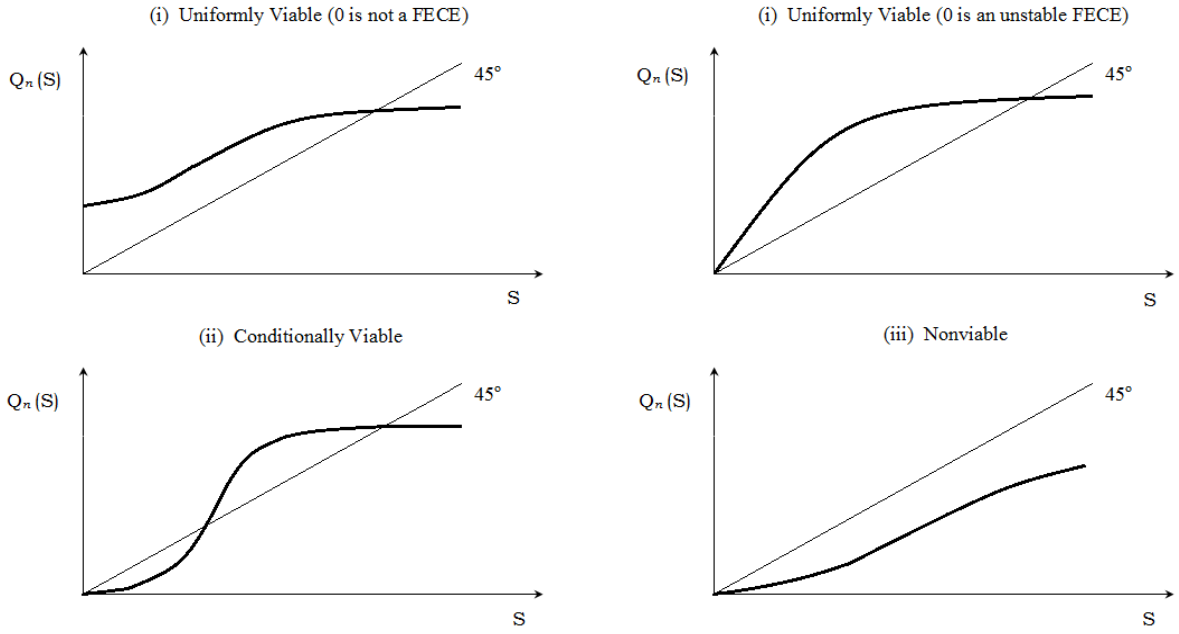
In other words, an industry is (i) uniformly viable if the dynamic process (6) converges to a non-trivial equilibrium from any initial expectation S_0 about the size of the network, (ii) conditionally viable if the same convergence takes place from high enough S_0, and (iii) nonviable if (6) always converges to 0. It is intuitive that the maximal (minimal) selection of $Q_n(.)$ is the most (least) favorable for the viability of the industry.

The next result provides sufficient conditions for the three possible viability outcomes, by directly linking them with the sufficient conditions for a nontrivial equilibrium provided in Theorem 4 (the proof is omitted due to its similarity with that of Theorem 4).

Proposition 6 *An industry is (i) uniformly viable if either Theorem 4(i) or (ii) holds; (ii) conditionally viable if Theorem 4(iii) holds; and (iii) nonviable if and only if zero output is the unique equilibrium.*

[19] Such orbits always converge by the Monotone Convergence Theorem, since $\overline{Q}_n(\cdot)$ is increasing. This limit FECE will always be a fixed point of the correspondence $Q_n(\cdot)$ since the latter is upper hemi-continuous (see the proof of Lemma 5 in Section 7), but may fail to be a fixed point of the selection $\overline{Q}_n(\cdot)$ in some cases where CM is not a point of continuity of $\overline{Q}_n(\cdot)$. This definition captures the essence of critical mass in this context, and is a suitable extension of the usual definition of CM to this more general setting where discontinuities are induced by multiple equilibria.

Figure 2: Industry viability



Thus, just like the existence of nontrivial equilibrium, viability depends in a crucial way on the stand alone-value of the good, on the strength of the network effects at the early start of the industry, on the strength of network effects away from the origin (i.e., after the build-up of some consumer base), as well as on the number of firms in the market. Last but not least, viability depends on the initial expectations level S_0. For nonviable industries, the latter dependence is only transient and the dynamics is actually of an ergodic nature, with the death of the industry being the ultimate outcome. A similar conclusion holds for uniformly viable industries in case of a unique non-trivial equilibrium; otherwise, S_0 determines which non-trivial equilibrium the industry will converge to.

In the most interesting scenario as captured by Proposition 6(ii), whenever the stand alone value is low and network effects are weak near the origin but strong away from it, the industry is conditionally viable, so the location of the initial expectations level S_0 relative to the critical mass emerges as the critical determinant of actual viability. This constitutes an extreme form of path dependence due to the all-or-nothing character of the final outcome: Historic factors (or early events) matter greatly for the ultimate fate of the industry. Such path dependence is commonly

associated with increasing returns in diverse contexts in economics (e.g., Arthur [5]). Here the increasing returns property lies in the demand-side externalities (Assumption $A5$), when these are strong enough as implied by the conditions in Proposition 6.

These results shed light on a commonly observed strategy that firms in network industries often follow as a way to create a stand alone value thereby inducing uniform viability: The bundling of multiple functions in one product, with at least one of them ensuring a positive stand alone value. Thus later fax machines often include a copier; and Blu-ray discs enable storage, recording, rewriting and playback of high-definition video, along with compatibility with CDs and DVDs. These super-products are far less vulnerable to the start up problem than pure network goods.

In order to derive some useful comparative statics of viability, we shall need to compare two different situations for the same industry. To this end, a simple option is to think of the size of the basin of attraction of zero as an inverse measure of industry viability.[20]

Definition 2 The viability of an industry increases if the critical mass (CM) decreases.

The next result, a central finding of this paper, captures the effects of two key determinants of industry viability. Here, exogenous technological progress (or process R&D) is modeled as a decrease in α for the cost function $\alpha C(.)$.

Theorem 7 *With more firms in the market and/or technological progress, i.e., as n increases and/or α decreases, (i) $\overline{Q}_n(.)$ shifts up; and (ii) the viability of an industry increases.*

Thus, market structure may play a critical role in industry viability: Having more firms around implies a lower critical mass would be needed to launch a given industry.[21] The underlying intuition is intimately connected to the FECE concept. Consider the natural question: If S_0 happens to be below the critical mass, what prevents the existing firms from attempting to act as if there were more of them by producing a higher output level in an effort to influence consumers' expectations of the

[20] Recall that viability (and thus CM) depends on the equilibrium selection under which the industry operates. The selections $\overline{Q}_n$ and $\underline{Q}_n$ correspond to the most and least optimistic scenarios in terms of viability. Nevertheless, one implication of Lemma 5 is that an industry is uniformly viable according to $\overline{Q}_n$ if and only if it is according to $\underline{Q}_n$.

[21] When CM is a fixed point of $\overline{Q}_n$, the result of Part (ii) corresponds to the Correspondence Principle (Echenique [12]). However, Theorem 7 does not make any such assumption about CM. We thus provide a novel and simple proof that works even when CM is not a point of continuity of $\overline{Q}_n$.

network size upwards? In a context where the appropriate solution concept is FECE, firms cannot commit to their desired output levels in a credible way, and, likewise, attempting to inflate their number by committing to a higher output would also not be credible, i.e., it would not constitute behavior compatible with the FECE concept.

That technological progress also lowers the critical mass that would be needed to launch an industry is more in line with standard intuition from ordinary markets.

In industries with multiple firms having their own versions of the same general good, this theorem provides a clear explanation as to why firms often settle for full compatibility between their products, instead of incompatibility. Their objective is to generate a single industry network that would be viable, when separate networks with one firm each would not be. The business strategy literature offers many case studies that can be instructively reviewed through the lens of the present results on viability, as we now argue.

Several notable failures at product launch by well-established firms confirm that the take off problem is a serious real-world concern for network industries. These flops include Picturephone by AT&T in the 1970s, digital compact cassettes by Philips and Matsushita in the 1980s, digital audiotapes by Sony in Japan in the late 1980s, early e-mail systems in the 1980s, and minidiscs, among other products. Rohlfs [29] identifies the failure to "interconnect" or develop one unified network of consumers as a critical ingredient behind most of these flops.

One well known success story is the case of Sony and Philips, two fierce competitors that jointly developed one industry standard for compact discs in the 1980s, and licensed the standard to other entrants on favorable terms. This led to an exemplary launch of their common standard (Shapiro [30]; Rohlfs [29]), despite the need to displace the existing inferior technology of LP records.

This comparative statics result can also shed some light on well known dynamic episodes of multiple attempts at product launch and market formation. The most instructive example is the fax industry, which took nearly one and a half centuries beyond the discovery of the initial technology in the 1840s to achieve true take off (Rohlfs [29]). After a number of false starts with firm-specific networks, it took several technological improvements of the fax machine and a significant drop in production costs for the industry to achieve true take off in the 1980s.[22] In particular, a critical

[22] These technological improvements include the speed of transmission and the suppression of noxious smells, quality

event in the evolution of this industry is the agreement achieved only in 1979 by the major firms to "interconnect" or to make their machines fully compatible, thus resulting in one very large network of users. This case study also indirectly provides some support for the relevance of FECE as a solution concept in network industries. Each subset of firms initially running a separate network failed to break out of its own small consumer base, which suggests that these firms were unable to have a decisive influence on the pessimistic expectations of their specific consumer base. Only upon interconnection or (agreement on a common standard) were there sufficiently many firms in one overall network to take advantage of the positive feedback effects and break through the initial stalemate. Thus in this illuminating story, the conjunction of the two factors described in Theorem 7 was needed to achieve long run viability.

Remark 2. Using insights from Theorems 4 and 7, two important threshold numbers of firms can be derived, those at which an industry undergoes a qualitative change in terms of viability. The first would cause the industry to switch from being nonviable to conditionally viable, and the second from the latter to uniformly viable, thus eliminating the risk of failed take off. Similar remarks on the effects of technological progress can be derived. This is left to the reader.
On the other hand, if $P_1(0,0) + P_2(0,0) \leq 0$, having more firms in the market can improve the viability of an industry but the industry can never attain uniform viability for any n! Indeed, it follows from Lemma 5 that in this case, one always has $Q_n'(0) < 1$ for all n, so that 0 is a locally stable equilibrium for all n. This result is not surprising in light of the established intuitive link between industry viability and the strength of network effects, as the condition $P_1(0,0)+P_2(0,0) \leq 0$ clearly stands for weak network externalities near 0.

The next example illustrates various aspects of the effects of n on viability in Theorem 7.

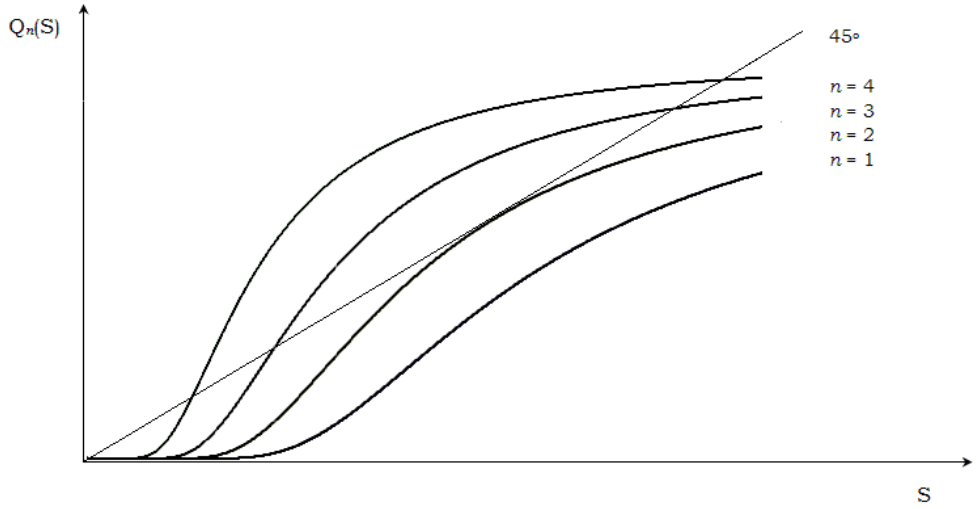
Example 1. Let $P(Z,S) = \exp\left(-\frac{2Z}{\exp(1-1/S)}\right)$ and assume no production costs.

Each firm's reaction function is $x(y,S) = (1/2)\exp(1-1/S)$. Hence, Cournot industry output given S is given by the function

$$Q_n(S) = (n/2)\exp(1-1/S)$$

upgrades that cannot be accounted for in the present model, other than as upward shifts in the demand function.

Figure 3: Viability and market structure



Setting $Q_n(S) = S$ for $n = 1, 2, 3, 4$, the FECE industry outputs are easily calculated as

$$Z_1 = \{0\}, Z_2 = \{0, 1\}, Z_3 = \{0, 0.457, 2.882\} \text{ and } Z_4 = \{0, 0.373, 4.311\}.$$

The graphs of the functions $Q_n(S)$ for $n = 1, 2, 3, 4$ are shown in Figure 3.

Here, the trivial equilibrium is stable for all values of n, at least locally. With monopoly ($n = 1$), the trivial equilibrium is the only one, so the industry is nonviable. With one extra firm, a nontrivial equilibrium appears and the industry becomes conditionally viable but only barely, since $Q_n(.)$ is tangent to the 45° line. This is a knife-edge (non-generic) situation where the stable (high output) equilibrium coincides with the unstable (low output) equilibrium, and with the critical mass. In this case, as one firm alone would simply be unable to achieve industry take-off, an incumbent monopolist would welcome one extra firm into the market as a matter of survival. Due to the knife-edge equilibrium, a monopolist might even opt for encouraging two other firms to enter, even though its profit can be seen to be lower with two competitors than with just one.[23] For any number of firms beyond three, the equilibrium set includes three points; the two extremal ones are stable

[23] Indeed, in a richer model (possibly including uncertainty), depending on the size of the critical mass and on their risk attitude, the firms might welcome entry beyond the threshold number $\widehat{n}$ that allows the industry to exit the nonviable state. This would shrink the critical mass, and thus would increase the probability of reaching the high steady state if the initial expectations level S_0 is viewed as a random variable.

and the intermediate one is unstable. This is the first example with closed-form solutions of the three-equilibrium constellation that is often portrayed as typical of many network industries.

It is easily verified that at the largest equilibrium per-firm profit decreases as n increases beyond 2 firms. So, of all possible market structures, the lowest per-firm profit is achieved by monopoly (zero profit) and the highest by duopoly, an outcome with no counterpart in ordinary markets. In addition, due to the non-emergence of the industry, *monopoly is clearly the worse scenario out of all possible market structures* for firms, consumers and thus society as well, but not because of the usual deadweight loss, but something far worse for all: The non-emergence of the industry!

This example is also instructive as to the co-opetitive nature of the FECE concept. Indeed, the cooperative aspect dominates the competitive aspect (in the strong sense of survival) when less than two firms are present in the market. With more than two firms, a higher number of firms carries the advantage of higher prospects for industry take-off (due to a lower critical mass), but it reduces per-firm profit. Thus, in the latter case, we uncover here another novel feature in network markets: That per firm profit alone is an insufficient indicator of firms "well-being", since it does not take into account the possibility of failure in take-off. Rather, a two dimensional index is needed for firms to be able to unambiguously rank different prospects.

Clearly, $Q_n(.)$ shifts up as n increases. As n increases beyond duopoly, viability increases since the basin of attraction of the zero equilibrium contracts, but uniformly viability is never attained as $P_1(0,0) + P_2(0,0) = 0$, in conformity with Remark 2 above.

Finally, this example may also be invoked to illustrate the use of Theorem 4 in proving existence of a *nontrivial* equilibrium. The main condition in Theorem 4(iii) is easily shown to boil down to

$$n \exp(1 - 1/S) \geq 2S \text{ for some } S,$$

which cannot hold for $n = 1$ but always holds for $n \geq 2$. Hence, in light of the above closed-form computations, we can conclude that for this example, the condition in Theorem 4(iii) turns out to be a necessary and sufficient condition for existence of a *nontrivial* equilibrium.‖

5 Number of Firms and Market Performance

This section studies the effects of market structure (or exogenous entry) on the equilibrium industry output, market price, per-firm output and profits, consumer surplus and social welfare. Amir and Lambson [4] and Amir [3] address similar questions for standard Cournot competition, and show that scale economies lead to counterintuitive results. This section shows that, under network effects, similar (and additional) reversals are typically much easier to come by, and that they can be generated solely by demand-side externalities instead of production scale economies. We provide sufficient conditions (or at least closed-form examples) for these counterintuitive outcomes.

The analysis that follows refers specifically to the largest equilibrium FECE, with outputs denoted by $\overline{Z}_n$ and $\overline{x}_n$, throughout. All the results in this section also apply to the smallest equilibrium, although these are trivial conclusions whenever the zero outcome is that equilibrium.

The new assumptions we invoke below depend on the signs of the functions (with domain $[0, nK]$)

$$\begin{aligned}\Delta_3(Z) &= P_1(Z,Z) + P_2(Z,Z) \text{ and} \\ \Delta_4(Z) &= \left[P(Z,Z) - C'(Z/n)\right]\left[P_{11}(Z,Z) + P_{12}(Z,Z)\right] - P_1(Z,Z)\left[P_1(Z,Z) + P_2(Z,Z)\right].\end{aligned}$$

$\Delta_3(Z)$ evaluates the total effect on market price of changing aggregate output along the fulfilled expectation path. We will provide some insight on $\Delta_4(Z)$ later. In what follows, let $I_n = [\overline{Z}_n, \overline{Z}_{n+1}]$.

The first result relates entry to equilibrium industry output and market price.

Theorem 8 *At the extremal equilibria*

(i) aggregate output satisfies $\overline{Z}_{n+1} \geq \overline{Z}_n$*; and*

(ii) $P_{n+1} \geq P_n$ *if* $\Delta_3(.) \geq 0$ *on* I_n*, and* $P_{n+1} \leq P_n$ *if* $\Delta_3(.) \leq 0$ *on* I_n*.*

That industry output increases with n is also true in standard Cournot competition [Amir and Lambson [4], Theorem 2.2(b)]. In the latter case this implies that market price decreases after new entry. As captured by Theorem 8(ii), the effect of entry on market price is ambiguous when network effects prevail. The reason is that when industry output increases firms must set the price low enough to attract the marginal consumer, but when more buyers join the network, consumers' willingness to pay increases. Thus the overall effect of entry on price depends on how strong the output effect

is relative to the network effect. As a consequence, the so-called property of quasi-competitiveness need not be hold here, in contrast to the standard Cournot game.

The next result deals with the effects of entry on per-firm outputs, and is important in terms of its implications on the performance of the industry (on profits and welfare). In what follows, interior equilibrium means $\overline{Z}_n, \overline{Z}_{n+1} < nK$.

Lemma 9 *At an interior equilibrium, per-firm outputs are such that*

(i) $\overline{x}_{n+1} \geq \overline{x}_n$ *if* $\Delta_4(.) \geq 0$ *on* I_n*; and*

(ii) $\overline{x}_{n+1} \leq \overline{x}_n$ *if* $\Delta_4(.) \leq 0$ *on* I_n*.*

In short, this result means that the scope for the business-stealing effect, which is nearly universal in standard Cournot oligopoly, is quite a bit narrower in the presence of network externalities. On the other hand, the scope for the opposite, or business-enhancing, effect is much broader here.

To shed light on this comment (and the meaning of $\Delta_4(.)$), assume no cost of production for simplicity, so that $\Delta_4(.) \geq 0$ reduces to

$$\left[P\left(Z,Z\right)P_{12}\left(Z,Z\right) - P_1\left(Z,Z\right)P_2\left(Z,Z\right)\right] + \left[P\left(Z,Z\right)P_{11}\left(Z,Z\right) - P_1^2\left(Z,Z\right)\right] \geq 0. \tag{7}$$

The first term is positive by $A5$, and log-convexity of $P\left(Z,S\right)$ in Z would make the second one positive as well. Thus, log-convexity is a sufficient (but not necessary) condition for the extremal selections of per-firm equilibrium output to increase after new entry whenever marginal costs are zero. [Amir and Lambson (2000), Theorem 2.3] requires log-convexity to guarantee the same result for standard Cournot competition. Thus, network effects facilitate this unusual outcome.

Based on Theorem 8 and Lemma 9, the following result deals with the effects of entry on per-firm equilibrium profits. Recall that in standard Cournot oligopoly, the only part of the conventional wisdom about the effects of competition that is universally valid is that per-firm profits decline with the number of competitors (Amir and Lambson [4] and Amir [3]). We now show that in the presence of network effects, this result can be easily reversed.

Theorem 10 *At an interior equilibrium, per-firm profits are such that*

(i) $\pi_{n+1} \geq \pi_n$ *if* $\Delta_3(.) \geq 0$ *and* $\Delta_4(.) \geq 0$ *on* I_n*; and*

(ii) $\pi_{n+1} \leq \pi_n$ *if* $\Delta_3(.) \leq 0$ *and* $\Delta_4(.) \leq 0$ *on* I_n.

The first result provides sufficient conditions for the firms in the market to prefer entry by new firms. It generalizes a result in Economides [13], based on a more specific formulation (also see Katz and Shapiro [19]). While at first surprising, this result has a simple intuition. As seen above, with strong network effects, the output increase in response to entry also shifts the inverse demand function up, thus pushing for a price increase. If the overall effect on the market price is positive and each firm increases own output, then the incumbent firms in the market are better-off with entry. So, again, strong network effects can overturn the usual competitive effect of entry.

A natural question arises when profits increase in n. Why can't the existing firms attempt to act as if there were more of them in order to each get higher profits at equilibrium? Since they would do so by producing a higher output level in an effort to influence consumers' expectations of the network size upwards, the answer is the same as for the start-up problem: The tacit lack of commitment power on the part of the firms, which is at the heart of the FECE concept.

These departures from standard Cournot competition reinforce the perception suggested by the viability results that FECE is a co-opetitive, rather than a purely non-cooperative, equilibrium concept (Brandenburg and Nalebuff [8]). Firms work together to build a common network base, and then compete with each other in serving it. Thus more firms can be helpful or detrimental to a firm, depending on the relative strengths of the network and the business-stealing effects.

Under conditions often imposed in the related literature, no second order effects on P, or $P_{11}(Z,Z) + P_{12}(Z,Z) = 0$, $\Delta_3(.) \geq 0$ ($\Delta_3(.) \leq 0$) suffices for per-firm output and profits to increase (decrease) with entry.[24]

This result identifies industry characteristics that make firms benefit from further entry by competitors. Both conditions in Theorem 10(i) can be interpreted in large part as saying that network effects must be strong enough. Recall that (7) is more likely to hold when $A5$ holds with sufficient slack, which means that price elasticity increases fast enough in the network size.

The next example highlights the implications of Theorem 10.

[24] $P_{11}(Z,Z) + P_{12}(Z,Z) = 0$ is satisfied if, for example, $P(Z,S) = h(S) - kZ$ with $h(.)$ an increasing function (as in Example 2), or $P(Z,S) = f(S-Z)$ with $f(.)$ increasing.

Example 2. Consider a Cournot oligopoly with no production costs and

$$P(Z,S) = \max\{a + bS^{\alpha} - Z, 0\} \text{ with } a \geq 0,\ b > 0 \text{ and } \alpha \in (0,1).$$

The reaction function of any given firm is $\widetilde{x}(y,S) = \max\{(a + bS^{\alpha} - y)/2, 0\}$. (Here we assume K is large enough.) From the first order condition, the symmetric equilibrium industry output is implicitly defined by $-Z_n(1+n) + na + nbZ_n^{\alpha} = 0$.

Setting $a = 10$, $b = 5$ and $\alpha = 4/5$, per-firm equilibrium profits for different values of n are

$$\begin{aligned} \pi_1 &\approx 14,561 < \pi_2 \approx 49,255 < \pi_3 \approx 67,316 < \pi_4 \approx 70,676 \\ \pi_5 &\approx 67,288 > \pi_6 \approx 61,520 > \pi_7 \approx 55,301 > \pi_8 \approx 49,404 > ... > \pi_{21} \approx 14,444. \end{aligned}$$

When the number of firms is small, $n = 1, 2$, or 3, incumbent firms will be better off if an extra firm enters the market. In particular, a monopolist would prefer to have a few competitors. However, when $n \geq 4$, firms would be worse-off upon further entry.

Consider a hypothetical situation where per-firm entry costs are $14,443$. Then a single firm in the market would barely make a positive profit, and potential entrants might decide to stay out if they based their assessment on standard oligopoly settings (due to profits just covering entry costs). Yet, the market could actually accommodate a full 21 firms at the unique free entry equilibrium! ‖

The last result describes the effects of entry on consumer surplus and social welfare (here, $A(.)$ denotes average cost, defined as usual by $A(x) = C(x)/x$ and $A(0) = C'(0)$).

Theorem 11 *At the highest equilibrium output*

(i) $CS_{n+1} \geq CS_n$ *if* $\Delta_3(.) \leq 0$ *on* I_n *or* $P_{12}(Z,S) \leq 0$ *for all* Z, S*; and*

(ii) $W_{n+1} \geq W_n$ *if* $P\left(Z, \overline{Z}_{n+1}\right) - P\left(Z, \overline{Z}_n\right) \geq A\left(\overline{x}_{n+1}\right) - A\left(\overline{x}_n\right)$ *for all* Z*, or* $\overline{x}_{n+1} \geq \overline{x}_n$.

In thinking about social and consumer welfare throughout, it is useful to keep in mind that since $P_2(Z,S) > 0$ and $\overline{Z}_{n+1} \geq \overline{Z}_n$ by Theorem 8(i), the inverse demand shifts up as the number of firms increases, i.e., $P\left(., \overline{Z}_{n+1}\right) \geq P\left(., \overline{Z}_n\right)$. Hence, the area under the inverse demand changes through two effects: The shift in the demand curve and the change in the equilibrium output.

As a consequence of the so-called property of quasi-competitiveness (price falls with the number of firms), which under similar conditions holds in the standard Cournot game, the first condition in Theorem 11(i) is always satisfied in the absence of network effects. The condition $P_{12} \leq 0$, which is consistent with $A5$, is always satisfied in the widely used cases of additively and multiplicatively separable inverse demand, so consumer surplus is well-behaved in much of the extant literature. In contrast, Example 3 (below) shows that consumer surplus can decrease with entry in network industries, even in a global sense. Katz and Shapiro [19] explain why this unusual effect might occur here: If the network externality is strong for the marginal consumer, then the increment in sales generated by the larger number of firms in the market, will increase willingness to pay for the product above that of the average consumer. As a consequence, the firms will be able to raise the market price by more than the increase in the average consumer's willingness to pay for the product and consumer surplus will fall.

The left hand side of the first condition in Theorem 11(ii) is always positive. So Theorem 11(ii) identifies two sufficient conditions for welfare to increase: Either one has decreasing or constant returns to scale ($A(.)$ is increasing) and decreasing per-firm output, or one has per-firm output increasing in n. Network effects play a key role in inducing these two conditions. First, they facilitate the demand shift and the increase in total output, which makes the first condition more likely to hold. As seen earlier, they also weaken the business-stealing effect, thereby easing the conditions under which per-firm output increases in n. Therefore, the effects of entry on welfare conform quite closely to standard intuition, and it would take a combination of strong economies of scale and weak network effects to reverse this result.

To recapitulate, while the scope for per-firm profits and consumer surplus to respond in a counter-intuitive way to entry is non-existent in standard Cournot (under the present assumptions), it is fairly broad under network effects. On the other hand, the usual result on social welfare is much harder to reverse for network industries.

Example 3 shows an interesting case in which both social welfare and industry profits increase with entry, but consumer surplus decreases, with all these effects holding globally.

Example 3. Consider an industry with zero costs and inverse demand function $P(Z,S) = \max\{a -$

$Z/S^3, 0\}$ with $a, K > 1$. The reaction function of a firm is then

$$x(y,S) = \begin{cases} \max\left\{\left(aS^3 - y\right)/2, 0\right\} & \text{if } \left(aS^3 - y\right)/2 < K \\ K & \text{if } \left(aS^3 - y\right)/2 \geq K \end{cases}.$$

Upon calculation, we find three FECE, with industry outputs: $Z_n = \left\{0, \sqrt{(n+1)/(na)}, nK\right\}$. We restrict consideration below to the highest equilibrium, $\overline{Z}_n = nK$.

From a simple computation, consumer surplus is $CS_n = 1/(2nK)$, assuming $a \geq 1/(nK)^2$. Hence consumer surplus globally decreases in n. This result is possible as the two sufficient conditions in Theorem 11(i) are violated, i.e., industry price $P_n = a - 1/n^2K^2$ increases in n and $P_{12}(Z,S) = 3/S^4 > 0$. Note that price globally increases here as network effects clearly dominate the effect of the law of demand, i.e., $P_1(Z,Z) + P_2(Z,Z) = 2/Z^3 > 0$.

Per-firm profit is $\pi_n = K[a - 1/(nK)^2]$, which globally increases in n! Despite consumer surplus and profit behaving in counter-intuitive fashion, corresponding social welfare $W_n = anK - 1/(2nK)$ globally increases in n, in line with intuition and Theorem 11(ii).

Hence, an unusual outcome prevails here, which is a full reversal with respect to conventional wisdom from non-network markets. The most preferred market structure is monopoly from the consumers' standpoint, and each firm would prefer to have as many rivals as possible in the industry! In addition, the interest of society is fully aligned with that of the firms, not consumers. ‖

6 Concluding Remarks

This paper has provided a thorough theoretical analysis of a static model of oligopolistic competition with non-additive network effects. A minimal complementarity structure on the model leads to industry output increasing in the rivals' output that a firm faces and in the expected network size, thus yielding in one broad stroke existence of symmetric equilibrium as well as some key characterization results with a comparative statics flavor. The so-called start up problem is extensively investigated, in terms of basic properties of the market primitives, and the strength of network effects. In particular, industry viability, a key concept in network markets for which we provide novel theoretical foundations, is shown to be enhanced by higher numbers of competitors in the market as well as by technological progress. The central feature here is a simple learning/adjustment dy-

namics that also serves as a theoretical foundation for the solution concept of fulfilled expectations Cournot equilibrium. We elaborate in some detail on the natural tendency for multiple equilibria, path dependence, and the importance of initial market expectations, features that emerge due to the presence of demand-side increasing returns driven by non-additive network effects.

As to the effects of market structure, sufficient conditions are derived for each dimension of market performance to increase or decrease with more competition. The tendency for counterintuitive effects, which is extensively characterized, is much stronger than in ordinary markets. Most notably, price and per-firm profits can both increase with the number of firms, with the latter effect having no counterpart in ordinary markets even under scale economies (Amir and Lambson [4]). Along with the need, often critical, for firms to join hands to successfully launch new network products, these results underscore the co-opetitive nature of the FECE concept: Firms are partners in setting expectations and building consumer base, but (business-stealing) competitors in serving that base.

Several instructive examples with closed-form solutions are constructed, one of which reflects exactly the prototypical three-equilibrium configuration that is broadly thought to capture the essence of the viability issue through expectation dynamics in telecommunications industries.

In terms of policy implications, by identifying precise and tight conditions for the various possible effects to hold, our results provide solid theoretical foundations for some well-known policy prescriptions that need revisiting for network markets (Shapiro [30]). The main departure from ordinary markets is the emergence of a start up problem, with potentially serious consequences for both firms and society. A successful launch of a new product with a small stand-alone value depends on various factors, including the usual ones, such as intrinsic quality, production costs, reputational aspects, and government participation. In addition, as the case studies reported in the business strategy literature confirm, interconnection amongst competitors or agreement on compatibility is quite often a critical determinant of success (Shapiro and Varian [31] and Rohlfs [29]). The results on viability provide a solid theoretical grounding to lend insight to the lessons on the start up problem gleaned from these case studies, in ways that apply to both successes and failures. One implication for product development is that, as a way out of the start up trap, firms ought to bundle multiple functions in network products, in order to ensure a sufficient stand alone value.

Our results on market performance also largely confirm Rohlfs [29]'s clear-cut conclusion that

interconnection is most often a win-win proposition for both firms and society.[25] When the effects of more competition can lead to multiple reversals of conventional intuition, the usual trade-offs between consumer surplus and producer surplus are no longer the norm, and many pillars of conventional wisdom about suitable public policy for such industries need re-examining. The presence of network effects might have unusual implications on firms' attitudes towards intellectual property rights and entry deterrence. Firms in possession of patents will have a much higher than usual incentive to engage in licensing to their rivals on rather generous terms (Shepard [32]; Shapiro [30]). Pooling of patents held by different firms is also to be expected. In terms of antitrust implications, various forms of pro-competitive cooperation amongst direct rivals should be allowed or even encouraged. This is particularly true concerning the often difficult and costly process of establishing a common standard needed for a new network industry to succeed.

In terms of public policy, government participation in network industry start ups can be crucial due to the major role it can play in terms of influencing market expectations (the S_0 variable) upwards. In addition, the interconnection process can raise such thorny and complex issues for the private actors that a positive coordinating role for government agencies often arises. Even initial subsidies might play a very constructive role. In one of his most instructive case studies, Rohlfs [29] reports that the unprecedented success of the ultimate network industry – the Internet – is largely due to the direct role played by the U.S. government via its DoD and NSF temporary subsidy programs, in terms of ensuring global interconnection.[26] Rohlfs' detailed account of this glorious episode of government intervention suggests that, without it, interconnection for the Internet – something usually taken for granted – could easily have failed or been substantially delayed.

The present analysis paves the way for further promising research in a number of interesting directions, including (i) the role of marketing in the start up problem, (ii) the scope for divisionalization in network industries in terms of both start up and profit incentives of firms, and (iii) the comparison with the case where firms possess commitment power in setting output levels.

[25] The one exception to this recommendation that he points out arises in industries wherein one firm has a substantial first-mover advantage, typically achieved by being substantially ahead of rivals in offering a new product.

[26] Interestingly, this real life regulation scenario fits the two-stage game with a market maker proposed in Section 2 as a purely game-theoretic foundation for FECE.

7 Proofs

This section provides the proofs for all the results of the paper, and also contains the statements and proofs of some useful intermediate results not given in the body of the paper.

The proof of Lemma 1 calls for an intermediate result.

Lemma 12 *$\widetilde{\pi}(Z, y, S)$ has the strict single-crossing property in $(Z; S)$.*

Proof of Lemma 12: First note that $\Delta_2(Z, y, S) > 0$ if and only if $\partial^2 \log P(Z, S) / \partial Z \partial S > 0$. We show that this condition implies that $\widetilde{\pi}(Z, y, S)$ has the strict single-crossing property in $(Z; S)$, i.e., that for any $Z > Z'$ and $S > S'$,

$$\widetilde{\pi}\left(Z, y, S'\right) \geq \widetilde{\pi}\left(Z', y, S'\right) \Longrightarrow \widetilde{\pi}(Z, y, S) > \widetilde{\pi}\left(Z', y, S\right). \tag{8}$$

Since $\partial^2 \log P(Z, S) / \partial Z \partial S > 0$, $\log P(Z, S) - \log P(Z', S) > \log P(Z, S') - \log P(Z', S')$, or

$$P(Z, S) / P\left(Z', S\right) > P\left(Z, S'\right) / P\left(Z', S'\right). \tag{9}$$

The left hand side of (8) can be rewritten as

$$(Z - y) P\left(Z, S'\right) - C(Z - y) \geq \left(Z' - y\right) P\left(Z', S'\right) - C\left(Z' - y\right). \tag{10}$$

Combining (9) and (10), we get

$$(Z - y) P(Z, S) P\left(Z', S'\right) / P\left(Z', S\right) - C(Z - y) > \left(Z' - y\right) P\left(Z', S'\right) - C\left(Z' - y\right). \tag{11}$$

Multiplying both sides of (11) by $P(Z', S) / P(Z', S')$ we obtain

$$(Z - y) P(Z, S) - \frac{P(Z', S)}{P(Z', S')} C(Z - y) > \left(Z' - y\right) P\left(Z', S\right) - \frac{P(Z', S)}{P(Z', S')} C\left(Z' - y\right). \tag{12}$$

By $A1$, $P(Z', S) / P(Z', S') > 1$ and, by $A2$, $C(Z - y) \geq C(Z' - y)$. Thus, (12) implies

$$(Z - y) P(Z, S) - C(Z - y) > \left(Z' - y\right) P\left(Z', S\right) - C\left(Z' - y\right), \tag{13}$$

which is just the right hand side of (8). Hence, (8) holds.□

Proof of Lemma 1: Since $\partial^2 \widetilde{\pi}(Z, y, S) / \partial Z \partial y = \Delta_1(Z, y, S) > 0$, by $A4$, the maximand in (2) has strictly increasing differences in (Z, y). Furthermore, the feasible correspondence $[y, y + K]$

is ascending in y.[27] Then, by Topkis's theorem (Topkis [35]), every selection from the argmax of $\widetilde{\pi}(Z, y, S)$, $Z(y, S)$, increases in y.

By Lemma 12, $\widetilde{\pi}(Z, y, S)$ has the strict single-crossing property in $(Z; S)$. In addition, the feasible correspondence $[y, y + K]$ does not depend on S. Then, by Milgrom and Shannon [26], every selection from the argmax of $\widetilde{\pi}(Z, y, S)$, $Z(y, S)$, is also increasing in S.□

Proof of Theorem 2: The proof proceeds in two steps. First, for each fixed S, consider the mapping B_S, a normalized cumulative best-response (Amir and Lambson [**?**])

$$\begin{array}{rcl} B_S : [0, (n-1)K] & \longrightarrow & [0, (n-1)K] \\ y & \longrightarrow & \dfrac{n-1}{n} Z' \end{array}$$

where $Z' = x' + y$ denotes a best-response output level by a firm to a joint output y by the other $(n-1)$ firms, given S. It is readily verified that the (set-valued) range of B_S is as given, i.e., if $Z' \in [y, y + K]$ and $y \in [0, (n-1)K]$, then $[(n-1)/n] Z' \in [0, (n-1)K]$, and that a fixed point of B_S is a symmetric Cournot equilibrium, and vice versa. By Lemma 1, every selection of $B_S(y)$ increases in y. By Tarski's fixed point theorem (Tarski [**?**]), B_S has a fixed point.

From [Amir and Lambson [**?**], Theorem 2.1], we know that no asymmetric equilibria exists.

The second step is to show that $Q_n(S)$, the set of Cournot equilibrium industry outputs when inverse demand is $P(\cdot, S)$, has fixed points. To this end, by Topkis's Theorem and $A5$, every selection from the argmax of $\widetilde{\pi}(Z, y, S)$, $Z(y, S)$ or $[n/(n-1)] B_S(y)$, is increasing in S. Hence, by [Milgrom and Roberts [25], Theorem 6], the extremal fixed points of $B_S(y)$, i.e., $\overline{y}_n(S)$ and $\underline{y}_n(S)$, are increasing in S. Since $\overline{Q}_n(S) = [n/(n-1)] B_S[(\overline{y}_n(S)]$ and $\underline{Q}_n(S) = [n/(n-1)] B_S[\underline{y}_n(S)]$, the extremal selections of the correspondence $Q_n : [0, nK] \longrightarrow [0, nK]$ are both increasing in S. Then, by Tarski's fixed point theorem, $\overline{Q}_n$, say, has a fixed point, which is easily seen to be a FECE.□

Proof of Lemma 3: By definition, an industry output of 0 is a FECE if $0 \in \widetilde{x}(0, 0)$. This holds if and only if $\pi(0, 0, 0) \geq \pi(x, 0, 0)\ \forall x \in [0, K]$, i.e., $0 \geq xP(x, 0) - C(x)$, $\forall x \in [0, K]$.□

The proof of Theorem 4 calls for several intermediate results. We first state a non-smooth version of the Implicit Function Theorem for increasing selections of $Q_n(\cdot)$. We then show that when 0 is part of the equilibrium set, $Q_n(S)$ is single-valued and right-differentiable at $S = 0$.

[27] Notice that with capacity constraints $Z(y, S) = \arg\max\{\widetilde{\pi}(Z, y, S) : y + K \geq Z \geq y\}$.

Lemma 13 *Let $\widetilde{Q}_n$ be an increasing selection of $Q_n(S)$. Then $\widetilde{Q}_n(S)$ is differentiable for almost all S, and, if $\widetilde{Q}_n(S) \in (0, nK)$ for $S > 0$, its slope is given by (here, $\widetilde{Q}_n$ stands for $\widetilde{Q}_n(S)$)*

$$\frac{\partial \widetilde{Q}_n(S)}{\partial S} = -\frac{nP_2(\widetilde{Q}_n, S) + \widetilde{Q}_n P_{12}(\widetilde{Q}_n, S)}{(n+1)\,P_1(\widetilde{Q}_n, S) + \widetilde{Q}_n P_{11}(\widetilde{Q}_n, S) - C''(\widetilde{Q}_n/n)}. \tag{14}$$

Proof of Lemma 13: If $\widetilde{Q}_n(S)$ is interior, it satisfies the first order condition (upon multiplying by n and writing $\widetilde{Q}_n$ for $\widetilde{Q}_n(S)$)

$$nP(\widetilde{Q}_n, S) + \widetilde{Q}_n P_1(\widetilde{Q}_n, S) - nC'(\widetilde{Q}_n/n) = 0. \tag{15}$$

Since $\widetilde{Q}_n(S)$ is increasing, it is differentiable almost everywhere (w.r.t. Lebesgue measure). Hence, differentiating both sides of (15) with respect to S on a subset of full Lebesgue measure and collecting terms, we get that for almost all S, (14) holds (since the derivatives on the RHS of (14) all exist from our smoothness assumptions).□

Proof of Lemma 5: We first show that if $0 \in Q_n(0)$, then $0 = Q_n(0)$, i.e., $Q_n(0)$ is a singleton. By Lemma 3 we know that $0 \in Q_n(0)$ if and only if

$$xP(x, 0) \leq C(x) \text{ for all } x \in [0, K]. \tag{16}$$

By *A*1, (16) implies that $xP(x+y, 0) < C(x)$ for all $x, y > 0$. In other words, 0 is a (strictly) dominant strategy in the standard Cournot game with $S = 0$. Hence $Q_n(0)$ is single-valued and $Q_n(0) = 0$.

It is convenient to divide the rest of the proof into two separate cases.

Case 1. $P(0, 0) = C'(0)$. Then the trivial outcome is an interior equilibrium. To show (5), take any sequence $S_k \downarrow 0$ such that $\widetilde{Q}_n$ is differentiable at S_k for all k (this is possible since the set of points of differentiability of an increasing function is a set of full Lebesgue measure, and thus a dense subset of the domain). In addition, since $\widetilde{Q}_n$ is increasing, it admits left and right limits at every point, so $\lim_{k\to\infty} \widetilde{Q}_n(S_k)$ exists. Treating S as a parameter and invoking the upper hemi-continuity (u.h.c) of the equilibrium correspondence for the Cournot game (see, e.g., Fudenberg and Tirole [16]), we conclude that $Q_n(.)$ is u.h.c.. Hence, $\lim_{k\to\infty} \widetilde{Q}_n(S_k) \in Q_n(0) = \{0\}$, so that by the earlier part of this proof, $\lim_{k\to\infty} \widetilde{Q}_n(S_k) = 0$.

Now consider (14) with $S = S_k$. By Assumption $A1$ and the fact that $\lim_{k\to\infty} \widetilde{Q}_n(S_k) = 0$, the right-hand side of (14) is right-continuous in S at 0. Taking limits as $k \longrightarrow \infty$, it follows that $\lim_{k\to\infty} \partial\widetilde{Q}_n(S_k)/\partial S$ exists and is equal to the RHS of (5). Since this argument is clearly independent of the sequence (S_k) chosen (out of the subset of full Lebesgue measure of the domain $[0, nK]$), we conclude that $\partial\widetilde{Q}_n(S)/\partial S\Big|_{S=0}$ exists, is continuous at 0, and given by (5).

Next observe that this entire argument for $\widetilde{Q}_n(\cdot)$ is also valid for the two selections $\overline{Q}_n(\cdot)$ and $\underline{Q}_n(\cdot)$, since these are both increasing. Hence, $\partial\overline{Q}_n(S)/\partial S\big|_{S=0} = \partial\underline{Q}_n(S)/\partial S\Big|_{S=0}$, with both being equal to the RHS of (5). A moment's reflexion will reveal that $\max\{\partial Q_n(S)/\partial S|_{S=0}\} = \partial\overline{Q}_n(S)/\partial S\big|_{S=0}$ and $\min\{\partial Q_n(S)/\partial S|_{S=0}\} = \partial\underline{Q}_n(S)/\partial S\Big|_{S=0}$. Hence $\max\{\partial Q_n(S)/\partial S|_{S=0}\} = \min\{\partial Q_n(S)/\partial S|_{S=0}\}$. It follows that, for the entire correspondence $Q_n(\cdot)$, $\partial Q_n(S)/\partial S|_{S=0}$ exists (in the sense of being single-valued), is continuous and given by the RHS of (5).

Case 2. $P(0,0) < C'(0)$. Then the trivial equilibrium is not interior. By $A1$, $P(0,S) < C'(0)$ for S sufficiently small, so $Q_n(S) = 0$ for all such S. It follows that $Q_n'(S) = 0$ for S small enough.□

We next show that any argmax of the fictitious objective $\Pi(Z,S)$ is an element of $Q_n(S)$.

Lemma 14 *Assume A1-A5 are satisfied and $C(.)$ is convex. Given any $n \in N$ and $S \in [0, nK]$, if $Z^* \in \arg\max\{\Pi(Z,S) : 0 \le Z \le nK\}$ then $Z^* \in Q_n(S)$.*

Proof of Lemma 14: We show that if Z^* is an argmax of $\Pi(Z,S)$, then Z^* is the industry output of a symmetric Cournot equilibrium with exogenous S. Let $Z^* = x^* + y^*$, with $x^* = Z^*/n$ and $y^* = (n-1)x^*$, and consider $Z' = x' + y^*$, with $x' \in [0, K]$. Then x' denotes a firm's possible deviation from its equilibrium output x^*. We show this unilateral deviation is never profitable.

Since Z^* is a maximizer of $\Pi(Z,S)$, then $\Pi(Z^*,S) \ge \Pi(Z',S)$, which is equivalent to say

$$\frac{n-1}{n}\int_0^{x^*+y^*} P(t,S)\,dt + x^*P(x^*+y^*,S) - nC(x^*) \ge$$
$$\frac{(n-1)}{n}\int_0^{x'+y^*} P(t,S)\,dt + \frac{(x'+y^*)}{n}P\left(x'+y^*,S\right) - nC\left(\frac{x'+y^*}{n}\right). \tag{17}$$

Then we have

$$\begin{aligned} x^{*}P\left(x^{*}+y^{*},S\right)-C\left(x^{*}\right) &\geq \frac{n-1}{n}\int_{0}^{x'+y^{*}}P\left(t,S\right)dt+\frac{\left(x'+y^{*}\right)}{n}P\left(x'+y^{*},S\right) \\ &\quad -nC\left(\frac{x'+y^{*}}{n}\right)-\frac{n-1}{n}\int_{0}^{x^{*}+y^{*}}P\left(t,S\right)dt+\left(n-1\right)C\left(x^{*}\right) \\ &\geq \frac{n-1}{n}\int_{x^{*}+y^{*}}^{x'+y^{*}}P\left(t,S\right)dt+\frac{\left(x'+y^{*}\right)}{n}P\left(x'+y^{*},S\right)-C\left(x'\right) \\ &\geq \frac{\left(n-1\right)\left(x'-x^{*}\right)}{n}P\left(x'+y^{*},S\right)+\frac{\left(x'+y^{*}\right)}{n}P\left(x'+y^{*},S\right)-C\left(x'\right) \\ &= x'P\left(x'+y^{*},S\right)-C\left(x'\right). \end{aligned}$$

The first inequality follows from (17), after rearranging terms. The second one holds as we assumed $C\left(.\right)$ is convex (and $y^{*}=\left(n-1\right)x^{*}$), and the last one by $A1$, $P_{1}\left(Z,S\right)<0$. Since x' is arbitrary, this argument shows that x^{*} is a symmetric Cournot equilibrium.□

Proof of Theorem 4: Part (i) holds because if the trivial outcome (zero output) is not part of the equilibrium set, then Theorem 2 guarantees there is a FECE with strictly positive industry output.

The proofs of Parts (ii) and (iii) both depend on the following argument. By the proof of Theorem 2, the maximal and minimal selections of $Q_{n}\left(S\right)$, $\overline{Q}_{n}\left(S\right)$ and $\underline{Q}_{n}\left(S\right)$, increase in S. Assume, for the moment, there exists an $S'\in\left(0,nK\right]$ such that $\overline{Q}_{n}\left(S'\right)\geq S'$. If we restrict attention to the values of S in $[S',nK]$, it follows that $\overline{Q}_{n}\left(S\right)\in[S',nK]$ because $\overline{Q}_{n}\left(.\right)$ is increasing and $\overline{Q}_{n}\left(S'\right)\geq S'$. Therefore, for all $S\in[S',nK]$, $\overline{Q}_{n}\left(S\right)$ is an increasing function that maps $[S',nK]$ into itself. Hence, by Tarski's fixed point theorem (Tarski [**?**]), there is an $S'\leq S''\leq nK$ such that $\overline{Q}_{n}\left(S''\right)=S''$. Since this condition implies $\overline{Q}_{n}\left(S''\right)$ is a strictly positive FECE, the existence of a nontrivial equilibrium reduces to showing there is at least one $S\in\left(0,nK\right]$ for which $\overline{Q}_{n}\left(S\right)\geq S$.

To prove Part (ii), we show $Q_{n}'\left(0\right)>1$. By Lemma 5, $Q_{n}'\left(0\right)>1$ if

$$P_{1}\left(0,0\right)+P_{2}\left(0,0\right)>\left[-P_{1}\left(0,0\right)+C''(0)\right]/n.$$

Then the existence of a nontrivial FECE follows by the argument in the previous paragraph, as Lemma 5 and the property $Q_{n}'\left(0\right)>1$, imply there exists a small $\varepsilon>0$ for which $\overline{Q}_{n}\left(\varepsilon\right)>\varepsilon$. This completes the proof of Part (ii).

The condition in Part (iii) guarantees there is some $S\in\left(0,nK\right]$ and some $Z'\geq S$ for which $\Pi\left(Z',S\right)\geq\Pi\left(Z,S\right)$ for all $Z\leq S$. As a consequence, the largest argmax of $\Pi\left(Z,S\right)$ must be larger

than S. Call this argmax Z''. Our proof follows because $Z'' \in Q_n(S)$, by Lemma 14, and this ensures there is an $S \in (0, nK]$ for which an element of $Q_n(S)$ is higher than S.□

Proof of Theorem 7: We will prove the result for a change in n, using the selection $\overline{Q}_n(S)$, or any other increasing selection. The proof for α, being almost identical, is omitted.

(i) The fact that $\overline{Q}_n(\cdot)$ shifts up as n increases follows from [Amir and Lambson [4], Theorem 2.2(b)], which shows that the largest Cournot equilibrium output increases in n. The proof here consists of applying this result at every exogenously given S.

(ii) Let $n' > n$, and CM' and CM denote the critical masses corresponding to $\overline{Q}_{n'}(S)$ and $\overline{Q}_n(S)$, respectively. Pick any $S_0 > CM$. By definition of CM, we know that the orbit $S_k = \overline{Q}_n(S_{k-1})$ starting from the given S_0 is a bounded monotonic sequence. Hence, by the Monotone Convergence Theorem, there is some $S_\infty > 0$ such that $\{S_k\} \uparrow S_\infty$, with S_∞ being a FECE industry output of the n-firm problem. From the same $S_0 > CM$, the orbit $S'_k = \overline{Q}_{n'}(S'_{k-1})$ is also bounded and monotonic, so there is some S'_∞ such that $\{S'_k\} \uparrow S'_\infty$, with S'_∞ being a FECE industry output of the n'-firm problem. Since $\overline{Q}_{n'}(\cdot) \geq \overline{Q}_n(\cdot)$, we have $S'_\infty \geq S_\infty > 0$. To recapitulate, we have shown that

$$\text{for any } S_0 > CM, \text{ we have } \{S'_k\} \uparrow S'_\infty > 0. \tag{18}$$

Since CM' is by definition the smallest initial expectation satisfying (18), then $CM' \leq CM$.□

Proof of Theorem 8: The mapping $B_S(.)$, defined in the proof of Theorem 2, increases in n. Hence, by [Milgrom and Roberts [25], Theorem 6], the extremal fixed points of $B_S(y)$, i.e., $\overline{y}_n(S)$ and $\underline{y}_n(S)$, are increasing in n, for each given S.

By Lemma 12, every selection from the argmax of $\widetilde{\pi}(Z, y, S)$, $Z(y, S)$ or $[n/(n-1)]\, B_S(y)$, increases in y. Then the extremal selections of the correspondence $Q_n : [0, nK] \longrightarrow [0, nK]$, $\overline{Q}_n(S) = [n/(n-1)]\, B_S[\overline{y}_n(S)]$ and $\underline{Q}_n(S) = [n/(n-1)]\, B_S[\underline{y}_n(S)]$, are both increasing in n. Hence, again by [Milgrom and Roberts [25], Theorem 6], the extremal fixed points of Q_n, $\overline{Z}_n$ and $\underline{Z}_n$, increase in n. This shows Part (i).

Part (ii) follows directly from the previous claim since $dP(Z, Z)/dZ = \Delta_3(Z)$.□

Proof of Theorem 9: At any interior equilibrium x_n must satisfy the first order condition

$$P(Z_n, Z_n) + x_n P_1(Z_n, Z_n) - C'(x_n) = 0. \tag{19}$$

Differentiating (19) with respect to n and rearranging terms we get

$$\frac{dx_n}{dn} = \frac{\{P_1(Z_n, Z_n) + P_2(Z_n, Z_n) + x_n[P_{11}(Z_n, Z_n) + P_{12}(Z_n, Z_n)]\}}{-P_1(Z_n, Z_n) + C''(x_n)} \frac{dZ_n}{dn}. \tag{20}$$

Substituting in (20) x_n by $[C'(Z_n/n) - P(Z_n, Z_n)]/P_1(Z_n, Z_n)$, and rearranging terms, we get

$$\frac{dx_n}{dn} = \frac{-1}{P_1(Z_n, Z_n)} \frac{\Delta_4(Z_n)}{-P_1(Z_n, Z_n) + C''(x_n)} \frac{dZ_n}{dn}. \tag{21}$$

It follows from $A1, A4$ and Theorem 8(i) that $\frac{dx_n}{dn}$ has the same sign as $\Delta_4(Z)$ on I_n.□

Proof of Theorem 10: Consider the following inequalities

$$\begin{aligned}
\pi_{n+1} &= \overline{x}_{n+1} P\left(\overline{x}_{n+1} + \overline{y}_{n+1}, \overline{Z}_{n+1}\right) - C\left(\overline{x}_{n+1}\right) \\
&\geq \overline{x}_n P\left(\overline{x}_n + \overline{y}_{n+1}, \overline{Z}_{n+1}\right) - C\left(\overline{x}_n\right) \\
&\geq \overline{x}_n P\left(\overline{x}_{n+1} + \overline{y}_{n+1}, \overline{Z}_{n+1}\right) - C\left(\overline{x}_n\right) \\
&\geq \overline{x}_n P\left(\overline{x}_n + \overline{y}_n, \overline{Z}_n\right) - C\left(\overline{x}_n\right) = \pi_n.
\end{aligned}$$

The first inequality follows by the Cournot equilibrium property. The second one is from $\overline{x}_{n+1} \geq \overline{x}_n$ (see Lemma 9) and $A1$. The third inequality holds as our assumptions imply $P\left(\overline{Z}_{n+1}, \overline{Z}_{n+1}\right) \geq P\left(\overline{Z}_n, \overline{Z}_n\right)$. Therefore, $\pi_{n+1} \geq \pi_n$. This shows Part (i). We omit the proof of Part (ii) as it is almost identical to the last one.□

Proof of Theorem 11: The first claim in Part (i) follows directly from Theorem 8.

The following steps prove the sufficiency of the second condition

$$\begin{aligned}
CS_{n+1} - CS_n &= \int_0^{\overline{Z}_{n+1}} \left[P\left(t, \overline{Z}_{n+1}\right) - P\left(\overline{Z}_{n+1}, \overline{Z}_{n+1}\right)\right] dt - \int_0^{\overline{Z}_n} \left[P\left(t, \overline{Z}_n\right) - P\left(\overline{Z}_n, \overline{Z}_n\right)\right] dt \\
&\geq \int_0^{\overline{Z}_n} \left[P\left(t, \overline{Z}_{n+1}\right) - P\left(\overline{Z}_{n+1}, \overline{Z}_{n+1}\right)\right] dt - \int_0^{\overline{Z}_n} \left[P\left(t, \overline{Z}_n\right) - P\left(\overline{Z}_n, \overline{Z}_n\right)\right] dt \\
&= \overline{Z}_n \left[P\left(\overline{Z}_n, \overline{Z}_n\right) - P\left(\overline{Z}_{n+1}, \overline{Z}_n\right)\right] \\
&\quad - \int_0^{\overline{Z}_n} \left\{\left[P\left(\overline{Z}_{n+1}, \overline{Z}_{n+1}\right) - P\left(\overline{Z}_{n+1}, \overline{Z}_n\right)\right] - \left[P\left(t, \overline{Z}_{n+1}\right) - P\left(t, \overline{Z}_n\right)\right]\right\} dt \\
&\geq \overline{Z}_n \left[P\left(\overline{Z}_n, \overline{Z}_n\right) - P\left(\overline{Z}_{n+1}, \overline{Z}_n\right)\right] \geq 0.
\end{aligned}$$

The first inequality follows directly from $P_1(Z, S) < 0$ and Theorem 8(i). The next step is obtained from the previous one by adding and subtracting $\int_0^{\overline{Z}_n} P\left(\overline{Z}_{n+1}, \overline{Z}_n\right) dt$, and rearranging terms. To

justify the second inequality notice that $P_{12}(Z,S) \leq 0$ is sufficient for

$$\int_0^{\overline{Z}_n} \left[P\left(t,\overline{Z}_{n+1}\right) - P\left(t,\overline{Z}_n\right)\right] dt \geq \int_0^{\overline{Z}_n} \left[P\left(\overline{Z}_{n+1},\overline{Z}_{n+1}\right) - P\left(\overline{Z}_{n+1},\overline{Z}_n\right)\right] dt.$$

Our last step is true since $P_1(Z,S) < 0$.

Hence, $P_{12}(Z,S) \leq 0 \ \forall Z,S \in [0,nK]$ is sufficient for $CS_{n+1} - CS_n \geq 0$, or $CS_{n+1} \geq CS_n$.

To prove the first claim of Part (ii) consider

$$\begin{aligned} W_{n+1} - W_n &= \int_0^{\overline{Z}_{n+1}} P\left(t,\overline{Z}_{n+1}\right) dt - \overline{Z}_{n+1} A\left(\overline{x}_{n+1}\right) - \left[\int_0^{\overline{Z}_n} P\left(t,\overline{Z}_n\right) dt - \overline{Z}_n A\left(\overline{x}_n\right)\right] \\ &\geq \int_0^{\overline{Z}_n} P\left(t,\overline{Z}_{n+1}\right) dt - \overline{Z}_n A\left(\overline{x}_{n+1}\right) - \left[\int_0^{\overline{Z}_n} P\left(t,\overline{Z}_n\right) dt - \overline{Z}_n A\left(\overline{x}_n\right)\right] \geq 0. \end{aligned}$$

The first inequality follows because $P\left(t,\overline{Z}_{n+1}\right) - A\left(\overline{x}_{n+1}\right) \geq 0$ for all $t \leq \overline{Z}_{n+1}$, and $\overline{Z}_{n+1} \geq \overline{Z}_n$ by Theorem 8(i). The second inequality holds by the assumed conditions.

To show the sufficiency of the second condition let us define $V_n(x,S) = \int_0^{nx} P(t,S)\, dt - nC(x)$. Notice $V_n(x,S)$ is concave in x since $n\left[nP_1(nx,S) - C''(x)\right] < 0$ by both $A1$ and $A4$. In addition,

$$\begin{aligned} \int_0^{\overline{Z}_{n+1}} P\left(t,\overline{Z}_{n+1}\right) dt &= \int_0^{n\overline{x}_{n+1}} P\left(t,\overline{Z}_{n+1}\right) dt + \int_{n\overline{x}_{n+1}}^{\overline{Z}_{n+1}} P\left(t,\overline{Z}_{n+1}\right) dt \\ &\geq \int_0^{n\overline{x}_{n+1}} P\left(t,\overline{Z}_{n+1}\right) dt + \overline{x}_{n+1} P\left(\overline{Z}_{n+1},\overline{Z}_{n+1}\right) \end{aligned} \tag{22}$$

where the inequality follows by $A1$. Next, consider the following steps

$$\begin{aligned} W_{n+1} - W_n &= \int_0^{(n+1)\overline{x}_{n+1}} P\left(t,\overline{Z}_{n+1}\right) dt - (n+1) C\left(\overline{x}_{n+1}\right) - \left[\int_0^{n\overline{x}_n} P\left(t,\overline{Z}_n\right) dt - nC\left(\overline{x}_n\right)\right] \\ &\geq \pi_{n+1} + \int_0^{n\overline{x}_{n+1}} P\left(t,\overline{Z}_{n+1}\right) dt - nC\left(\overline{x}_{n+1}\right) - \left[\int_0^{n\overline{x}_n} P\left(t,\overline{Z}_{n+1}\right) dt - nC\left(\overline{x}_n\right)\right] \\ &= \pi_{n+1} + V_n\left(\overline{x}_{n+1},\overline{Z}_{n+1}\right) - V_n\left(\overline{x}_n,\overline{Z}_{n+1}\right) \\ &\geq \pi_{n+1} + \left[\partial V_n\left(\overline{x}_{n+1},\overline{Z}_{n+1}\right)/\partial x\right]\left(\overline{x}_{n+1} - \overline{x}_n\right) \\ &= \pi_{n+1} + n\left[P\left(n\overline{x}_{n+1},\overline{Z}_{n+1}\right) - C'\left(\overline{x}_{n+1}\right)\right]\left(\overline{x}_{n+1} - \overline{x}_n\right) \\ &\geq \pi_{n+1} + n\left[P\left((n+1)\,\overline{x}_{n+1},\overline{Z}_{n+1}\right) - C'\left(\overline{x}_{n+1}\right)\right]\left(\overline{x}_{n+1} - \overline{x}_n\right) \geq 0. \end{aligned}$$

The first inequality follows from inequality (22), $A1$ and Theorem 8(i), and the second one by the concavity of $V_n(x,S)$ in x. The third inequality holds by $A1$ and because we assumed $\overline{x}_{n+1} \geq \overline{x}_n$, and the last one by the Cournot property.□

References

[1] R. Amir, Cournot oligopoly and the theory of supermodular games, Games Econ. Behav. 15 (1996a), 132-148.

[2] R. Amir, Sensitivity analysis of multisector optimal economic dynamics, J. Math. Econ. 25 (1996b), 123-141.

[3] R. Amir, Market structure, scale economies and industry performance, CORE 2003/65, 2003.

[4] R. Amir, V. Lambson, On the effects of entry in Cournot markets, Rev. Econ. Stud. 67 (2000), 235-254.

[5] B. Arthur, Increasing Returns and Path Dependence in the Economy, University of Michigan Press, Ann Arbor, 1994.

[6] B. Bensaid, J.-P. Lesne, Dynamic monopoly pricing with network externalities, Int. J. Ind. Organ. 14 (1996), 837-855.

[7] T. Bergstrom, H. Varian, Two remarks on Cournot equilibria, Econ. Letters 19 (1985), 5-8.

[8] A. Brandenburg, B. Nalebuff, Co-opetition, New York: Doubleday, 1996.

[9] J. Boone, A new way to measure competition, Econ. J. 118 (2008), 1245-1261.

[10] J. Chen, U. Doraszelski, J. Harrington, Avoiding market dominance: product compatibility in markets with network effects, RAND J. Econ. 40 (2009), 455-485.

[11] A. Dhebar, S. Oren, Optimal dynamic pricing for expanding networks, Management Science 4 (1985), 336-351.

[12] F. Echenique, Comparative statics by adaptive dynamics and the correspondence principle, Econometrica 70 (2002), 833-844.

[13] N. Economides, Network externalities, complementarities, and invitations to enter, Europ. J. Polit. Economy 12 (1996), 211-233.

[14] N. Economides, C. Himmelberg, Critical mass and network evolution in telecommunications, Toward a Competitive Telecommunication Industry, G. W. Brock (ed.), 1995, 47-66.

[15] A. Edlin, C. Shannon, Strict monotonicity in comparative statics, J. Econ. Theory 81 (1998), 201-219.

[16] D. Fudenberg, J. Tirole, Game Theory, MIT Press, 1991.

[17] A. Granas, J. Dugundji, Fixed Point Theory, Springer, New York, 2003.

[18] S. Hoernig, Existence of equilibrium and comparative statics in differentiated goods Cournot oligopolies, Int. J. Ind. Organ. 21 (2003), 989-1019.

[19] M. Katz, C. Shapiro, Network externalities, competition and compatibility, Amer. Econ. Rev. 75 (1985), 424-440.

[20] M. Katz, C. Shapiro, System competition and network effects, J. Econ. Perspect. 8 (1994), 93-115.

[21] N. Kwon, Characterization of Cournot equilibria in a market with network effects, Manchester Sch. 75 (2007), 151-159.

[22] H. Leibenstein, Bandwagon, snob, and Veblen effects in the theory of consumers' demand, Quart. J. Econ. 64 (1950), 183-207.

[23] J. Liebowitz, S. Margolis, Network externality: an uncommon tragedy, J. Econ. Perspect. 8 (1994), 133-150.

[24] C. Matutes, X. Vives, Competition for deposits, fragility, and insurance, J. Finan. Intermediation 5 (1996), 184-216.

[25] P. Milgrom, J. Roberts, Rationalizability, learning, and equilibrium in games with strategic complementarities, Econometrica 58 (1990), 1255-1278.

[26] P. Milgrom, C. Shannon, Monotone comparative statics, Econometrica 62 (1994), 157-180.

[27] S. Oren, S. Smith, Critical mass and tariff structure in electronic communications markets, Bell J. Econ. 12 (1981), 467-487.

[28] J. Rohlfs, A theory of interdependent demand for a communications service, Bell J. Econ. and Management Science 5 (1974), 16-37.

[29] J. Rohlfs, Bandwagon Effects in High-Technology Industries, MIT Press, Cambridge, 2001.

[30] C. Shapiro, Antitrust in network industries, Department of Justice, http://www.usdoj.gov/atr/public/speeches/0593.htm#N_9_, 1996.

[31] C. Shapiro, H. Varian, Information Rules: A Strategic Guide to the Network Economy, Harvard Business School Press, Boston, 1998.

[32] A. Shepard, Licensing to enhance demand for new technologies, RAND J. Econ. 18 (1987), 360-368.

[33] O. Shy, The Economics of Network Industries, Cambridge University Press, 2001.

[34] M. Spence, Product selection, fixed costs, and monopolistic competition, Rev. Econ. Stud. 43 (1976), 217-235.

[35] D. Topkis, Minimizing a submodular function on a lattice, Operations Research 26 (1978), 305-321.

[36] D. Topkis, Supermodularity and Complementarity, Princeton University Press, 1998.

[37] T. Veblen, The Theory of the Leisure Class: An Economic Study of Institutions, London: Macmillan, 1899.

[38] X. Vives, Nash equilibrium with strategic complementarities, J. Math. Econ. 19 (1990), 305-321.

[39] X. Vives, Oligopoly Pricing: Old Ideas and New Tools, The MIT Press, Cambridge, MA, 1999.